The time-symmetric re-formulation of quantum mechanics, weak values and the classical limit of quantum mechanics

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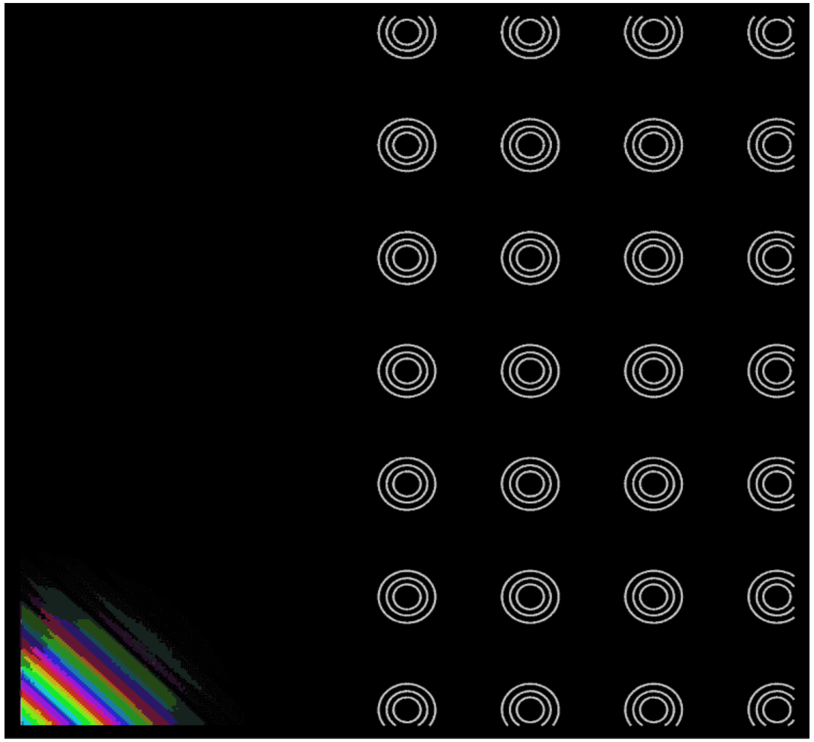
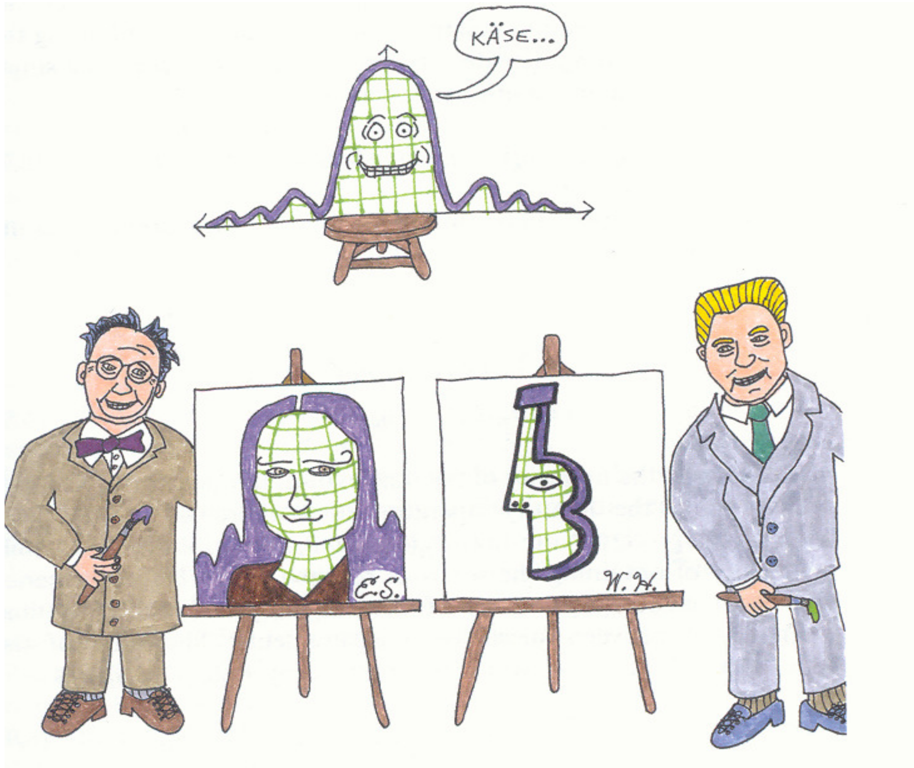


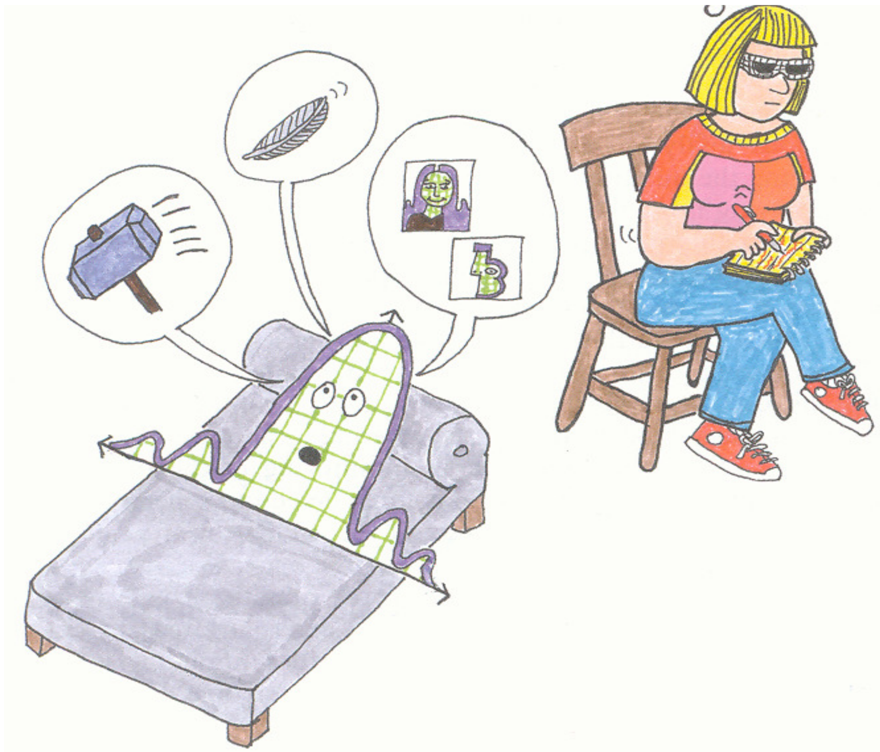
Courtesy Thaller

EmQM15 Emergent Quantum Mechanics
Vienna, Austria October 24, 2015

Paradigms shield themselves against falsification as part of *normal science*

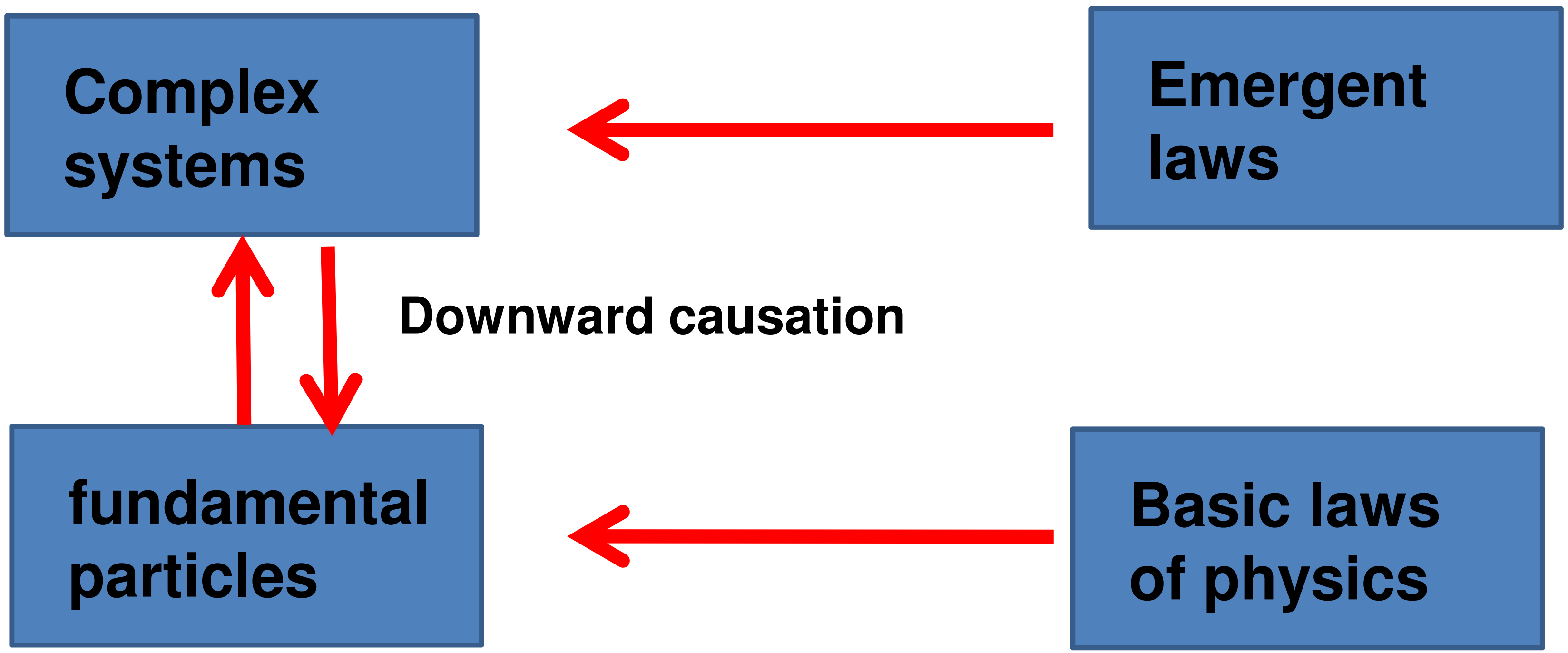
Thomas Samuel Kuhn (1922 –1996):
The paradigm determines how you see the facts! *The Structure of Scientific Revolutions*





Strong vs Weak emergence

- **Strong emergence: higher levels have causal efficacy over lower levels; whole-part causation**
- **No room at the bottom–the laws of physics @ micro-level completely determine everything (?)**
- **New emergent laws lead to over-determination**



Methodology to explore Emergent QM:
start with more intuitive axioms for QM

Quantum Mechanics

Nonlocality ☮ Relativity

Axioms for quantum mechanics

• Physical states are normalized vectors $\psi(\mathbf{r})$, $\Psi(\mathbf{r},t)$, $|\psi\rangle$, $|\Psi(t)\rangle$.

• Measurable physical quantities – "observables" – correspond to *Hermitian* or (*self-adjoint*) operators on the state vectors.

• If a system is an eigenstate $|a\rangle$ with eigenvalue a of an observable $\hat{A}$, then a measurement of $\hat{A}$ on $|a\rangle$ will yield a.

• Conversely, if a measurement of $\hat{A}$ on any state yields a, the measurement leaves the system in an eigenstate $|a\rangle$.

• The probability that a system in a normalized state $|\psi\rangle$ can be found in the state $|\varphi\rangle$ is $|\langle\varphi|\psi\rangle|^2$.

• The time evolution of a quantum state $|\Psi(t)\rangle$ is given by

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle =$$

where $\hat{H}$ is the Hamiltonian (kinet
of the system in the state $|\Psi(t)\rangle$.

• The wave function of identical fe
be antisymmetric under exchange
function of identical bosons (spin
under exchange of any pair of ther

•
•
•

Methodology to explore Emergent QM:
start with more intuitive axioms for QM

- Those axioms recalls a Woody Allen joke:

  This guy goes to a psychiatrist and says, "Doc, my brother's crazy – he thinks he's a chicken!"

  The doctor says, "Well, why don't you turn him in?"

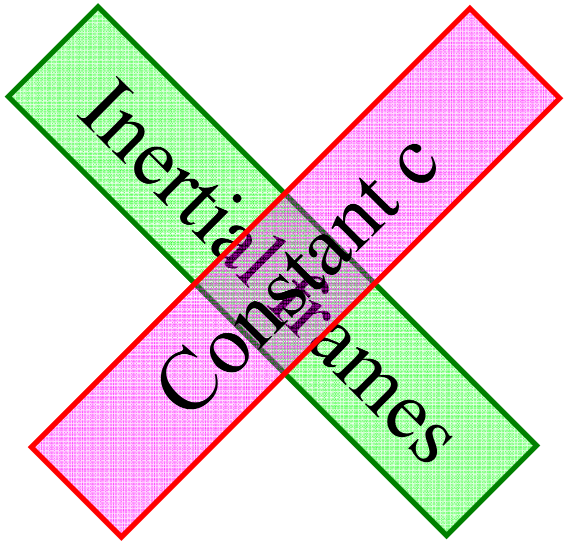
  The guy says, "I would, but I need the eggs!"

- We say, "Quantum theory is crazy – but we n ... "
- Not intuitive: "It's like trying to derive specia ... from the wrong axioms." – Yakir Aharonov
  - ❖ Fast objects contract in the direction of th
  - ❖ Moving clocks slow down
  - ❖ Observers determine the results of measu

Methodology to explore Emergent QM:
start with more intuitive axioms for QM

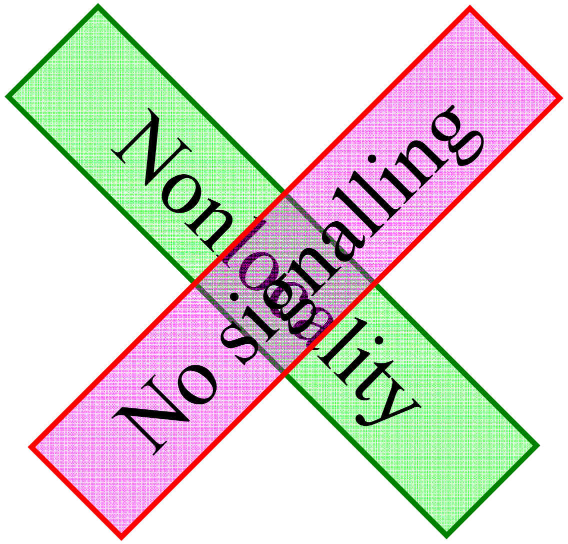
- **Question: What, indeed, is so "special" about special relativity?**

- **Answer: The two axioms so nearly contradict each other that only a unique theory reconciles them.**



Methodology to explore Emergent QM:
start with more intuitive axioms for QM

Y. Aharonov and (independently) A. Shimony:
Quantum mechanics, as well, reconciles two things that nearly contradict each other:



- **Can we derive *a part of* quantum mechanics from these axioms?**
- **Aharonov: Quantum mechanics must include uncertainty.**

Methodology to explore Emergent QM:
start with more intuitive axioms for QM

~~Quantum Mechanics~~

$\Downarrow$

~~Nonlocality $\oplus$ Relativity~~

Why uncertainty?

Traditional answer: nature is capricious

Methodology to explore Emergent QM:
start with more intuitive axioms for QM

Nonlocality ☮ Relativity

$\Downarrow$?

Quantum Mechanics

Can we *derive* quantum mechanics from these two axioms?

1. What is nonlocality?

    Nonlocal correlations?
    Aharonov-Bohm effect?
    “Modular” dynamical variables?

2. What does “no signaling” mean?

    What is left of “no signaling” in the limit $c \to \infty$ of nonrelativistic quantum mechanics?

Methodology to explore Emergent QM: start with more intuitive axioms for QM

Nonlocality ☮ Relativity

⬇ ?

Quantum Mechanics

Can we *derive* quantum mechanics from these two axioms?

1. What is nonlocality?

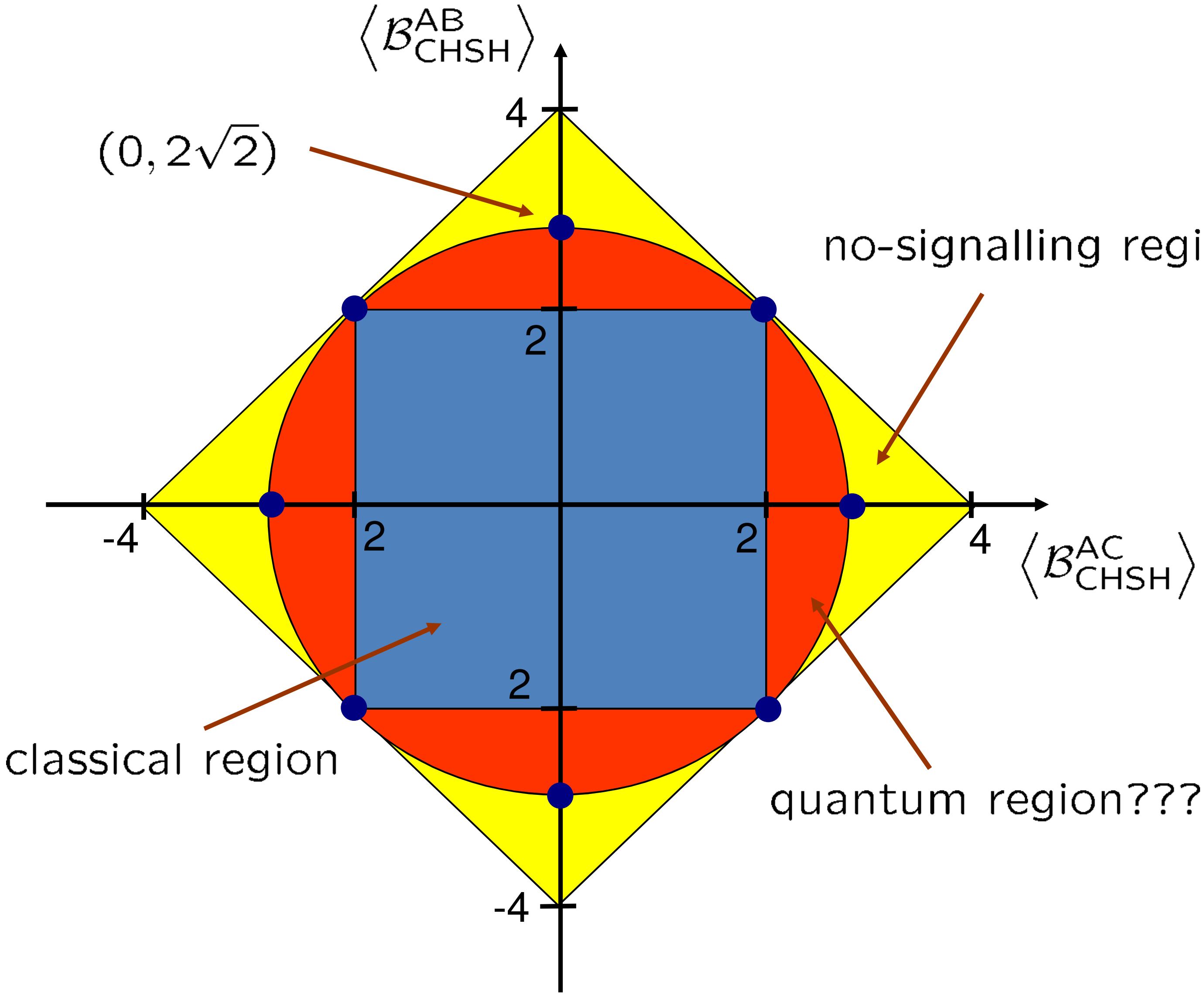
   Nonlocal correlations?

