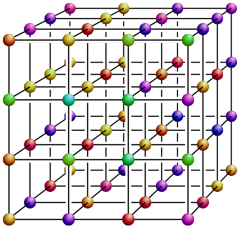
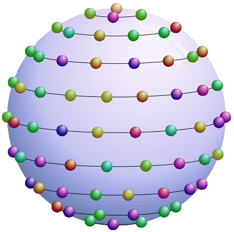
EmQM15
Emergent Quantum Mechanics
OCTOBER 23rd–25th 2015 VIENNA

Physics Utrecht
EMMEΦ

How quantization of gravity leads to a discrete space-time



Gerard ’t Hooft

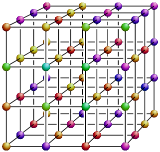


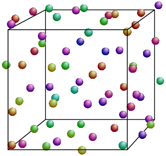
October 25, 2015

The Planck Units:

$$\begin{aligned} h/2\pi &= \hbar = 1.0546 \times 10^{-34} && \mathrm{kg\ m^2\ sec^{-1}} \\ G_N &= 6.674 \times 10^{-11} && \mathrm{m^3\ kg^{-1}\ sec^{-2}} \\ c &= 2.99792458 \times 10^{-8} && \mathrm{m/sec} \end{aligned}$$

$$\begin{aligned} L_{\mathrm{Planck}} &= \sqrt{\frac{\hbar G_N}{c^3}} = 1.616 \times 10^{-33} && \mathrm{cm} \\ M_{\mathrm{Planck}} &= \sqrt{\frac{\hbar c}{G_N}} = 21.76 && \mu\,\mathrm{g} \\ E_{\mathrm{Planck}} &= M_{\mathrm{Planck}} c^2 = 1.221 \times 10^{28} && \mathrm{eV} \end{aligned}$$





The idea that quantum mechanics might find its roots in a classical theory – "hidden variables" – is old and well-known.

Also the difficulties with representing entangled states and accounting for the violation of Bell's inequalities are well known.

Several people (L.Vervoort 2013, G. 't H 2014) pointed out that assumptions made by Bell included the assumption that spacelike correlations are insignificant, whereas such an assumption does not obviously hold true (superdeterminism, conspiracy)

One then notices that hidden variables do include some very satisfactory features:

- Very natural description and explanation of "wave function collapse" and a natural resolution to the "measurement problem"
- Very natural explanation of Born's probability interpretation of $|\psi|^2$
- Exact validity of QM mathematics over the full range between sub-microscopic and classical scales

Superdeterminism comes about by imposing a conservation law:

Classical states (*both* the classical configurations at the sub-microscopic scale, *and* the classical configurations at large scales) are represented by states in Hilbert space that from an *orthonormal basis*. These are the ontic states.

One constructs a Hamiltonian H such that their evolution is faithfully reproduced by the Schrödinger equ.

Then the *template states* are defined to be quantum superpositions of ontic states, obeying the same Schrödinger equ.

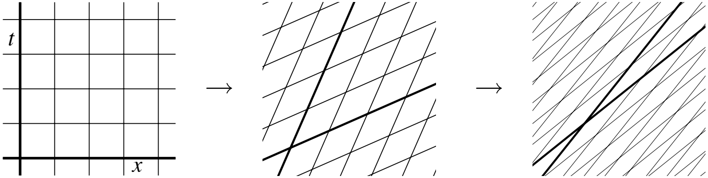
Conservation law: *Ontic states evolve into ontic states,*
templates evolve into templates

But today, we do not know which states are ontic. While Alice and Bob can <u>*only*</u> produce ontic states.

But even though there seems to be no fundamental obstacle, constructing a deterministic model that generates a local quantum field theory is still beyond our abilities.

If Nature indeed is an information processing machine, we need to understand that information (bits & bytes) is discrete. No continuum can exist. This is hard to implement in a QFT.

Strong Lorentz boosts:



F. Dowker: the only Lorentz invariant distribution of points in space-time is the *random* distribution (random sprinkling)

However, we can add to that:

The "sprinkling" does not have to be *entirely* random; the points may be correlated, by Lorentz invariant correlation functions. For instance: let $\phi(x)$ be the distribution function of points x_n,

$$\phi(x) = \sum_n \delta^4(x - x_n)\ , \qquad \langle\phi(x)\rangle = \varrho\ ,$$

$$\langle\phi(x)\,\phi(y)\rangle \;=\; F(x-y) \;=\; F\big((x-y)^2\big) \;\; > \;0$$

At large time differences, $(x-y)^2$ may still be small (nearly light-like separations); the correlations will then still be large, but they will be very non-local ($|\vec{x}-\vec{y}|$ is then large), and thus unnoticeable.

Proposal: the physics is in these correlation functions.

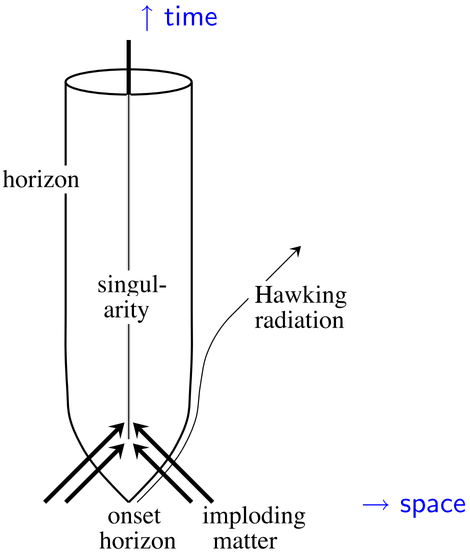
The *density* $\varrho(x)$ of points is fixed. It is tempting to suggest that $\varrho(x)$ is to be identified with the overal factor of the metric tensor:

$$g_{\mu\nu}(x) \equiv g(x)^{1/4}\,\hat{g}_{\mu\nu}(x)\ , \qquad g = \det(g_{\mu\nu}) = \varrho^8(x)$$

This means that time and distance are determined by the dynamics (spontaneously broken local scale invariance),
but the *light cones* are fundamental.

Conformal gravity

Light rays in black hole physics: much more essential than
distance- or *time*- scales.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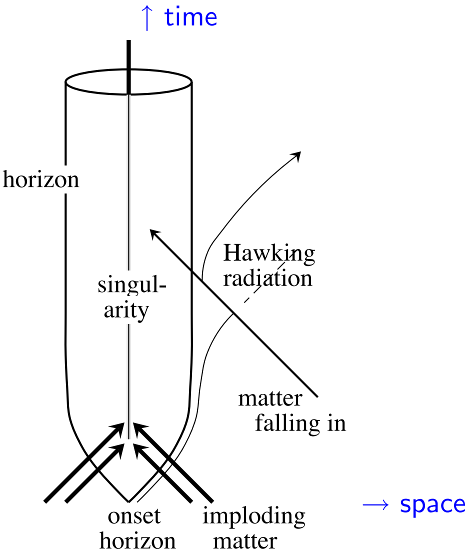
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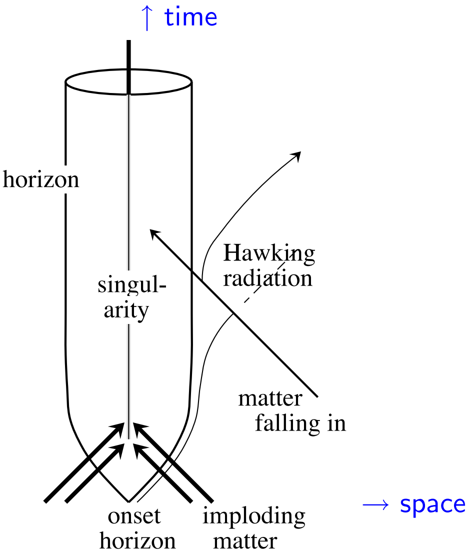

Shapiro shift

Matter going in will gravitationally deform its surrounding space-time.