2. What does "no signaling" mean?

   "No signaling" at any speed!

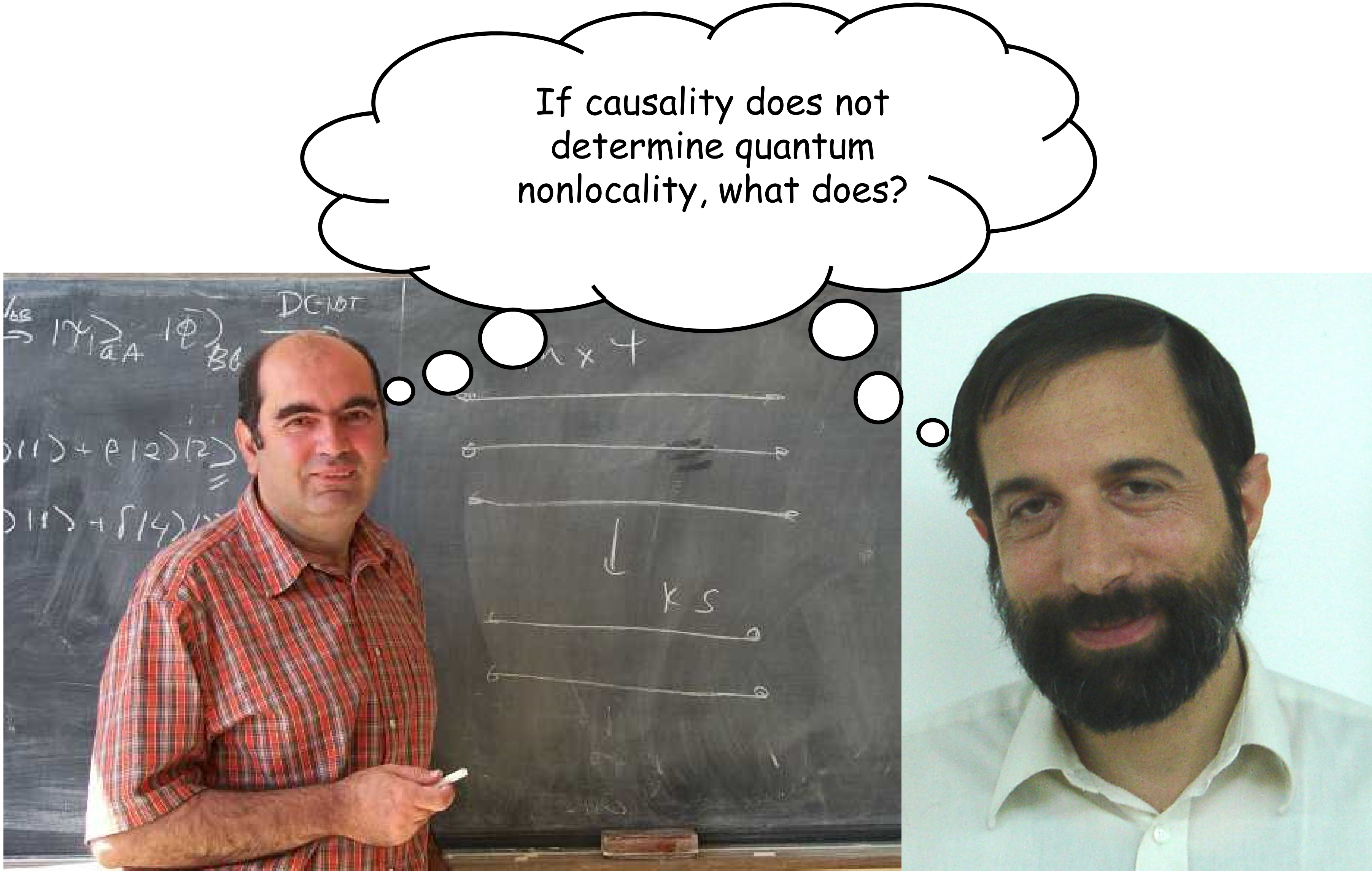
S. Popescu and D. Rohrlich, Found. Phys. 24 (1994) 379

Monogamy of CHSH correlations



S Popescu and D Rohrlich, Foundations of Physics, **24**, 379, (1994).

Super-quantum Nonlocal Correlations



S Popescu and D Rohrlich, Foundations of Physics, **24**, 379, (1994).

Many new results from PR impacting computation/information

Non-trivial communication complexity

G. Brassard, H. Buhrman, N. Linden, A. A. Methot, A. Tapp and F. Unger, Phys. Rev. Lett., 96 250401, (2006).

No Advantage for Nonlocal Computation

N. Linden, S. Popescu, A. J. Short, and A. Winter, Phys. Rev. Lett. 99, 180502 (2007).

Information Causality

M. Pawlowski, T. Paterek, D. Kaszlikowski, V. Scarani, A. Winter, and M. Zukowski, Nature 461, 1101 (2009).

Local orthogonality

T. Fritz, A. B. Sainz, R. Augusiak, J. B. Brask, R. Chaves, A. Leverrier and A. Acín, Nature Communications 4, 2263 (2013) .

Etc, etc, etc, etc

Methodology to explore Emergent QM:
start with more intuitive axioms for QM

Nonlocality ☮ Relativity

⬇ ?

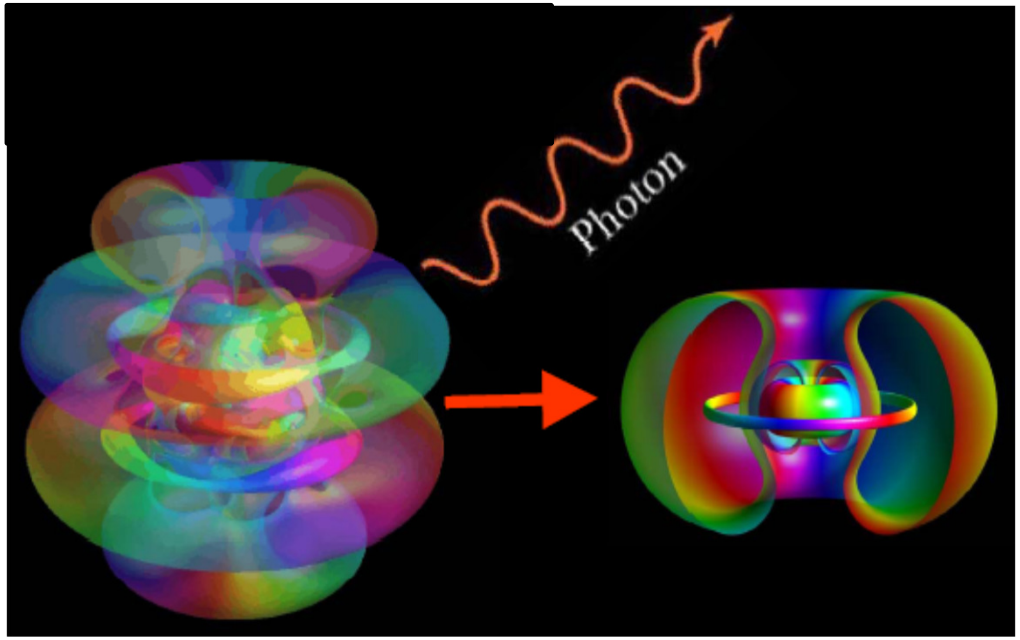
Quantum Mechanics

Quantum uncertainty (ontic) can be derived from:

- nonlocality: relevance of future to present
- causality
- free will

Indeterminancy: playing dice

- 1^{st} atom decays in 1 min
- 2^{nd} atom (identical to 1^{st} atom) decays in 1 hour
- There was no difference between them in the beginning, but they behave differently later



Einstein: "God does not play dice."

Time-Symmetric formulation of Quantum Mechanics TSQM

- **We ask: “Why does God play dice?”**
- **~~Traditional answer: nature is capricious~~**
- **Alternative: allows quantum mechanics to independently select both the initial and final states of a single system**
- **TSQM: the state of a system at a given moment is described by two wave-functions, one evolving from the past to the future, and one evolving from the future to the past**

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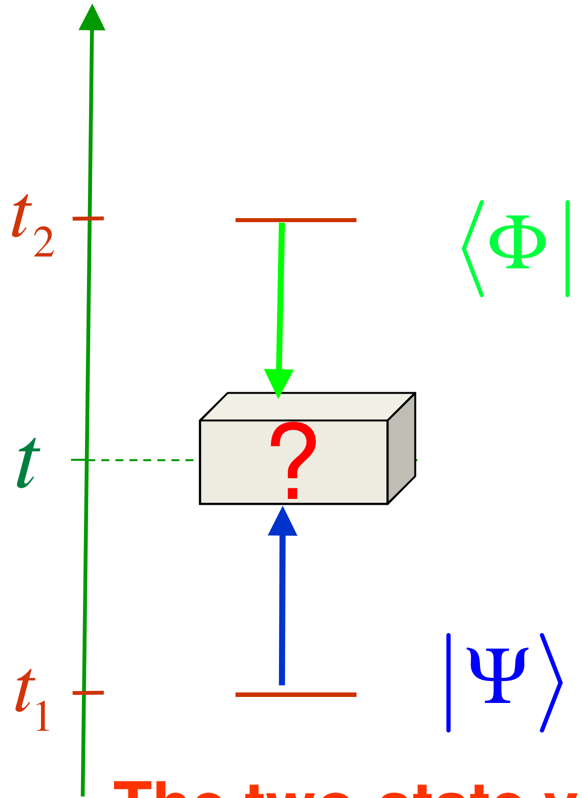
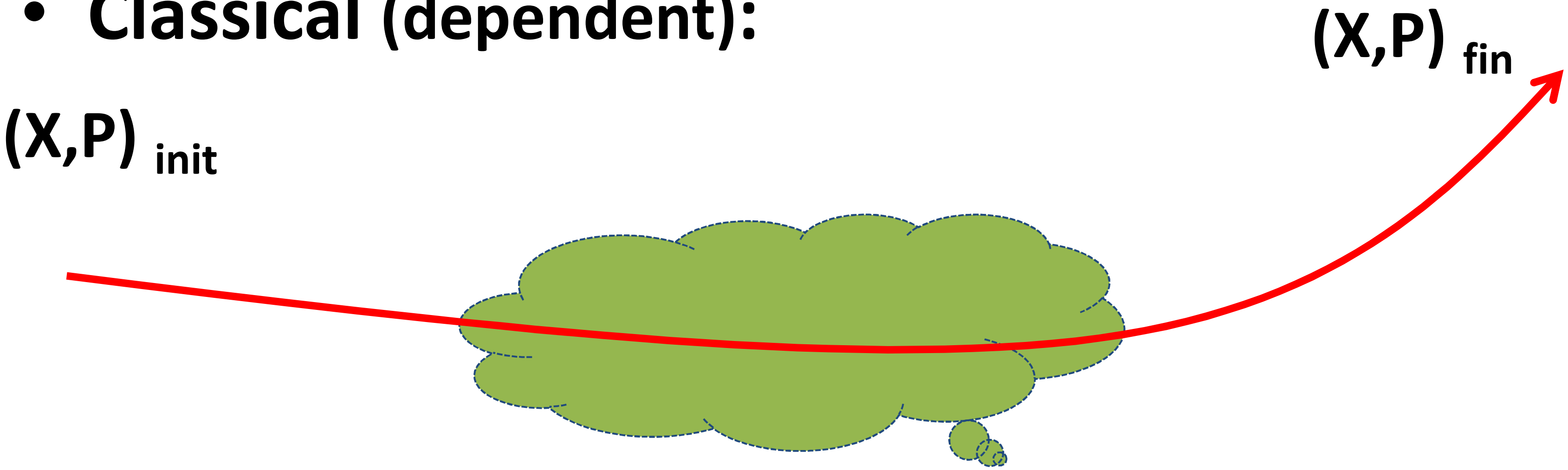


The two-state vector

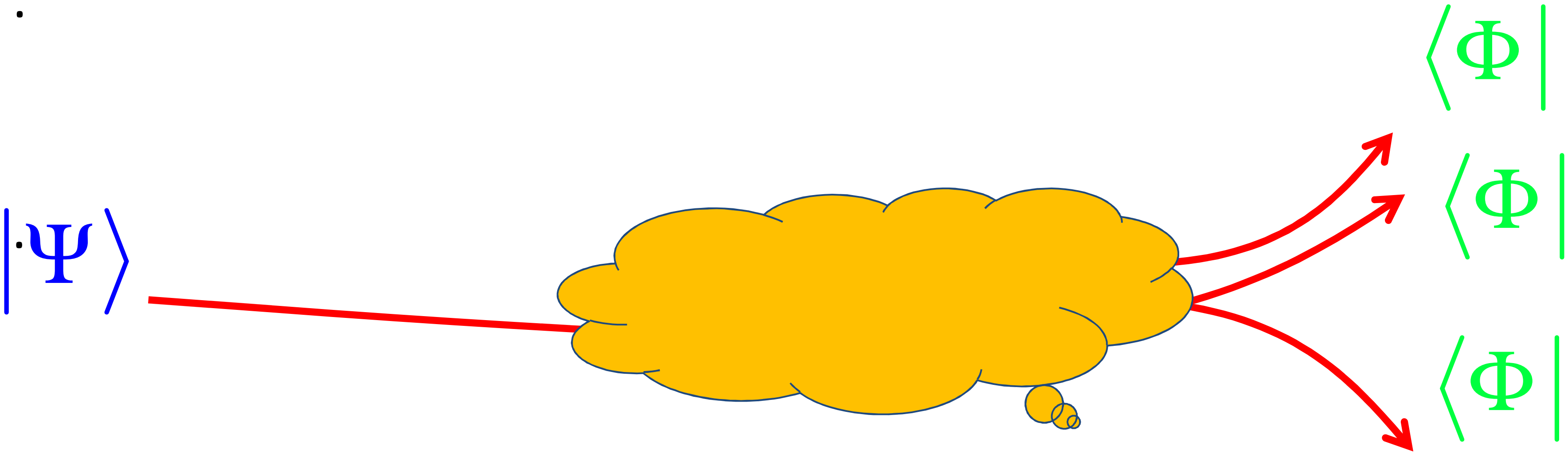
$$\langle\Phi| \quad |\Psi\rangle$$

Boundary conditions: classical vs quantum

- **Classical (dependent):**



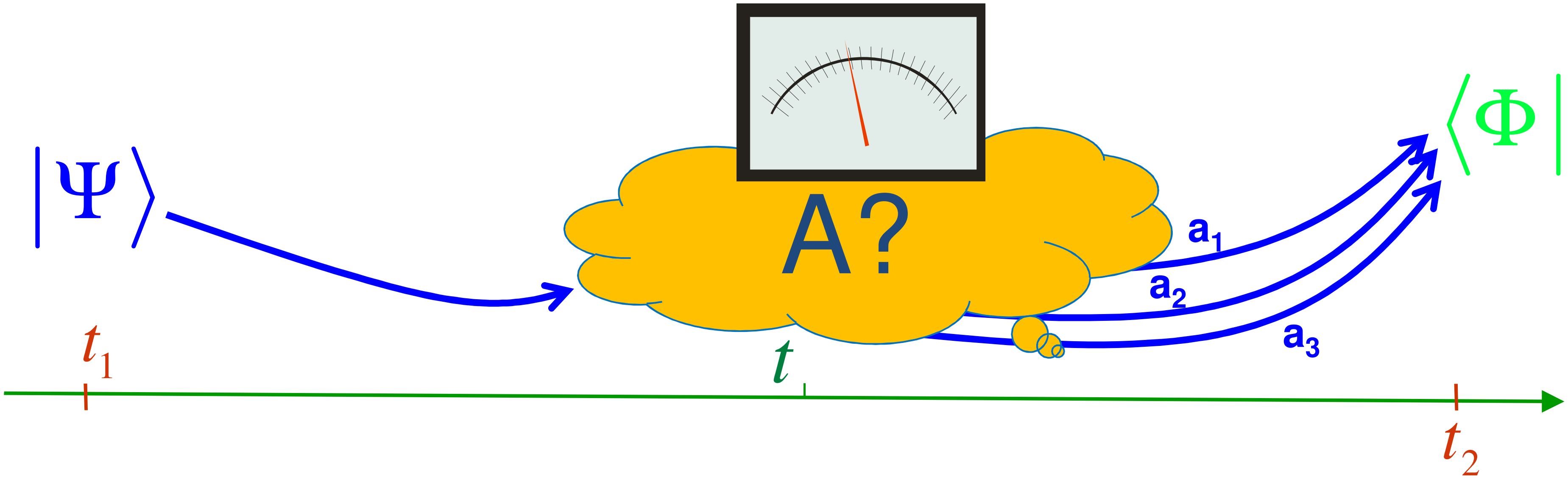
- **Quantum (independent):**



Boundary conditions: quantum

What can we say about the system at intermediate time, *t* by using strong measurements?