This effect can be calculated precisely (standard gravity):

Let $p^{\mu}_{\text{in}}(\theta,\varphi)$ be the momentum distribution of the in-particles, and

$\delta x^{\mu}_{\text{out}}(\theta,\varphi)$ the displacement of the out-partcles, then

$$\delta x^{\mu}_{\mathrm{out}}(\theta,\varphi) = 8\pi G \int \mathrm{d}^2\Omega'\, f(\theta,\varphi,\theta'\varphi')\, p^{\mu}_{\mathrm{in}}(\theta',\varphi')\ .$$

$f(\Omega,\Omega')$ is Green function: $\Delta f(\Omega,\Omega') = -\delta^2(\Omega,\Omega')$.

Hawking radiation has a complicated (quantum) structure. Whatever the actual distribution of these particles, the entire configuration is *displaced* by an angle-dep. function $\delta x^{\mu}(\theta,\varphi)$.

This is how information is imparted by in-particles onto out-particles.

This leads to an S-matrix. But it implies that all particles are geometrical (*like in string theory*), and it *only depends on the light-geodesics* (so that the overall factor of the metric tensor $g_{\mu\nu}(x)$ does not enter in the calculations

(all particles were treated as massless).

Calculation of the black hole entropy

Expand both the momentum distribution $p^\mu_{\rm in}(\theta,\varphi)$ and the displacements $\delta x^\mu_{\rm out}(\theta,\varphi)$ in

partial waves (transverse Fourier transform)

(for simplicity, $\Omega=(\theta,\varphi)\ \to\ (x,y)\equiv\tilde x\ ,\quad (\ell,m)\to r\tilde k$):

$$p^\mu_{\rm in}(\tilde x)=\frac{1}{2\pi}\int\mathrm{d}^2\tilde k\,\hat p^\mu_{\rm in}(\tilde k)\,e^{i\tilde k\cdot\tilde x}\ ,\quad \delta x^\mu_{\rm out}(\tilde x)=\frac{1}{2\pi}\int\mathrm{d}^2\tilde k\,\hat x^\mu_{\rm out}(\tilde k)\,e^{i\tilde k\cdot\tilde x}$$

$$\tilde k^2\,\hat x^-_{\rm out}(\tilde k)=8\pi G\,\hat p^-_{\rm in}(\tilde k)\ ,\quad \tilde k^2\,\hat x^+_{\rm in}(\tilde k)=-8\pi G\,\hat p^+_{\rm out}(\tilde k)$$

$$[\hat x^-_{\rm out}(\tilde k),\,\hat p^+_{\rm out}(\tilde k')]=[\hat x^+_{\rm in}(\tilde k),\,\hat p^-_{\rm in}(\tilde k')]=i\delta^2(\tilde k-\tilde k')$$

Tortoise coordinates[1] at given $\tilde{k}$:

$$x^+_{\rm in} \equiv \pm e^{\,\varrho} \ , \quad p^-_{\rm in} \equiv \pm e^{\,\omega}$$

Introduce wave functions $\psi(\pm,\varrho)$, and relate the operators ϱ to the operators ω by Fourier transforming ψ, and using the commutation rules between x^μ and p^μ. In a short-hand notation:

$$\psi(\pm,\varrho) \ = \frac{1}{\sqrt{2\pi}} \sum_{\pm} \int_{-\infty}^{\infty} \mathrm{d}u \, A(\pm,u) \hat{\psi}(\pm, u-\varrho) \ ;$$

$$A(\pm,u) = e^{\,\frac{1}{2}u \,\pm\, i\,e^u}$$

And now, Fourier transform in the Tortoise coordinate:

[1] The interpretation of the signs $\pm$ is still enigmatic.

$$\tilde{\psi}^{\pm}_{\rm out}(\kappa) = e^{-i\kappa \log \frac{8\pi G}{\tilde{k}^2}} \Big(\tilde{A}^{+}(\kappa)\tilde{\psi}^{\pm}_{\rm in}(-\kappa) + \tilde{A}^{-}(\kappa)\tilde{\psi}^{\mp}_{\rm in}(-\kappa) \Big)$$

$$A^{+}(\kappa) = \tfrac{1}{\sqrt{\pi}}\Gamma(\tfrac{1}{2} - i\kappa)(\cosh\frac{\pi\kappa}{2} - i\sinh\frac{\pi\kappa}{2})$$

$$A^{-}(\kappa) = \tfrac{1}{\sqrt{\pi}}\Gamma(\tfrac{1}{2} - i\kappa)(\sinh\frac{\pi\kappa}{2} - i\cosh\frac{\pi\kappa}{2})$$

Thus, in-going waves bounce against a kind of "brick wall" to become out-going waves. Take inverse Hawking temp. $\beta \to 2\pi$ At given $\tilde{k}$:

$$e^{-\beta F(\tilde{k})} = \tfrac{1}{\pi\beta}(2\log\Lambda + \log\beta + \gamma + \log(\tilde{k}^2/\,8\pi G)))$$

$$U = \tfrac{\partial}{\partial\beta}(\beta F)\ , \quad S = \beta(U - F)\ , \quad \beta \to 2\pi$$

Λ is size of box around black hole.

$\int \mathrm{d}^2\tilde{k} S(\tilde{k})$ diverges ($S(\tilde{k})$ only depends on $\log\log(\tilde{k}^2)$)

To reproduce Hawking's entropy, a cut-off is needed at $|\tilde{k}|$ somewhere near the Planck energy:

$$|\tilde{k}^2| \le C^2 M_{\mathrm{Planck}}^2 \ , \quad \ell \le C M_{\mathrm{Planck}}\, r$$

This is a Brillouin zone – a perfect circle due to rotational symmetry. Only a random distribution of points in 2-space can have a circular boundary in $\tilde{k}$ space. *Transverse coordinates* (coord. of bh. horizon) *must be discrete*, and random.

G. Dvali: the *total* number of Hawking particles emitted by black hole: $\approx (M_{BH}/M_{\mathrm{Planck}})^2$, that is, one per unit L_{Planck}^2 of the black hole horizon. No lattice finer that the Planck scale will ever be necessary!

Conclusion:

To reproduce Hawking's entropy on the horizon of a black hole, a random lattice is needed on the horizon, at the size scale of the Planck length.

In the *longitudinal* direction ((r, t)-direction), no cut-off is needed, but a cut-off would not do harm there.

arXiv: 1204.4926
arXiv: 1205.4107
arXiv: 1207.3612
arXiv: 1405.1548 v2 (version v3, totally revised, coming soon?)
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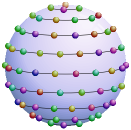
THE END



CAI

The Cellular Automaton Interpretation of Quantum Mechanics

If the Hamiltonian of the world happens to be that of an automaton, we can identify observables called *Beables*.

beables $\mathcal{B}_i(t)$ are ordinary quantum operators that happen to obey $[\mathcal{B}_i(t), \mathcal{B}_j(t')] = 0$.

The eigenstates of $\mathcal{B}_i(t)$ at a given time t form a *basis*, called the *ontological* (ontic) *basis.*

In a given quantum theory, it's not known how to construct an ontic basis.

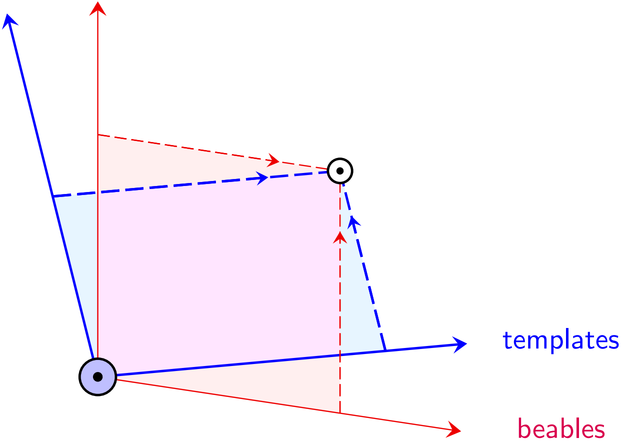
But one can come very close . . .

The CAI *assumes* that it exists. Its ontic states can be constructed from the ordinary quantum states.

If the beables can be constructed *more or less locally* from the known states, then we have a classical, "hidden variable theory".

The use of Templates

Hydrogen atom, plane waves of in- or out-particles, *etc.*



The states we normally use to do quantum mechanics are called *template states*. They form a basis of the kind normally used. This is a unitary transformation. Templates are quantum superpositions of ontic states and *vice versa*.

They all obey Schrödinger's equation!