$$Pr(a_j, t|\Psi, t_1; \Phi, t_2) = \frac{|\langle\Phi|U_{t\to t_2}|a_j\rangle\langle a_j|U_{t_1\to t}|\Psi\rangle|^2}{\sum_n |\langle\Phi|U_{t\to t_2}|a_n\rangle\langle a_n|U_{t_1\to t}|\Psi\rangle|^2}$$



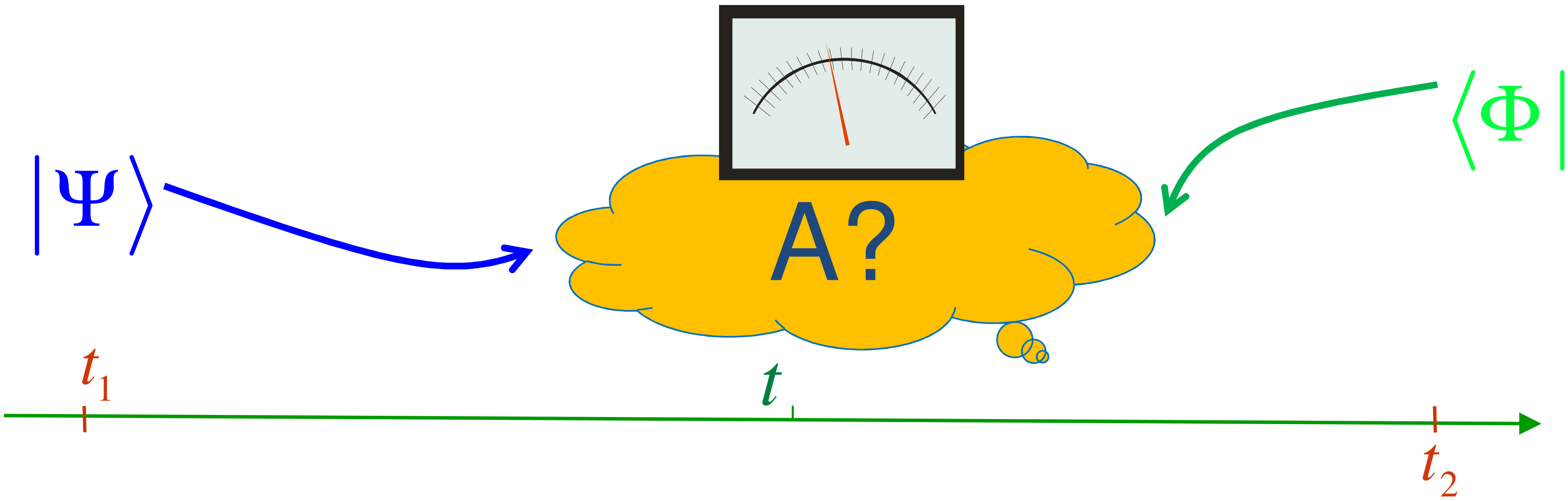
Y. Aharonov, P. G. Bergman and J. L. Lebowitz, *Phys. Rev.* **134**, 1410 (1964)

Boundary conditions: quantum

What can we say about the system at intermediate time, *t*?

Strong measurement: Aharonov-Bergmann-Lebowitz (ABL) formula

$$Pr(a_j, t|\Psi, t_1; \Phi, t_2) = \frac{\left|\langle U_{t_2\to t}\Phi|a_j\rangle\langle a_j|U_{t_1\to t}|\Psi\rangle\right|^2}{\sum_n \left|\langle U_{t_2\to t}\Phi|a_n\rangle\langle a_n|U_{t_1\to t}|\Psi\rangle\right|^2}$$



Y. Aharonov, P. G. Bergman and J. L. Lebowitz, *Phys. Rev.* **134**, 1410 (1964)

Time-Symmetric formulation of QM (TSQM)

To be useful and interesting, any re-formulation of QM should meet several criteria, for example:

YES

1) TSQM is consistent with standard QM

$$\langle \Psi_2 | U_{t \to t_2} = \langle U^{\dagger}_{t \to t_2} \Psi_2 | = \langle U_{t_2 \to t} \Psi_2 |$$

$$U^{\dagger}_{t \to t_2} = \left\{ e^{-iH(t_2 - t)} \right\}^{\dagger} = e^{iH(t_2 - t)} = e^{-iH(t - t_2)} = U_{t_2 \to t}$$

2) TSQM brings out features in QM re ***emergence*** that were missed before (e.g. weak values)

3) TSQM stimulated discoveries in other fields re ***emergence***

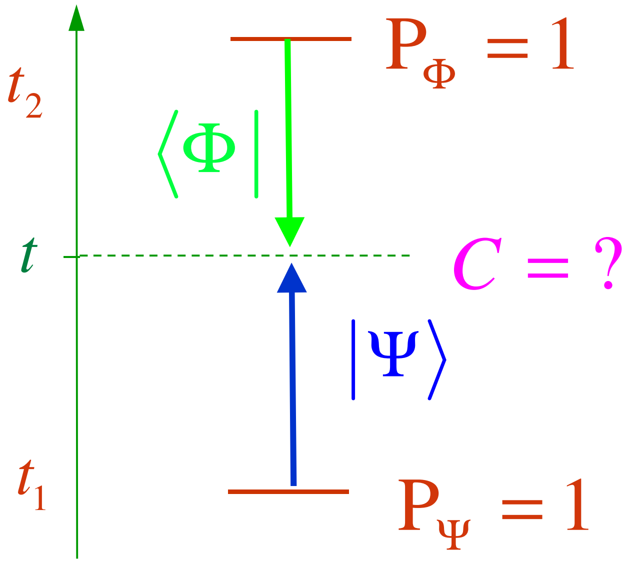
4) TSQM suggests generalizations re ***emergence***

2 The two-state vector description of a quantum system

Measurements performed on a pre- & post-selected system described by the two-state vector:

$$\langle \Phi | \quad | \Psi \rangle$$

Strong measurement: The Aharonov-Bergmann-Lebowitz (ABL) formula:



$$\mathrm{Prob}(C = c) = \frac{\left| \langle \Phi | P_{C=c} | \Psi \rangle \right|^2}{\sum_i \left| \langle \Phi | P_{C=c_i} | \Psi \rangle \right|^2}$$

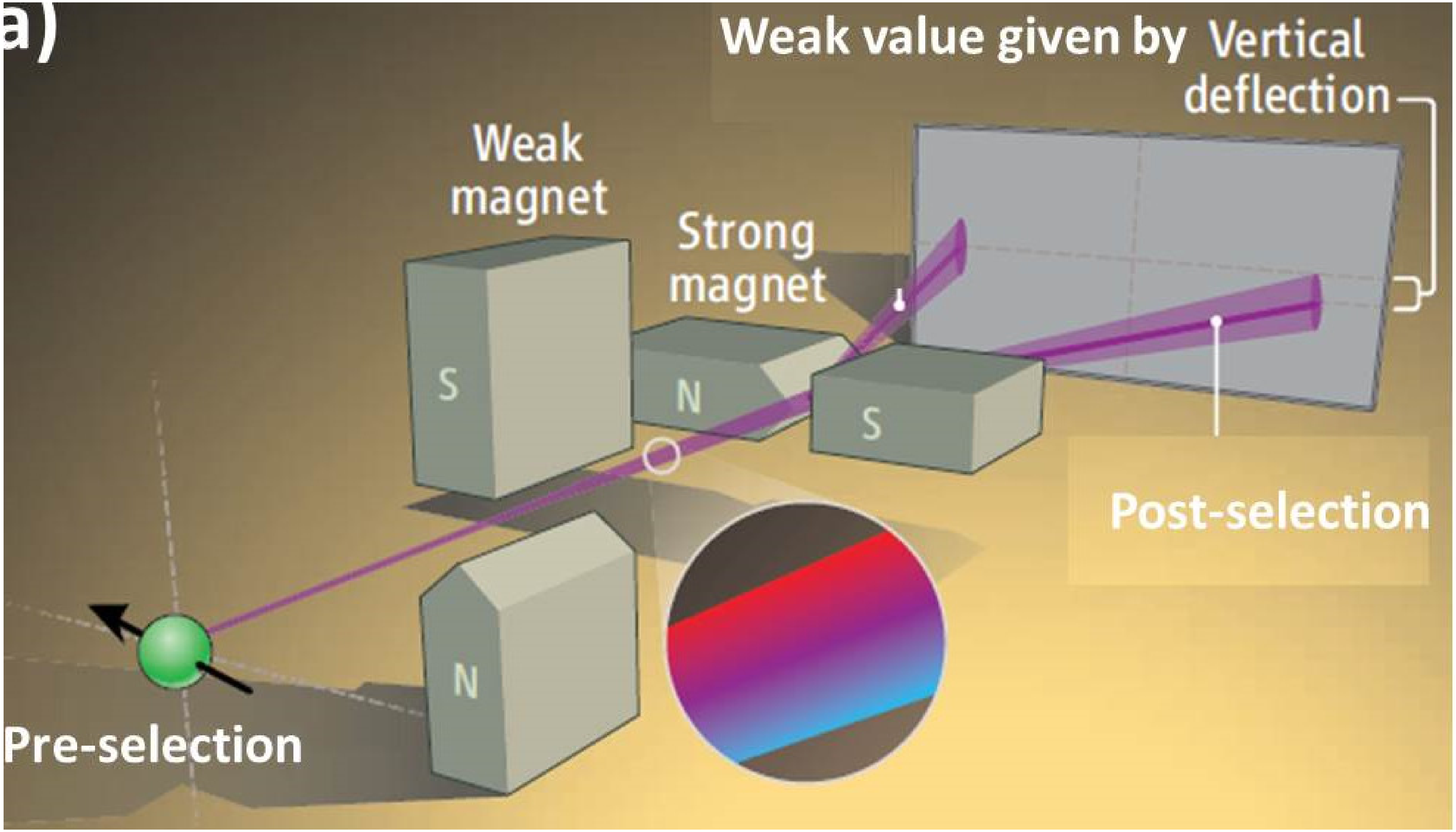
Weak measurement: The Aharonov-Albert-Vaidman effect:
Phys. Rev. Lett. 60, 1351 (1988).

Weak value

$$C_w \equiv \frac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}$$

Graphic courtesy Vaidman

2.a Weak Measurements - spin

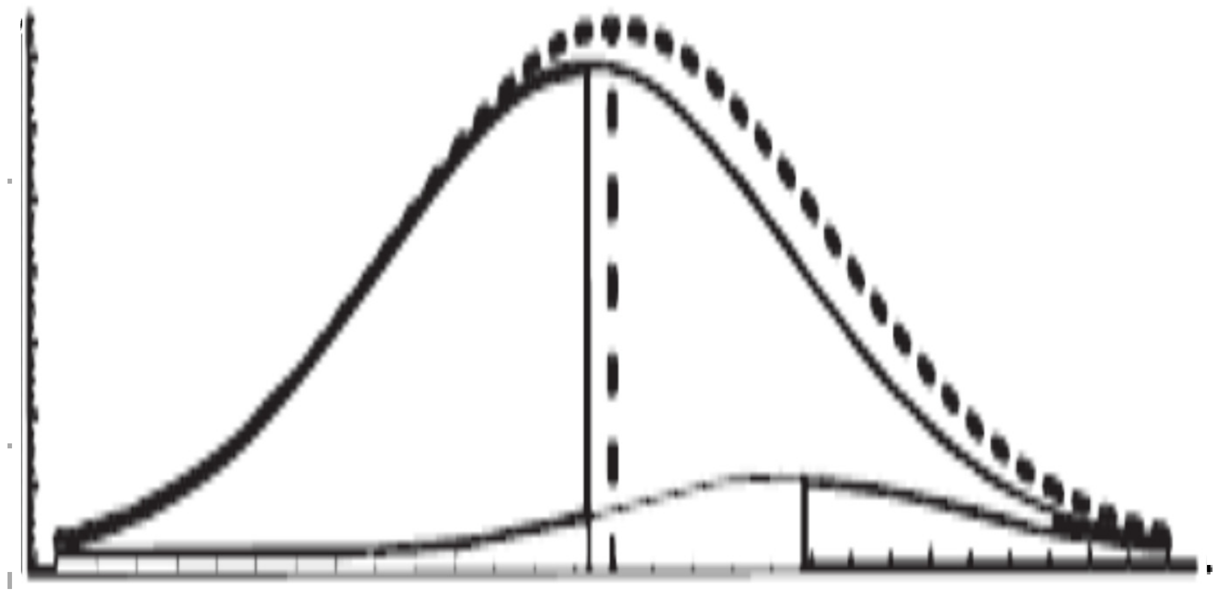


Y Aharonov, D Albert, L Vaidman, *Phys. Rev. Lett.* 60 (1988), 1351

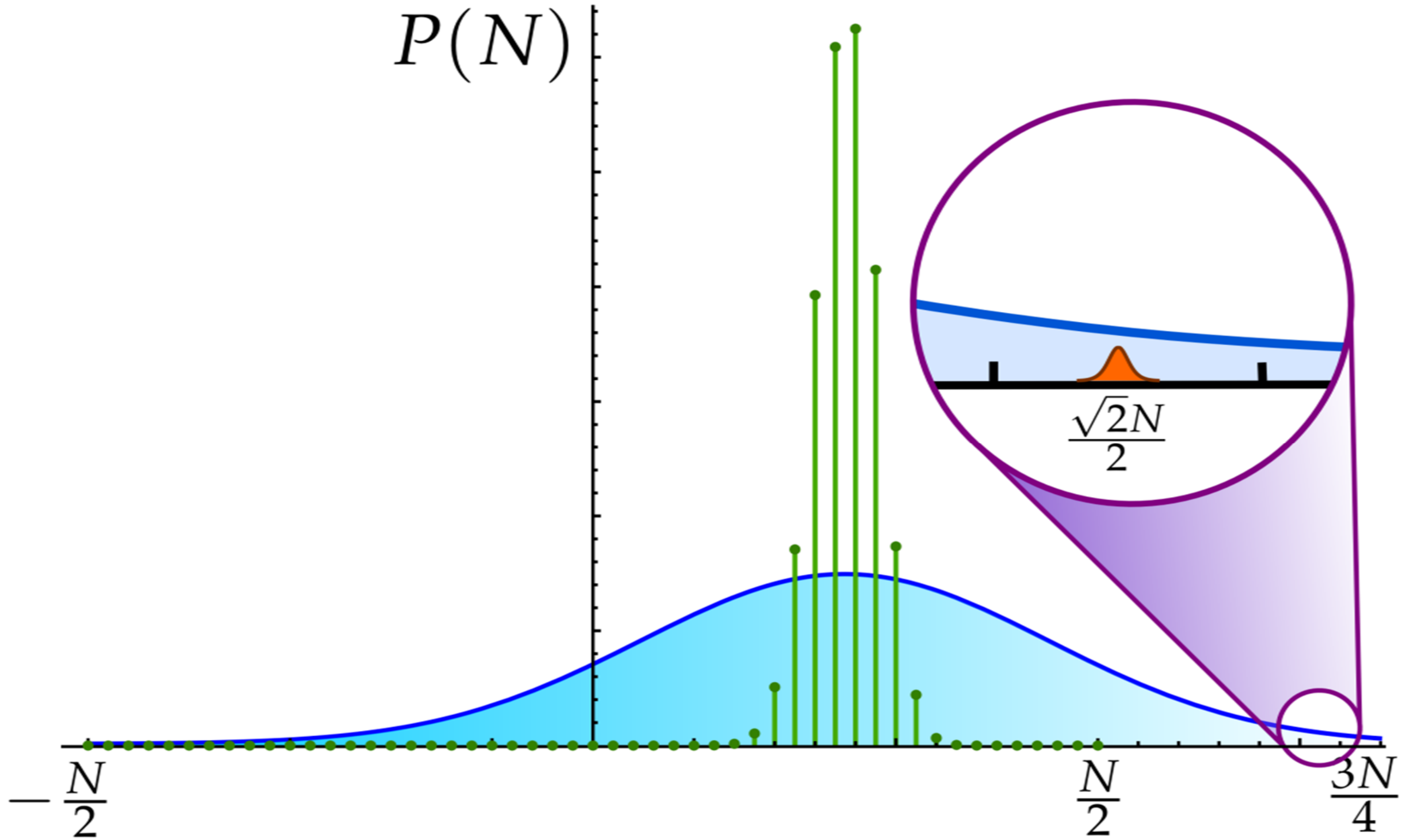
2 Weak values and causality: game of errors

$$\langle \Psi | A | \Psi \rangle = \sum_i P(\Phi_i | \Psi \;) \frac{\langle \Phi_i | A | \Psi \rangle}{\langle \Phi_i | \Psi \rangle}$$