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In a quantum calculation, we may assume the intial state to be an ontic state, $|\psi\rangle_{\rm ont}$. This state will be some *superposition of template states* $|k\rangle_{\rm template}$:

$$|\psi\rangle_{\rm ont} = \sum_k \alpha_k |k\rangle_{\rm template} \tag{1}$$

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In practice, we use some given template state of our choice. It will be related to the ontic states by

$$|k\rangle_{\rm template} = \sum_n \lambda_n |n\rangle_{\rm ont} \ , \tag{2}$$

where

$|\lambda_n|^2$ are the probabilities that we actually have ontic state $|n\rangle_{\rm ont}$.

Classical states

How are the *classical* states related to the *ontic* states?

Imagine a *planet*. The interior is very different from the local vacuum state. Vacuum state has *vacuum fluctuations*.

Take 1 mm^3 of matter inside the planet. Using statistics, looking at the ontic states, we may establish, with some probability, $P(\delta V) = \varepsilon > 0$, that the fluctuations are different from vacuum.

Combining the statistics of billions of small regions inside the planet, we can establish *with certainty* that there is a planet, by looking at the ontic state: $1 - P(V) = (1 - \varepsilon)^{V/\delta V} = e^{-\varepsilon V/\delta V}$

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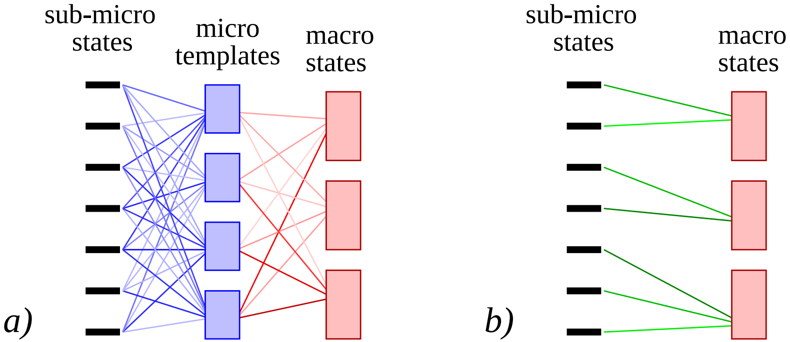
But what holds for a planet should then be true for all classical configurations, hence:

$\boxed{\textit{All classical states are ontological states!}}$

Classical states do not superimpose.


sub-micro states
micro templates
macro states
a)
sub-micro states
macro states
b)

The Born probabilities are now found to coincide with the probabilistic distributions reflecting the unknown details of the initial states.

And that's exactly how probabilities arise in an "ordinary" classical deterministic theory.

Ontological states form an orthonormal set: superpositions of ontological states are never ontological states themselves.
The universe is in an ontological state.

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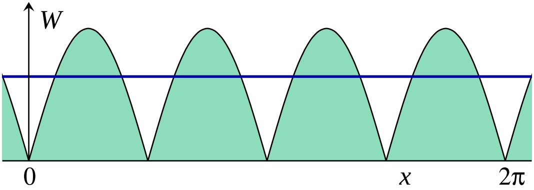
Classically, the probabilities of the different outcomes of an experiment reflect the uncertainties in the initial state

Quantum mechanically, we get *the same* probabilities, but now they are the Born probabilities!

In Bell's experiment, Alice and Bob are assumed not to be correlated with the photons emitted by the source.
But strong correlations should be expected everywhere, also in the vacuum fluctuations.

The mouse dropping function.

$$W(a,b,c) \;=\; \tfrac{1}{2\pi^2}|\sin(4c-2a-2b)|$$



$$\begin{aligned} c &= \text{joint polarisations entangled particles} \\ a &= \text{filter polarisation chosen by Alice} \\ b &= \text{filter polarisation chosen by Bob} \\ x &= 2c-a-b \end{aligned}$$

What happened according to the CAI ?

"Conspiracy" is ridiculous, unless there is an exact, physical, conservation law.