Weak value sum rule



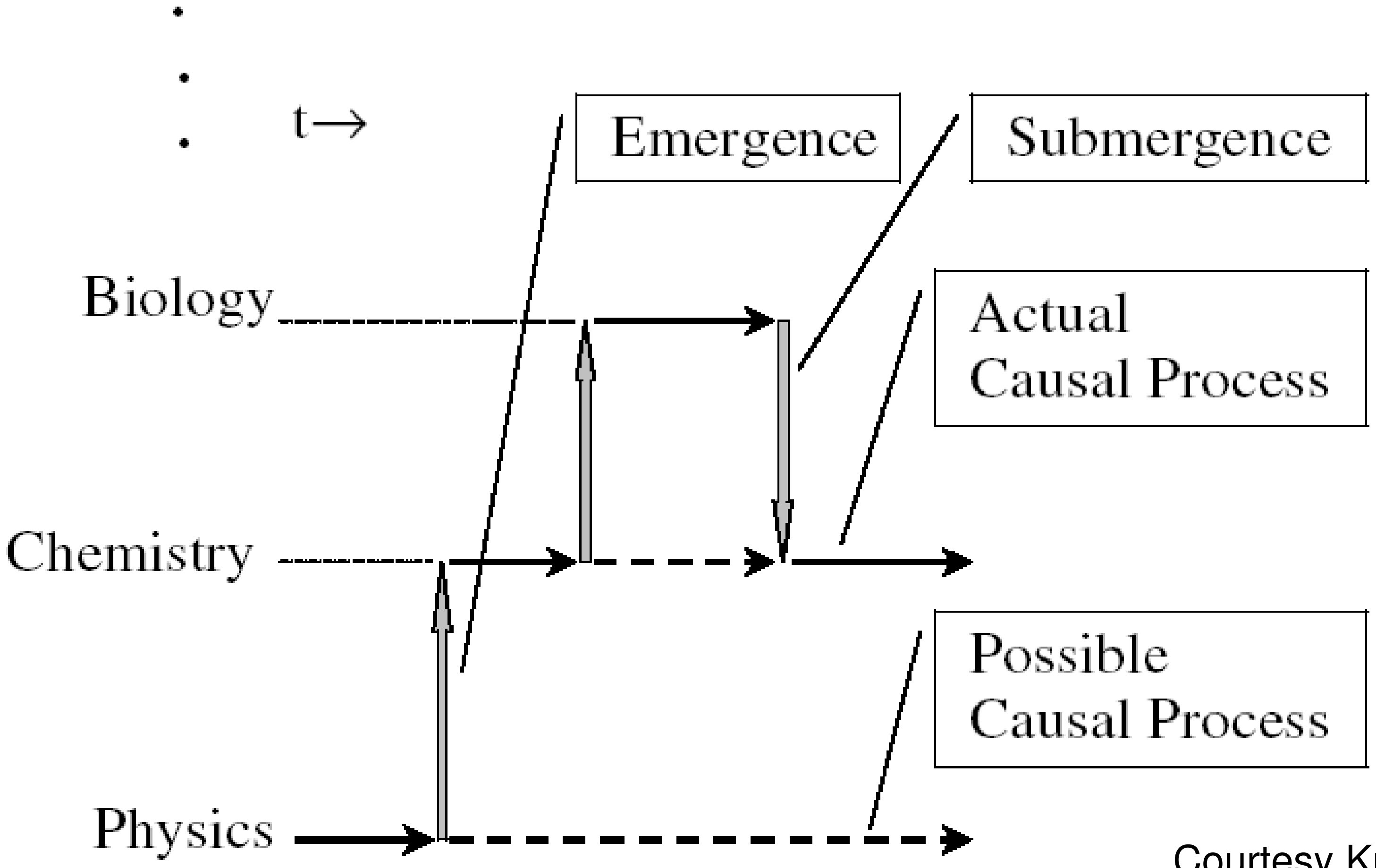
2 Weak values and causality: game of errors



- Probability to obtain weak value as an error of the measuring device is greater than the probability to obtain weak value
- Uncertainty (playing of dice) derived from nonlocality in time, causality
- Led to new approach to information we call “weak information”

2.a Emergence through hierarchical entanglement

- Using entanglement with a higher hierarchical level, there may be time intervals with no independently evolving sub-systems
- I.e. properties & functions depend on whole

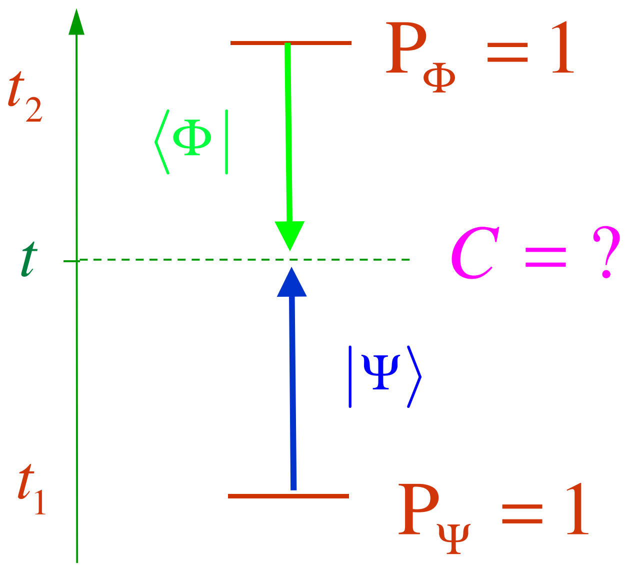


Courtesy Kronz

2.a Emergence through hierarchical entanglement

- Shimony states:
  - "Atoms thus exhibit form in Aristotle's sense, and even have the tendency to maintain this form, which phenomenologically is like his final cause. But the Aristotelian form is achieved by Democritean means -- by interactions among the electrons and the nucleus, which leave these building blocks intact."
- Leading him to:
  - "The parts-wholes problem has an ontological aspect, which concerns the properties of the components and the composite system without explicit consideration of how knowledge of them is obtained. Among the ontological questions are the following: Is there an ultimate set of entities which cannot be subdivided and which are therefore 'atomic' in the etymological sense? If the properties of the components are fully specified, together with the laws governing their interactions, are the properties of the composite system then fully determined? In particular, are there properties of composite systems which are radically different from those of the components, and which might properly be characterized as 'emergent,' also definable in terms of the later? Do composite systems belong, always or for the most part, to 'natural kinds?' Is the existence of natural kinds explicable in terms of the laws governing the components? Are both the possible taxonomy and the actual taxonomy of natural kinds thus explicable? Is there a hierarchy of 'levels of descriptions' -- i.e., microscopic, macroscopic, and possibly intermediate -- such that laws can be formulated concerning a coarser level without explicit reference to the properties at a finer level of description?"

2.a General feature of PPS: failure of product rule



$$\boxed{C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}}$$

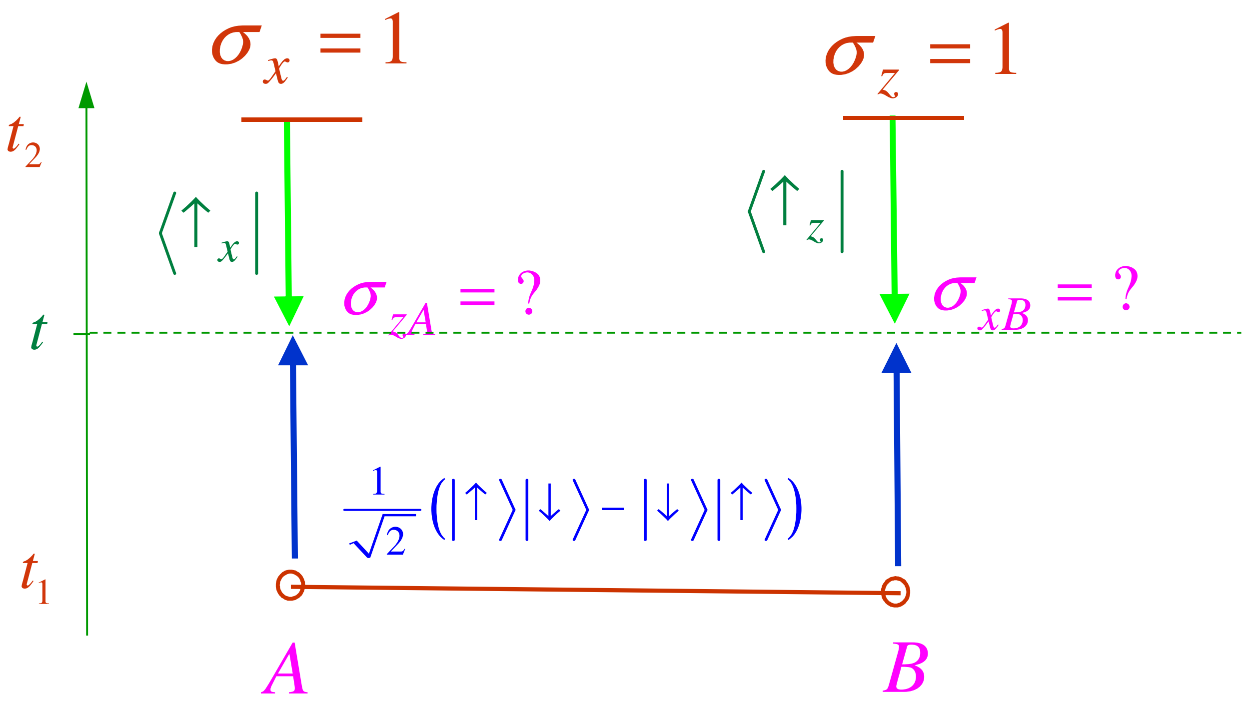
$$(A+B)_w = A_w + B_w$$

$$(AB)_w \neq A_w B_w$$

The weak value of a product of observables is not equal to the product of their weak values. In some sense, new properties emerge with complexity!

2.a Non-locality of EPR from perspective of TSQM

$$\langle\uparrow_x|_A\langle\uparrow_z|_B \quad \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A|\downarrow\rangle_B-|\downarrow\rangle_A|\uparrow\rangle_B\right)$$



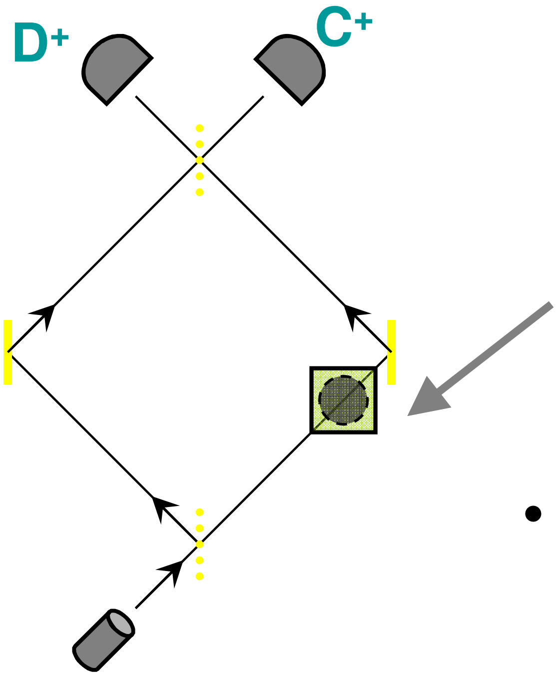
$$\sigma_{zA}=-1$$

$$\sigma_{xB}=-1$$

$$\sigma_{zA}\sigma_{xB}=-1$$

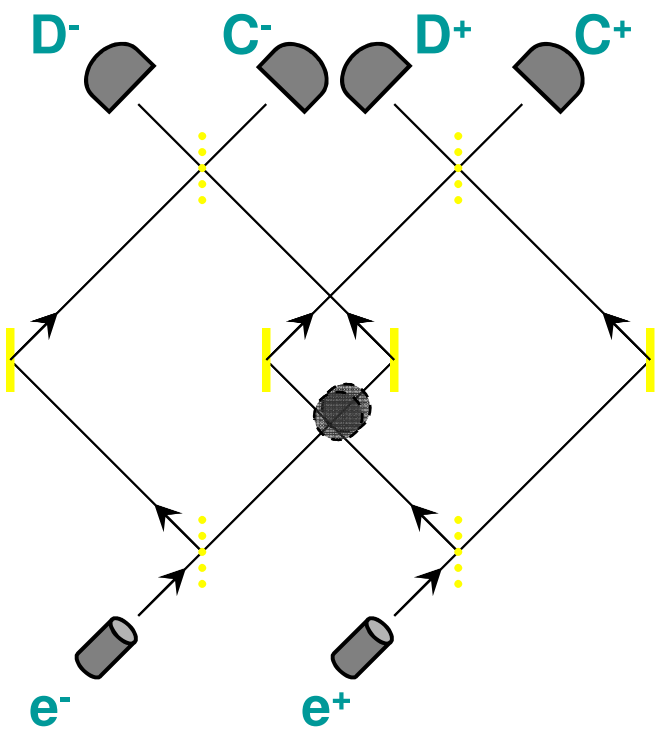
$$\mathrm{Prob}(\sigma_{zA}\sigma_{xB}=-1)=\frac{\left|\langle\uparrow_x|\langle\uparrow_z|\mathrm{P}_{(\sigma_{zA}\sigma_{xB}=-1)}\left(|\uparrow\rangle|\downarrow\rangle-|\downarrow\rangle|\uparrow\rangle\right)\right|^2}{\left|\langle\uparrow_x|\langle\uparrow_z|\mathrm{P}_{(\sigma_{zA}\sigma_{xB}=1)}\left(|\uparrow\rangle|\downarrow\rangle-|\downarrow\rangle|\uparrow\rangle\right)\right|^2+\left|\langle\uparrow_x|\langle\uparrow_z|\mathrm{P}_{(\sigma_{zA}\sigma_{xB}=-1)}\left(|\uparrow\rangle|\downarrow\rangle-|\downarrow\rangle|\uparrow\rangle\right)\right|^2}=1$$

2.a Elitzur & Vaidman Interaction Free Measurements



- However, the phase difference between the left and right path can be altered by the presence of an object in which case detector D+ may be triggered

- If detector D+ is triggered one may then conclude:
  - The particle was not blocked by the object
  - The object must have been in the path



PARADOX: they should never reach the detectors because e+ and e- overlap and therefore annhilate

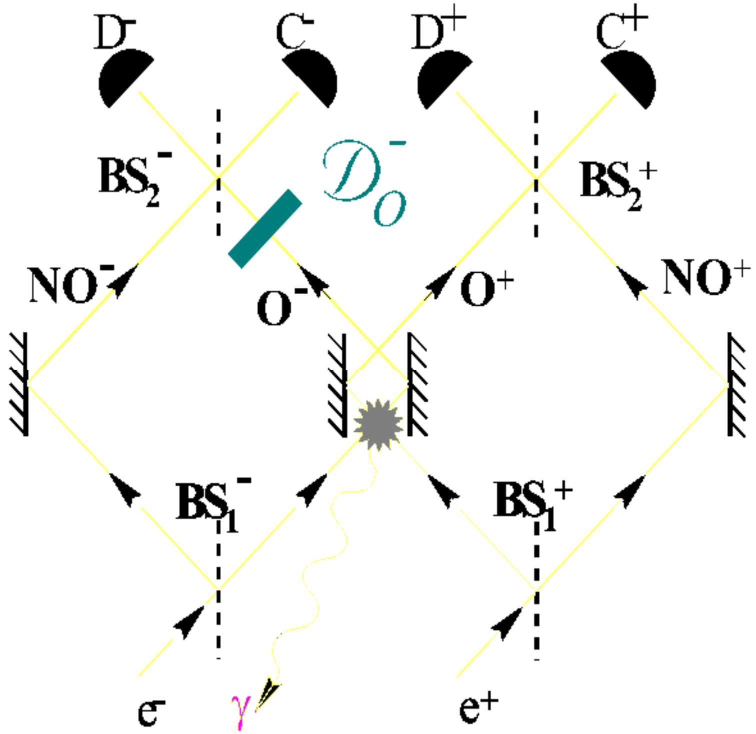
- With 2 MZI's, there is overlap region
- Each MZI can be said to measure whether or not the other MZI's particle is in the overlapping path, otherwise nothing would have disturbed the electron, and the electron couldn't have ended in D^-
- But, if detectors D^- and D^+ both click then intuition leads to a paradox.
- If D^- clicks, then e+ must have gone through the overlapping arm:

$$D^- \rightarrow e^+ \text{ overlapping arm}$$

- Conversely if D^+ clicks then e- must have gone through overlapping arm

$$D^+ \rightarrow e^- \text{ overlapping arm}$$

Hardy's Paradox



- Suppose we try to measure the position of e^- by inserting a detector D_0^- in the overlapping arm of the e^- MZI.
- e^- is always in the overlapping arm, however D_0^- disturbs e^- & e^- could end up in the D^- detector even if no e^+ is present in the overlapping arm
- Cannot infer from D^- that e^+ was in the overlapping arm disturbing e^-. ***The paradox disappears.***

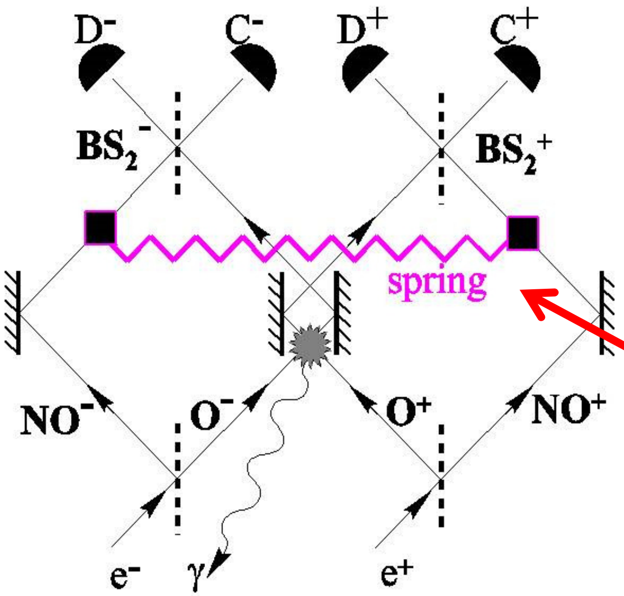
$$D^+ \implies e^- \mathrm{O}, \mathcal{D} \implies e^+ \mathrm{NO}$$

$$D^- \not\Longrightarrow e^+ \mathrm{O}$$

2.b Weak measurements and counterfactuals

Aharonov, Botero, Popescu, Reznik, JT, "Revisiting Hardy's Paradox: Counterfactual Statements, Real Measurements, Entanglement and Weak Values" *Physics Letters A*, v301, 130

Paradoxical reality implied counter-factually has new experimentally accessible consequences in terms of weak measurements, which allow us to test - to some extent - assertions that have been otherwise regarded as counter-factual

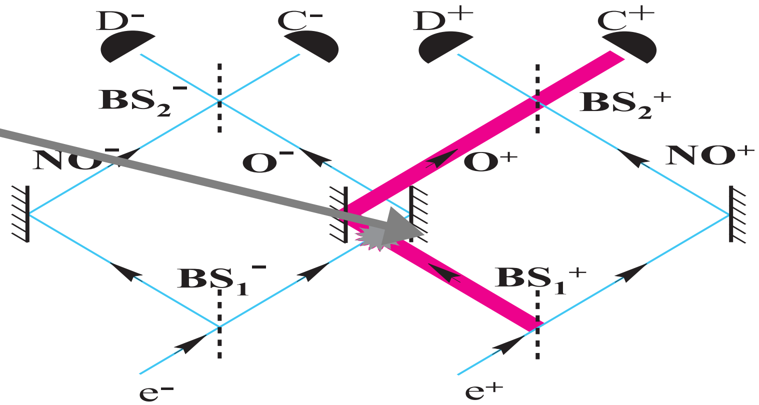


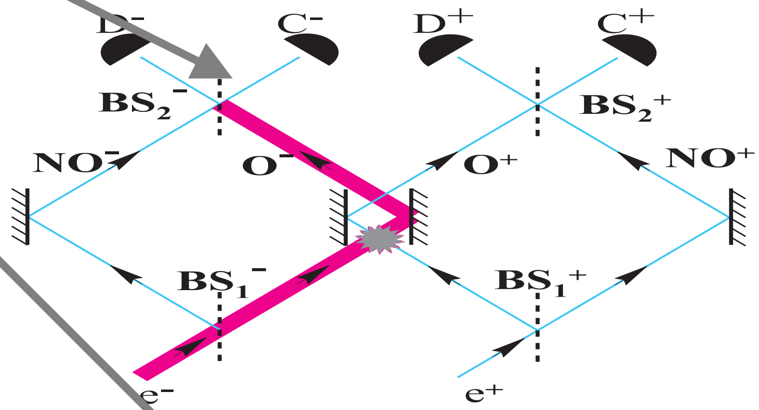
weak value of 'number' of particle-pairs is negative

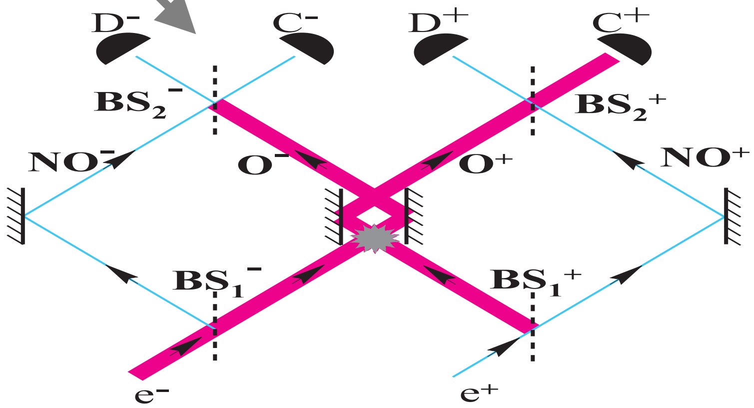
Experiments: J.S. Lundeen and A.M. Steinberg *Phys Rev Lett* **102**:020404, (2009)
K. Yokota, T. Yamamoto, M. Koashi, N. Imoto *New Journal of Physics* **11** (2009) 033011

2.a Weak values obey a simple intuitive & self-consistent logic

- **If there is 1 e+ in overlapping arm**
- **& 1 e- in other overlapping arm**
- **How can there be ZERO e+e- pairs in both overlapping arms?**
- **The particles are both definitely in the overlapping arms, but they are not there together?**
- **Ans: the weak value of a product of observables is not equal to the product of their weak values.**



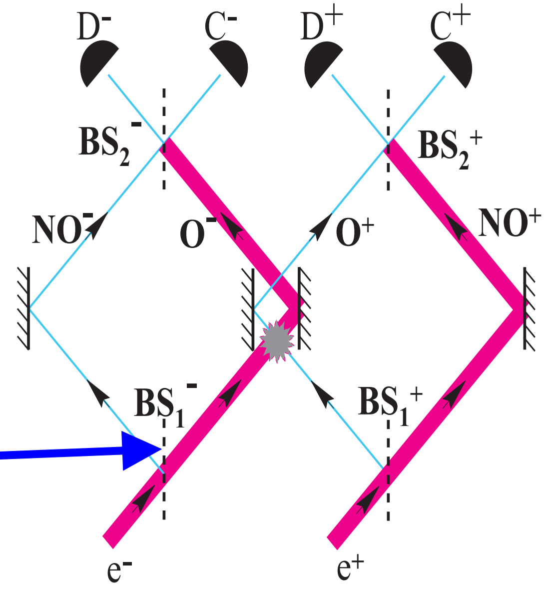




Aharonov, Botero, Popescu, Reznik, Tollaksen, *Physics Letters A* 301 (3-4): 130-138, 2001.

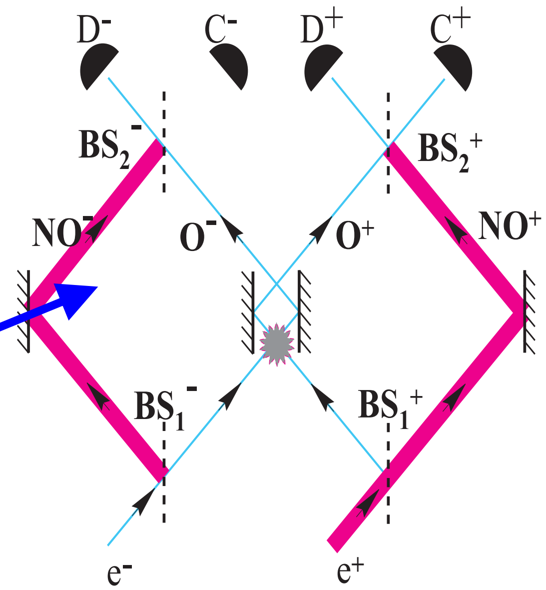
2.a Weak values obey a simple intuitive & self-consistent logic

- But we also have the statements:
  - e^- must be in the overlapping arm otherwise e^+ couldn't have ended at D- &
  - e^+ must be in the non-overlapping arm since there was no annihilation
  - These & the opposite are confirmed



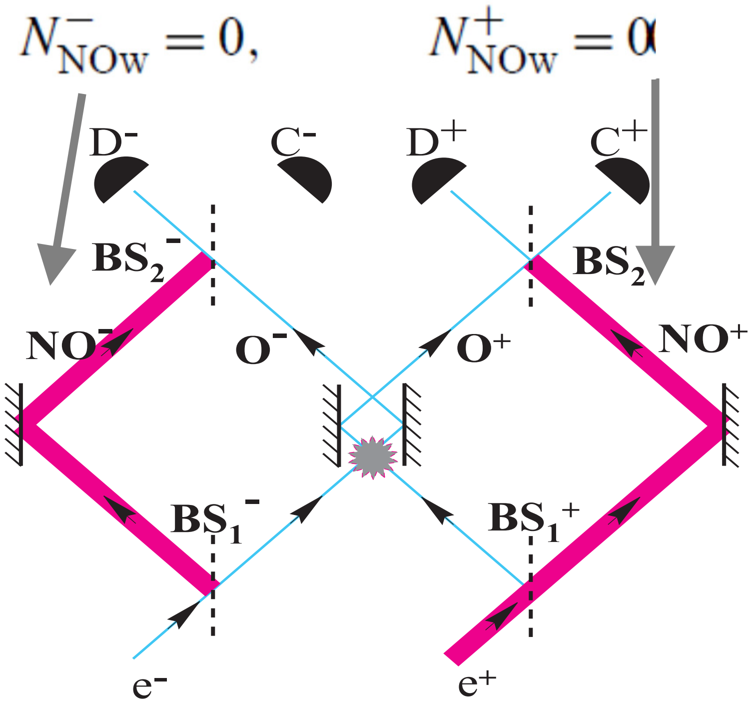
- These 2 statements are at odds w/ the fact that there is just one electron-positron pair

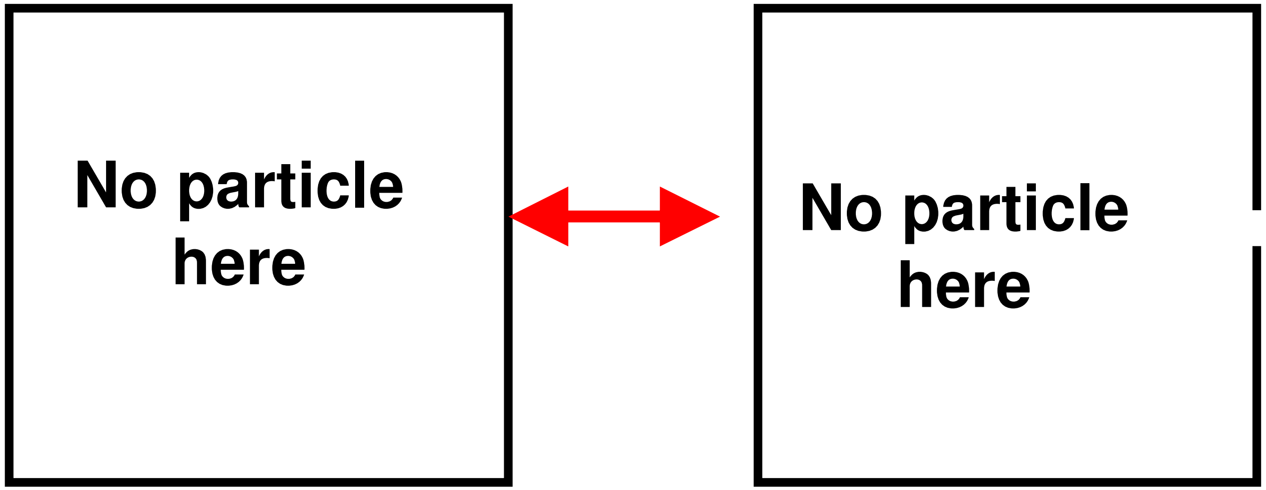
- QM solves the paradox in a remarkable way - it tells us that there is minus one electron-positron pair in the non-overlapping arms which brings the total down to a single pair!



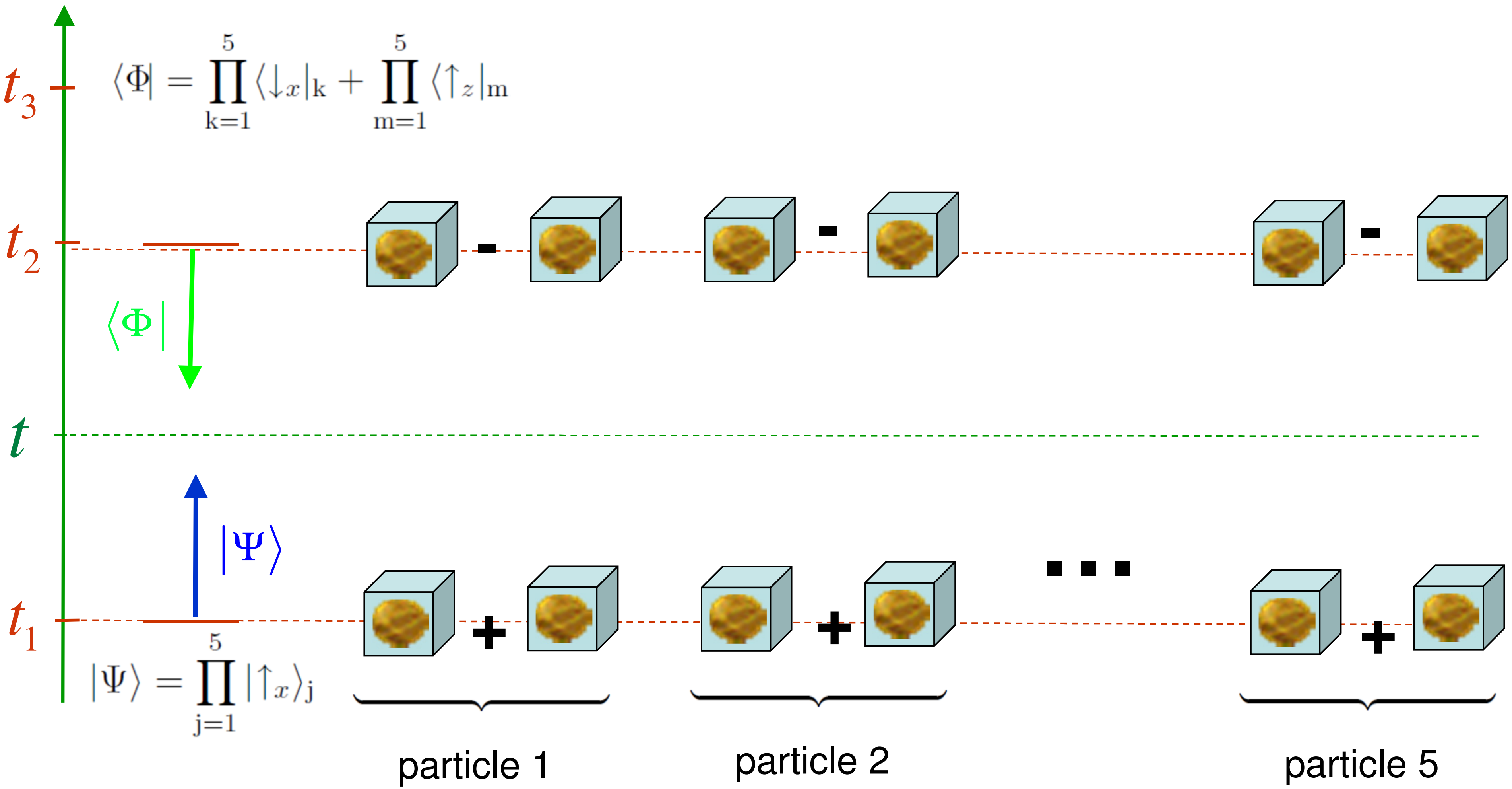
Atom of Emergence/Holism

- While there is no particle in either outer path
- Nevertheless, the interaction between the 2 paths tells us that there is minus one electron-positron pair in the non-overlapping arms