We have the *ontology conservation law* :
Ontic states evolve into ontic states.

$$ {}^{\text{class}}_{\text{template}}\langle \ell | k \rangle_{\text{template}} = \sum_k \lambda_n \, {}^{\text{class}}\langle \ell | n \rangle_{\text{ont}} $$

If Alice makes an infinitesimal modification of her settings, the *classical* state will change $\rightarrow$ all ontic states will change:

$$ {}^{\text{class}}_{\text{template}}\langle \ell + \delta\ell | k \rangle_{\text{template}} = \sum_k \lambda_m \, {}^{\text{class}}\langle \ell + \delta\ell | m \rangle_{\text{ont}} $$

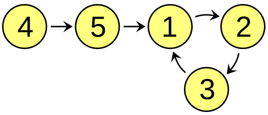
All Alice's ontological states $|m\rangle_{\text{ont}}$ are now different from all $|n\rangle_{\text{ont}}$ that she had before.

Time reversibility

The cellular automata discussed above were constructed such that they are time reversible. The evolution operator $U(t)$ is then a pure permutator, and its representation in Hilbert space is unitary $\Rightarrow$

The Hamiltonian is hermitean.

Black hole physics: non time-reversibility? Let's investigate.



$$U(\delta t) \stackrel{?}{=} \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} .$$

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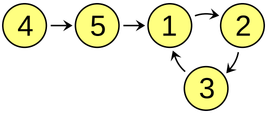

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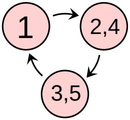
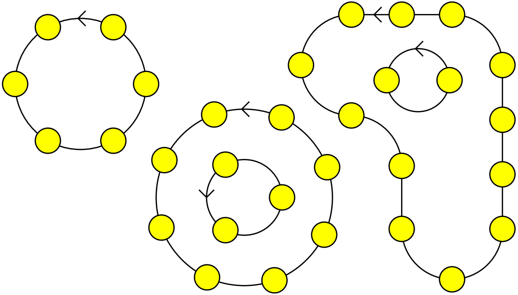
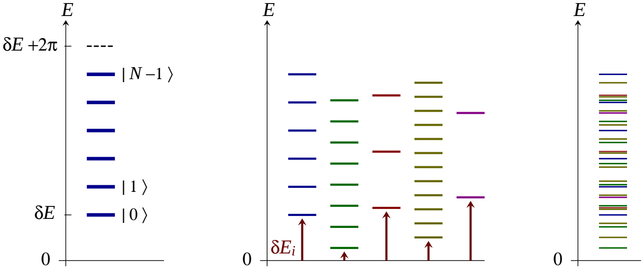
Introduce: *info-equivalence classes:* $(5) \approx (3)$, $(4) \approx (2)$



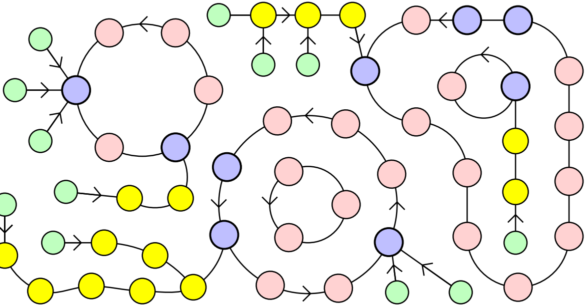
$$U(\delta t) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} .$$

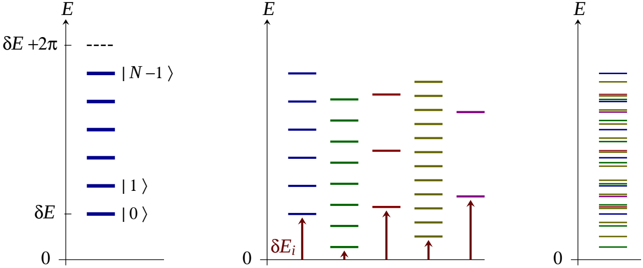
The generic, finite, deterministic, time reversible model:





The generic, finite, deterministic, time *non* reversible model:





The info-equivalence classes act as local gauge equivalence classes

Maybe they *are* local gauge equivalence classes!

By construction, these equivalence classes are time-reversible.

So, *in spite of info-loss*, the quantum theory will be time-reversible: PCT ivariance in QFT.

The classical, ontological states are not time reversible!

Therefore, the classical states carry an explicit arrow of time!
The quantum theory does not!