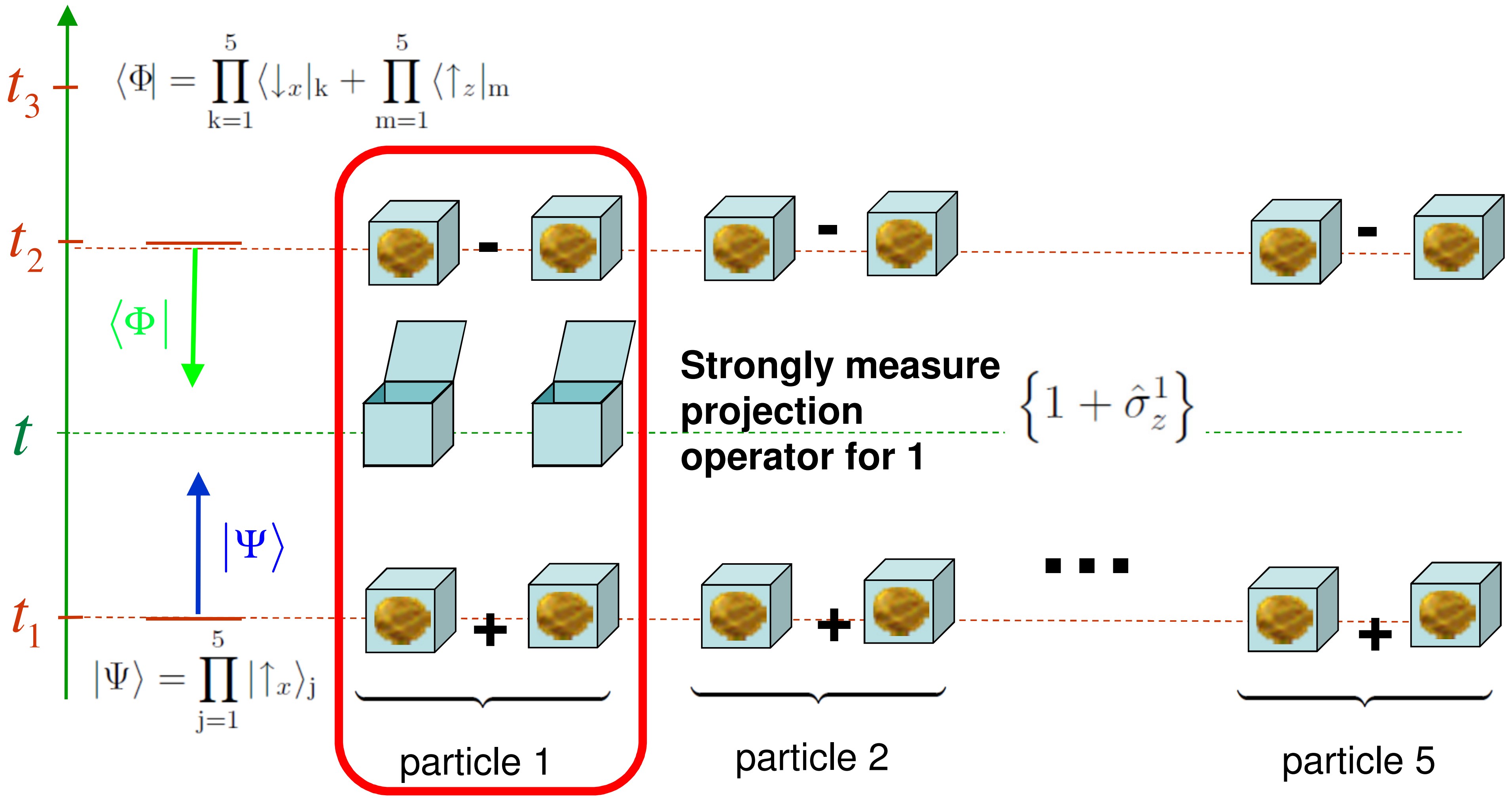




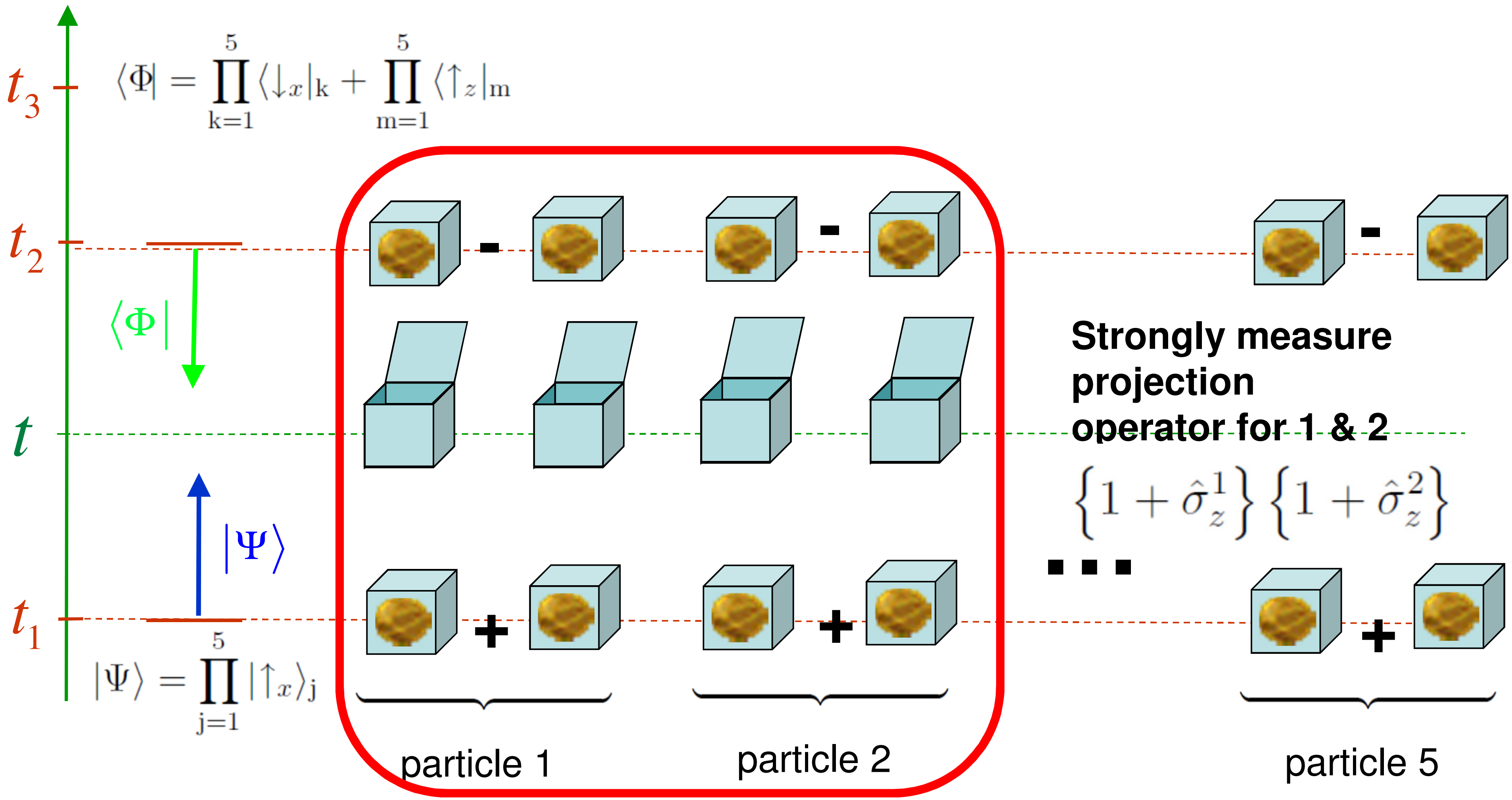
2.a Generalization of Atom of Emergence/Holism



2.a Generalization of Atom of Emergence/Holism

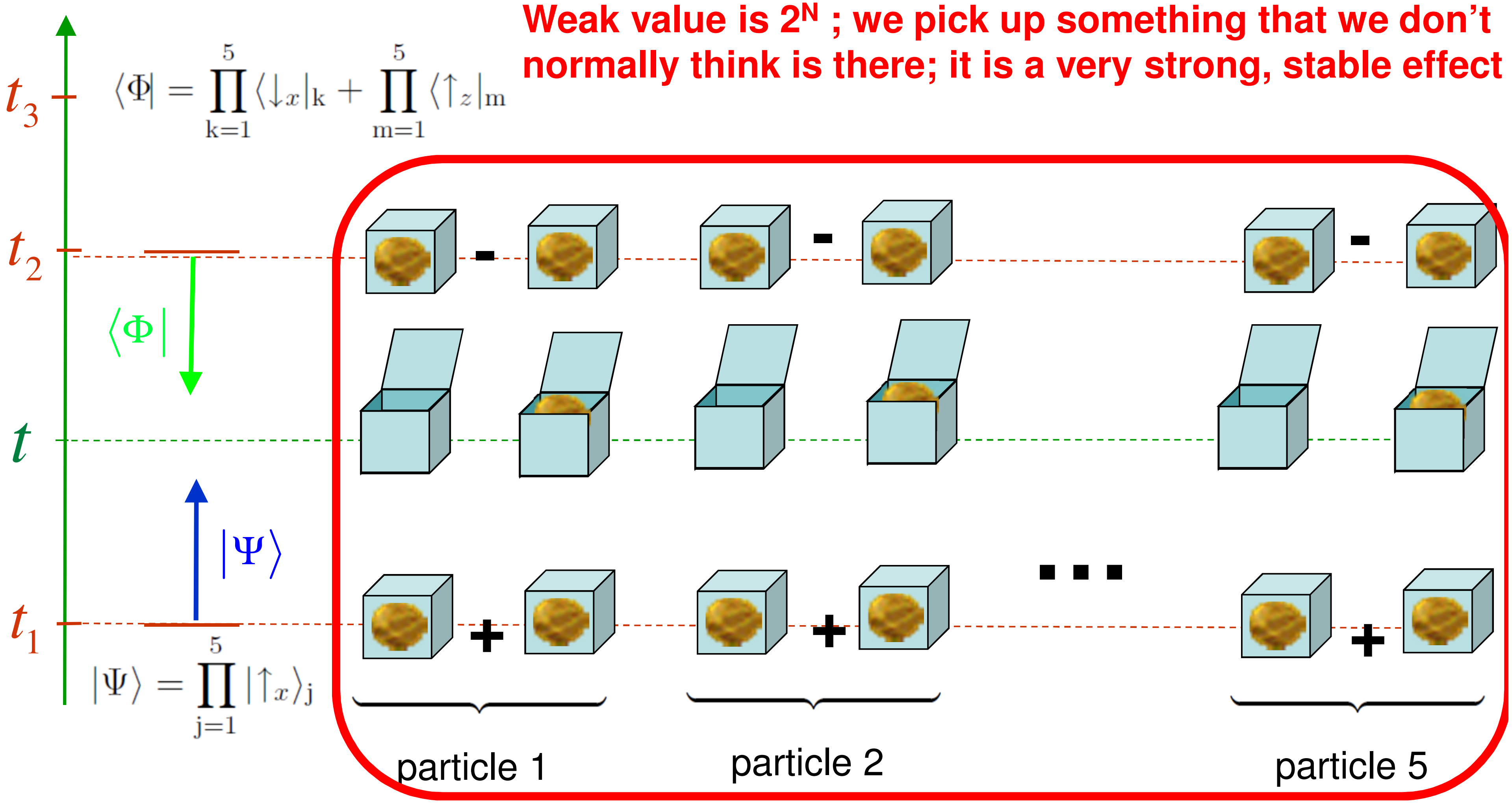


2.a Generalization of Atom of Emergence/Holism



2.a Generalization of Atom of Emergence/Holism

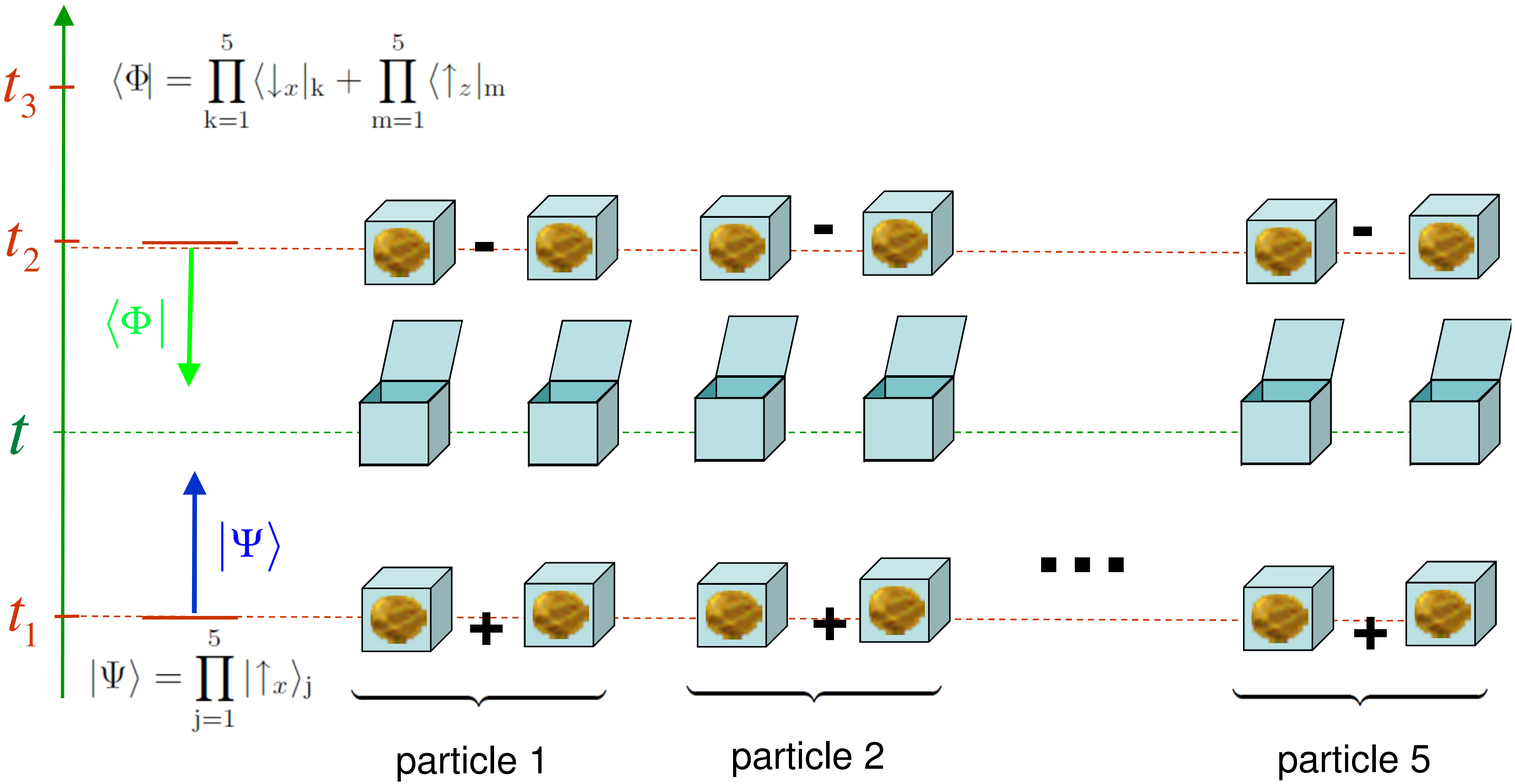
Weak value is 2^N ; we pick up something that we don't normally think is there; it is a very strong, stable effect



Only when strongly measure projection operator for all 5

$$\left\{1+\hat{\sigma}_z^1\right\}\left\{1+\hat{\sigma}_z^2\right\}\left\{1+\hat{\sigma}_z^3\right\}\left\{1+\hat{\sigma}_z^4\right\}\left\{1+\hat{\sigma}_z^5\right\}$$

2.a Generalization of Atom of Emergence/Holism



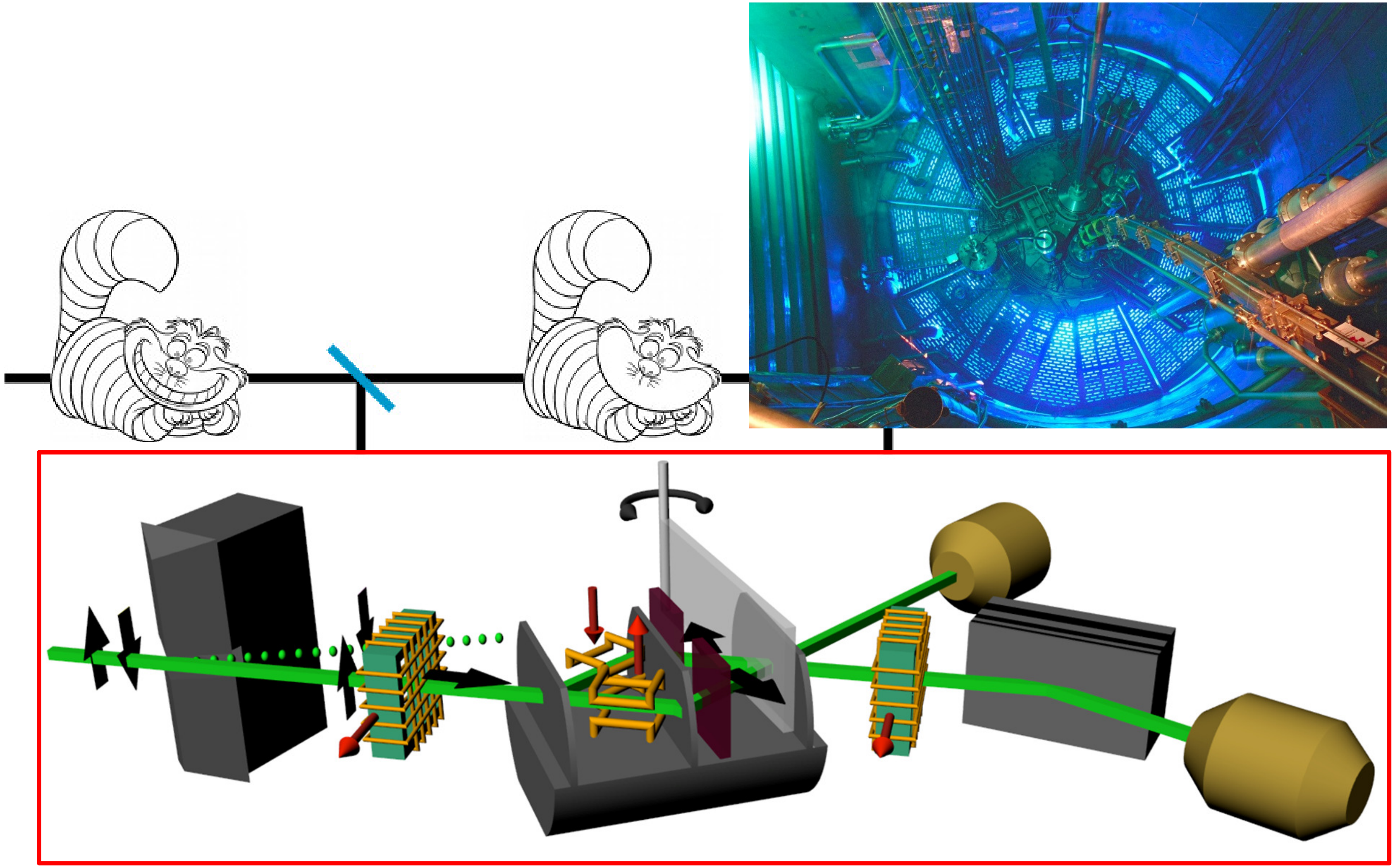
2.b Quantum Cheshire Cat

- **“Well I've often seen a cat without a grin," thought Alice “but a grin without a cat! It’s the most curious thing I ever saw in all my life”**

  - **As if you were “separating a particle from its properties”**
  - **what seemed to be a “whole inseparable system" can in fact be physically separated into distinct parts residing at different locations. This opens a new door on the relationship between wholes and parts**
  - **New computational/information resource**

Experiment: Denkmayr, Geppert, Sponar, Lemmel, Matzkin, JT, Hasegawa, ***Nature Comm* (2014)**
Theory: JT 2001 PhD thesis; Aharonov & Rohrlich 2005; Aharonov, Y., Popescu, S., Rohrlich, D., & Skrzypczyk, P. (2013). Quantum cheshire cats. *New Journal of Physics*, *15*(11), 113015.

2.b Quantum chesire cat–experimental verification

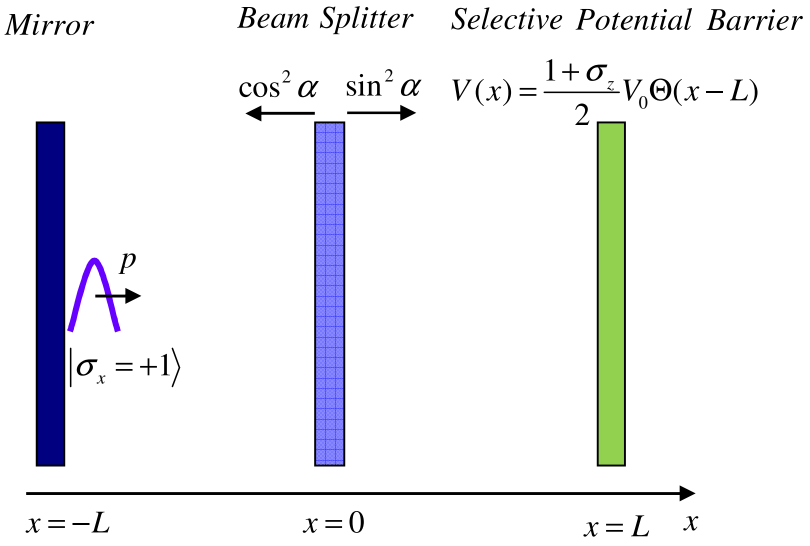


Experiment: Denkmayr, Geppert, Sponar, Lemmel, Matzkin, JT, Hasegawa, ***Nature Comm*** ***(2014)***
Theory: JT 2001 PhD thesis; Aharonov & Rohrlich 2005; Aharonov, Y., Popescu, S., Rohrlich, D., & Skrzypczyk, P. (2013). Quantum cheshire cats. *New Journal of Physics*, *15*(11), 113015.

2.b ...Meets Quantum Zeno Effect

- How can we understand this separation?
- Aharonov, Cohen and Popescu recently suggested (arXiv:1510.03087) to combine this effect with the Quantum Zeno effect.
- The latter enables to carefully monitor the time evolution of the “smile”, giving rise to a massless current of spin!
- Furthermore, this current leads to an observable effect on its remote owner.

2.b A Massless Current of Spin



The electron starts with momentum p, such that $\Delta p << p << \sqrt{2mV_0}$

Its state is given by $|\psi\rangle = |L\rangle|\sigma_x = +1\rangle$

Upon hitting the beam splitter:

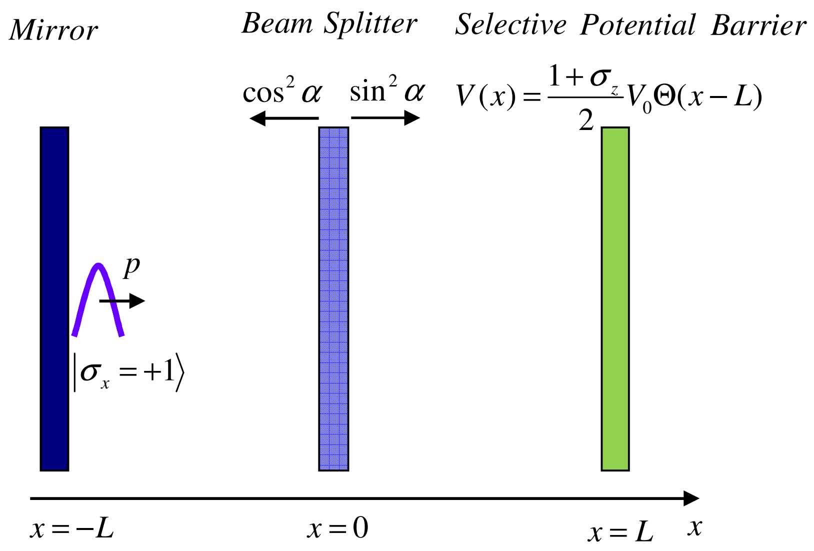
$$|L\rangle \to \cos\alpha|L\rangle + \sin\alpha|R\rangle, \qquad \alpha << 1$$
$$|R\rangle \to -\sin\alpha|L\rangle + \cos\alpha|R\rangle$$

Now there are two cases:

1. The electron has spin-z up and therefore feels the potential
2. The electron has spin-z down and therefore does not feel the potential

Y. Aharonov, E. Cohen, S. Popescu, arXiv:1510.03087

2.b A Massless Current of Spin



1. <u>The electron has spin-z up and therefore feels the potential</u>

After *n* period times

$$|\psi(n\tau)\rangle = \cos(n\alpha)|L\rangle|\sigma_z=+1\rangle + \sin(n\alpha)|R\rangle|\sigma_z=+1\rangle$$

Hence if $n\alpha \approx \frac{\pi}{2}$ the electron **would move to the right side**.

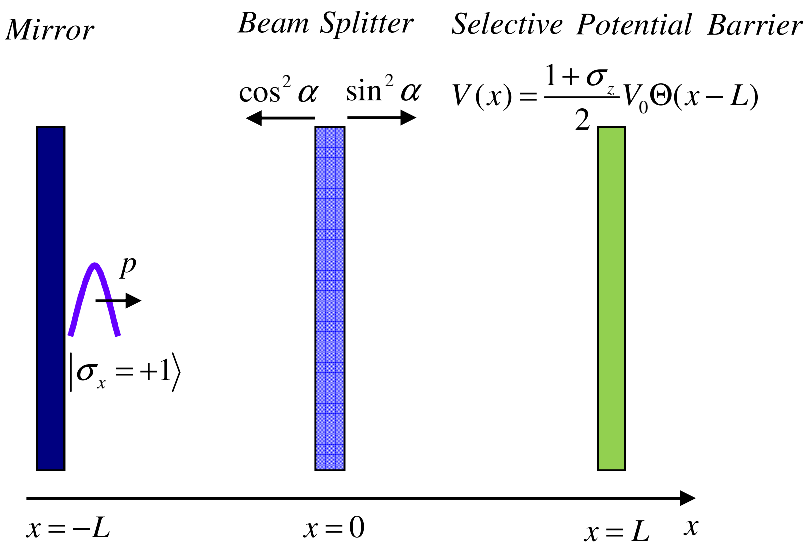
2. <u>Electron has spin-z down and therefore does not feel the potential</u>

After *n* period times the electron **would stay on the left** (the amplitudes did not sum up coherently).

Y. Aharonov, E. Cohen, S. Popescu, arXiv:1510.03087

2.b

A Massless Current of Spin

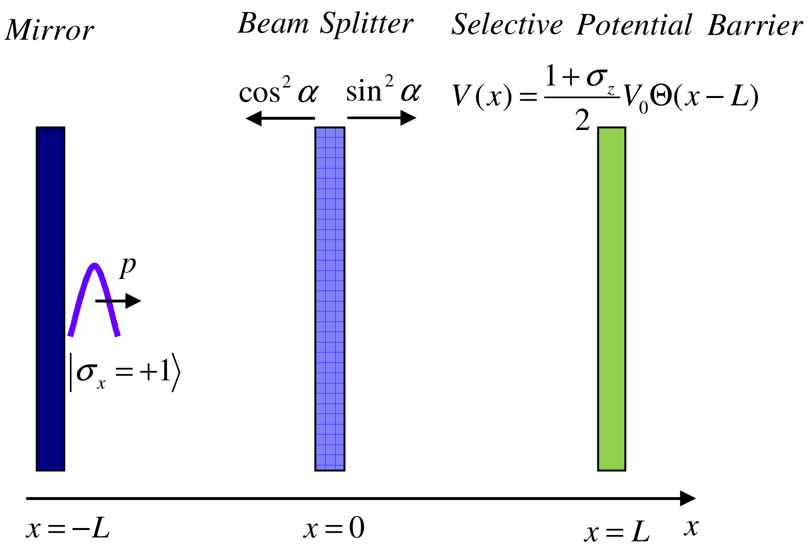


Therefore, if after *n* period times we find the electron on the left, we immediately conclude that $|\sigma_z=-1\rangle$.

But how can its spin change if it was all the time at the left side? **The electron's spin left its mass behind and traveled to the right side!**

Y. Aharonov, E. Cohen, S. Popescu, arXiv:1510.03087

2.b A Massless Current of Spin



The weak values tell the full story:

$\langle\sigma_z\rangle_w=-1$ ⇒ The spin along the z-direction was up at all times

$\langle\pi_R\sigma_x(n\tau)\rangle_w=\sin\alpha\sin(n\alpha),\ \langle\pi_L\sigma_x(n\tau)\rangle_w=\cos\alpha\cos(n\alpha)$ ⇒

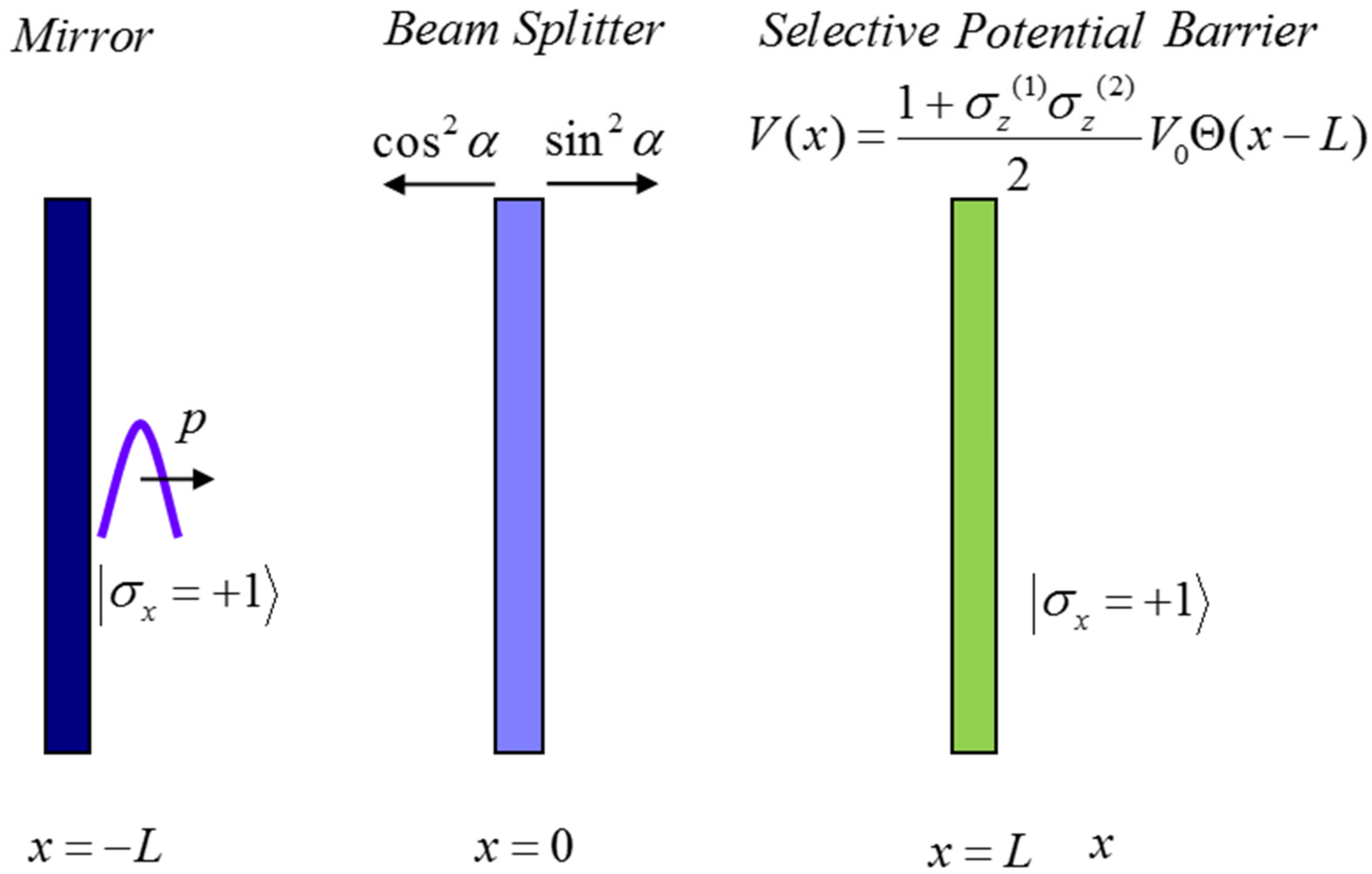
The spin along the x-direction moved from left to right, as indicated also by its time deri $\langle\pi_R\sigma_y(n\tau)\rangle_w=-i\sin\alpha\sin(n\alpha)$

Local current of massless spin can account for seemingly nonlocal effects!

Y. Aharonov, E. Cohen, S. Popescu, arXiv:1510.03087

Emergent Entanglement

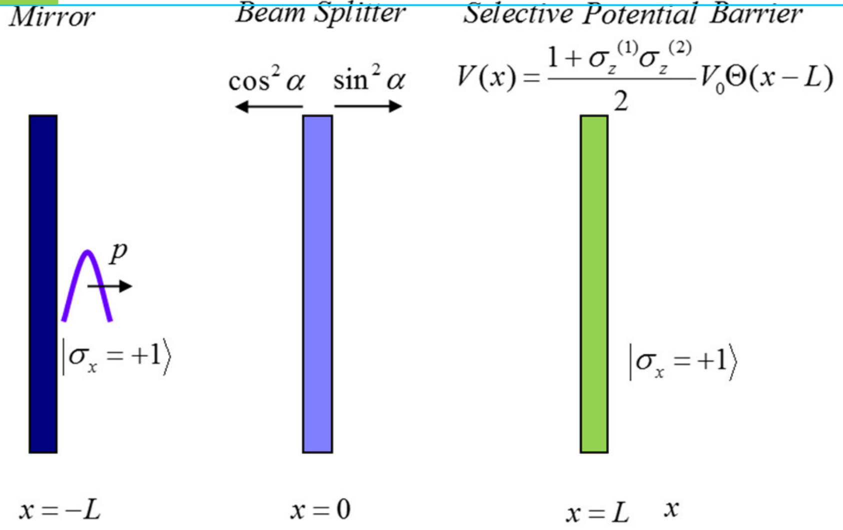
When the selective potential barrier has spin, some more surprises are expected!



Y. Aharonov, E. Cohen, S. Popescu, arXiv:1510.03087

2.b

Emergent Entanglement



A product state

$$|\psi\rangle = |L\rangle|\sigma_x^{(1)}=+1\rangle|\sigma_x^{(2)}=+1\rangle$$

If the spins are aligned, the right amplitude is coherently built

Otherwise, the particle stays on the left

Becomes Maximally entangled!

$$|\phi\rangle = |L\rangle\frac{|\sigma_z^{(1)}=+1\rangle|\sigma_z^{(2)}=-1\rangle+|\sigma_z^{(1)}=-1\rangle|\sigma_z^{(2)}=+1\rangle}{\sqrt{2}}$$

Here the particle and barrier become entangled by virtue of a massless current of entangled spins!

Time-Symmetric formulation of QM (TSQM)

To be useful and interesting, any re-formulation of QM should meet several criteria, for example:

YES

1) TSQM is consistent with standard QM

$$\langle\Psi_2|U_{t\to t_2} = \langle U^\dagger_{t\to t_2}\Psi_2| = \langle U_{t_2\to t}\Psi_2|$$

$$U^\dagger_{t\to t_2} = \left\{e^{-iH(t_2-t)}\right\}^\dagger = e^{iH(t_2-t)} = e^{-iH(t-t_2)} = U_{t_2\to t}$$

YES

2) TSQM brings out features in QM re ***emergence*** that were missed before (e.g. weak values)

3) TSQM stimulated discoveries in other fields re *emergence*

4) TSQM suggests generalizations re ***emergence***

3.b Weak Values and Contextuality

$$
\begin{array}{cccc}
\hat{\sigma}_x^1 & \hat{\sigma}_x^2 & \hat{\sigma}_x^1\hat{\sigma}_x^2 & =1 \\
\hat{\sigma}_y^2 & \hat{\sigma}_y^1 & \hat{\sigma}_y^1\hat{\sigma}_y^2 & =1 \\
\hat{\sigma}_x^1\hat{\sigma}_y^2 & \hat{\sigma}_x^2\hat{\sigma}_y^1 & \hat{\sigma}_z^1\hat{\sigma}_z^2 & =1 \\
=1 & =1 & =-1 &
\end{array}
$$

- **Novel proof that pre-and-post-selected QM is contextual**
- **Weak value signature that can be tested experimentally**
- **Conjectured that anomalous weak values constitute proofs of the incompatibility of quantum theory with non-contextual ontological models**

- JT, *Journal of Physics A*, 40 (2007) 9033-9066)

3.b Weak Values and Contextuality

Published in Phys. Rev. Lett. 113, 200401 (2014)

Anomalous Weak Values Are Proofs of Contextuality

Matthew F. Pusey*
Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, ON N2L 2Y5, Canada
(Dated: November 12, 2014)

The average result of a weak measurement of some observable A can, under post-selection of the measured quantum system, exceed the largest eigenvalue of A. The nature of weak measurements, as well as the presence of post-selection and hence possible contribution of measurement-disturbance, has led to a long-running debate about whether or not this is surprising. Here, it is shown that such "anomalous weak values" are non-classical in a precise sense: a sufficiently weak measurement of one constitutes a proof of contextuality. This clarifies, for example, which features must be present (and in an experiment, verified) to demonstrate an effect with no satisfying classical explanation.

doi:10.1038/nature13460

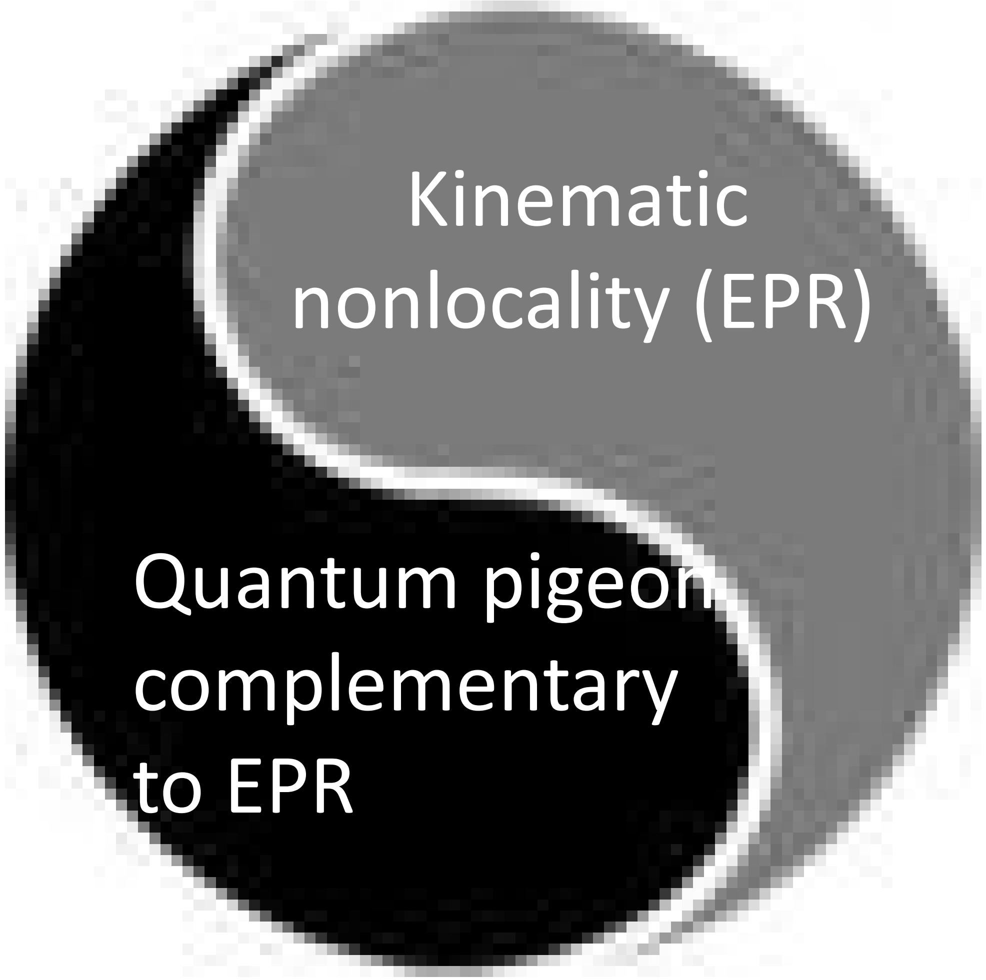
Contextuality supplies the 'magic' for quantum computation

Mark Howard[1,2], Joel Wallman[2], Victor Veitch[2,3] & Joseph Emerson[2]

Quantum computers promise dramatic advantages over their classical counterparts, but the source of the power in quantum computing has remained elusive. Here we prove a remarkable equivalence between the onset of contextuality and the possibility of universal quantum computation via 'magic state' distillation, which is the leading model for experimentally realizing a fault-tolerant quantum computer. This is a conceptually satisfying link, because contextuality, which precludes a simple 'hidden variable' model of quantum mechanics, provides one of the fundamental characterizations of uniquely quantum phenomena. Furthermore, this connection suggests a unifying paradigm for the resources of quantum information: the non-locality of quantum theory is a particular kind of contextuality, and non-locality is already known to be a critical resource for achieving advantages with quantum communication. In addition to clarifying these fundamental issues, this work advances the resource framework for quantum computation, which has a number of practical applications, such as characterizing the efficiency and trade-offs between distinct theoretical and experimental schemes for achieving robust quantum computation, and putting bounds on the overhead cost for the classical simulation of quantum algorithms.

M Waegell, JT "Contextuality, Pigeonholes, Cheshire Cats, Mean Kings, and Weak Values," arXiv: 1505.00098

3.d Quantum pigeonhole principle & nature of quantum correlations



In a pre- and post-selected scenario, you can put as many pigeons as you want in only two pigeonholes and guarantee that no two pigeons are in the same pigeonhole.

3.d Quantum pigeonhole principle & nature of quantum correlations

- Consider three particles and two boxes, denoted L (left) and R (right).
- To start our experiment, we prepare each particle in a superposition of being in the two boxes,

$$|+\rangle = \frac{1}{\sqrt{2}}\Big(|L\rangle + |R\rangle\Big).$$

The overall state of the three particles is therefore

$$|\Psi\rangle = |+\rangle_1|+\rangle_2|+\rangle_3.$$

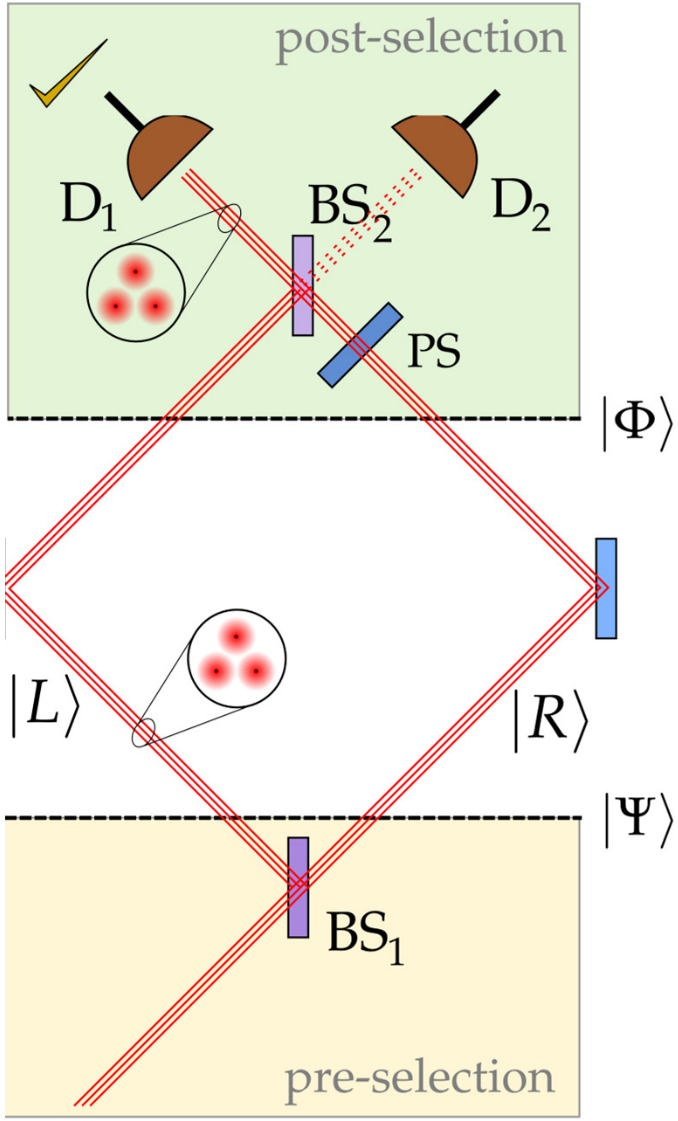
- Now, it is obvious that in this state any two particles have non-zero probability to be found in the same box. We want however to show that there are instances in which we can guarantee that no two particles are together; we cannot arrange that to happen in every instance, but, crucially, there are instances like that.
- To find those instances we subject each particle to a second measurement: we measure whether each particle is in the state:

$$|+i\rangle = \frac{1}{\sqrt{2}}\Big(|L\rangle + i|R\rangle\Big) \;\; or \;\; |-i\rangle = \frac{1}{\sqrt{2}}\Big(|L\rangle - i|R\rangle\Big)$$

(these are two orthogonal states, so there is an operator whose eigenstates they are - we measure that operator).

- The cases we are interested in are those in which all particles are found in $|+i\rangle$, i.e. the final state

$$|\Phi\rangle = |+i\rangle_1|+i\rangle_2|+i\rangle_3.$$



Aharonov, Colombo, Popescu, Sabadini, JT; arXiv1407.3194

3.d Quantum pigeonhole principle & nature of quantum correlations

- Importantly, neither the initial state nor the finally selected state contain any correlations between the position of the particles. Furthermore, both the preparation and the post-selection are done independently, acting on each particle separately.

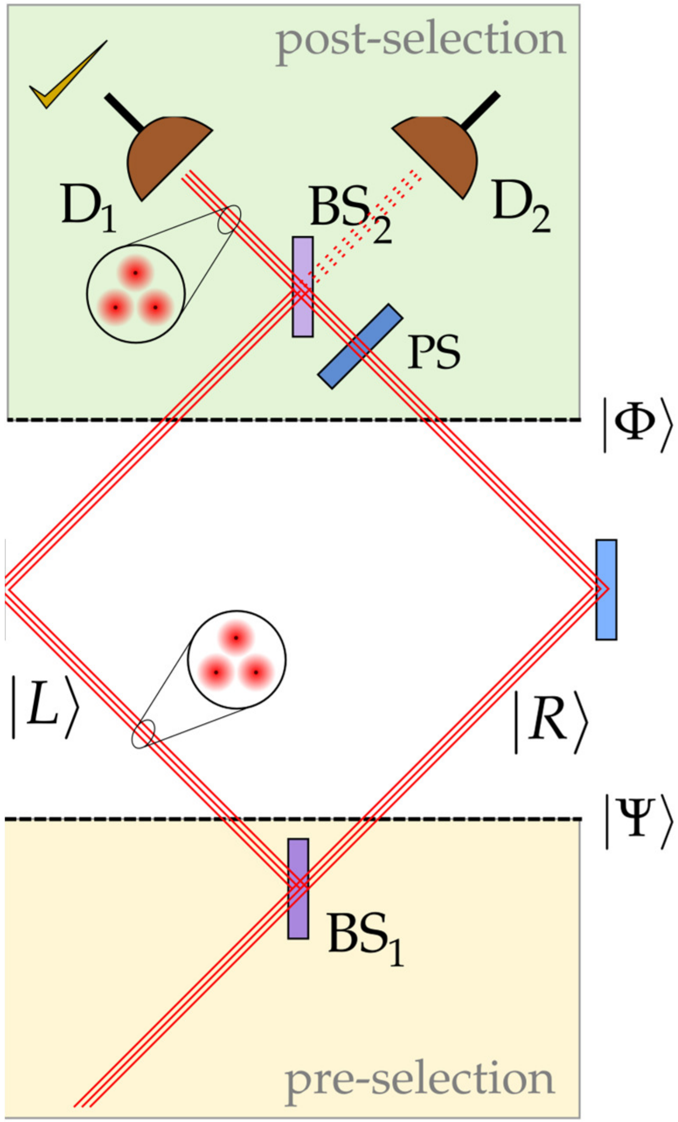
- Let us now check whether two of the particles are in the same box. Since the state is symmetric, we could focus on particles 1 and 2 without any loss of generality - any result obtained for this pair applies to every other pair.

- Particles 1 and 2 being in the same box means the state being in the subspace spanned by $|L\rangle_1|L\rangle_2$ and $|R\rangle_1|R\rangle_2$; being in different boxes corresponds to the complementary subspace, spanned by $|L\rangle_1|R\rangle_2$ and $|R\rangle_1|L\rangle_2$. The projectors corresponding to these subspaces are

$$\begin{aligned}\Pi_{1,2}^{same} &= \Pi_{1,2}^{LL} + \Pi_{1,2}^{RR}\\ \Pi_{1,2}^{diff} &= \Pi_{1,2}^{LR} + \Pi_{1,2}^{RL}\end{aligned}$$

where

$$\begin{aligned}\Pi_{1,2}^{LL} &= |L\rangle_1|L\rangle_2\,{}_1\langle L|_2\langle L|, \quad \Pi_{1,2}^{RR} = |R\rangle_1|R\rangle_2\,{}_1\langle R|_2\langle R|,\\ \Pi_{1,2}^{LR} &= |L\rangle_1|R\rangle_2\,{}_1\langle R|_2\langle L|, \quad \Pi_{1,2}^{RL} = |R\rangle_1|L\rangle_2\,{}_1\langle L|_2\langle R|.\end{aligned}$$



Aharonov, Colombo, Popescu, Sabadini, JT; arXiv1407.3194

Quantum pigeonhole principle & nature of quantum correlations

- On the initial state alone, the probabilities to find particles 1 and 2 in the same box and in different boxes are both 50%. On the other hand, given the results of the final measurements, we always find particles 1 and 2 in different boxes. Indeed, suppose that at the intermediate time we find the particles in the same box. The wavefunction then collapses (up to normalisation) to

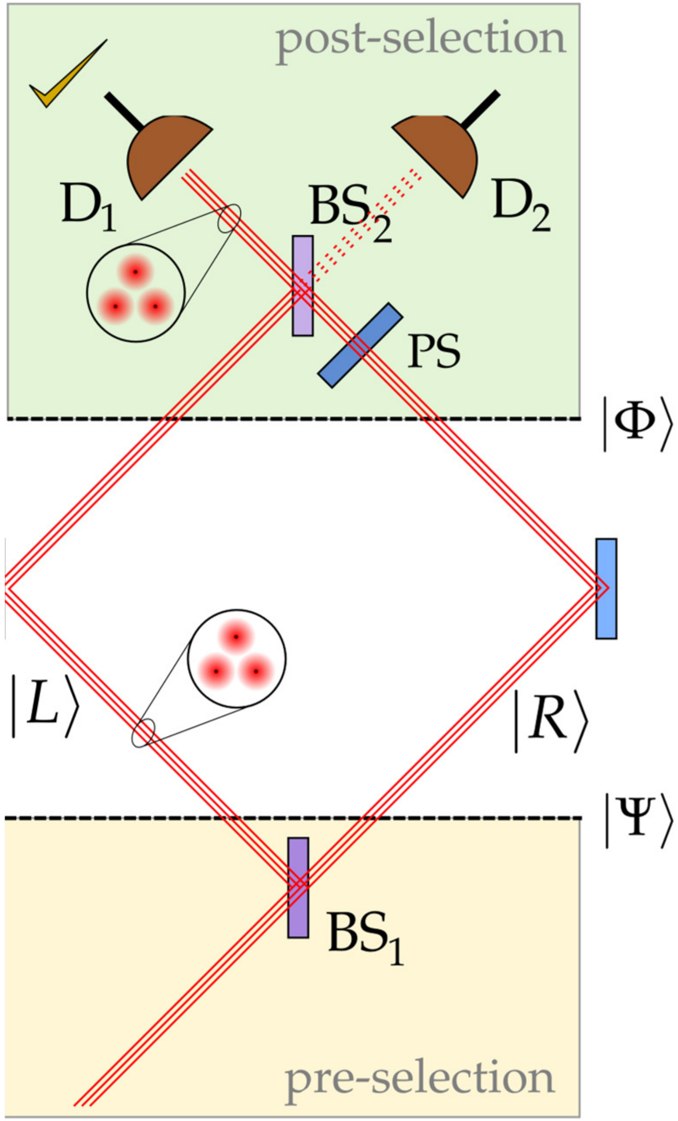
$$|\Psi'\rangle = \Pi_{1,2}^{same}|\Psi\rangle = \frac{1}{2}(|L\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2)|+\rangle_3$$

  which is orthogonal to the post-selected state i.e.

$$\langle\Phi|\Pi_{1,2}^{same}|\Psi\rangle = 0.$$

- Hence in this case the final measurements cannot find the particles in the state $|\Phi\rangle$. Therefore the only cases in which the final measurement can find the particles in the state $|\Phi\rangle$ are those in which the intermediate measurement found that particles 1 and 2 are in different boxes.

- Crucially, as noted before, the state is symmetric under permutation, hence what is true for particles 1 and 2 is true for all pairs. In other words, given the above pre- and post-selection, we have three particles in two boxes, yet no two particles can be found in the same box - our quantum pigeonhole principle.



Aharonov, Colombo, Popescu, Sabadini, JT; arXiv1407.3194

3.b

Kochen-Specker contextuality can be localized and observed through weak measurements

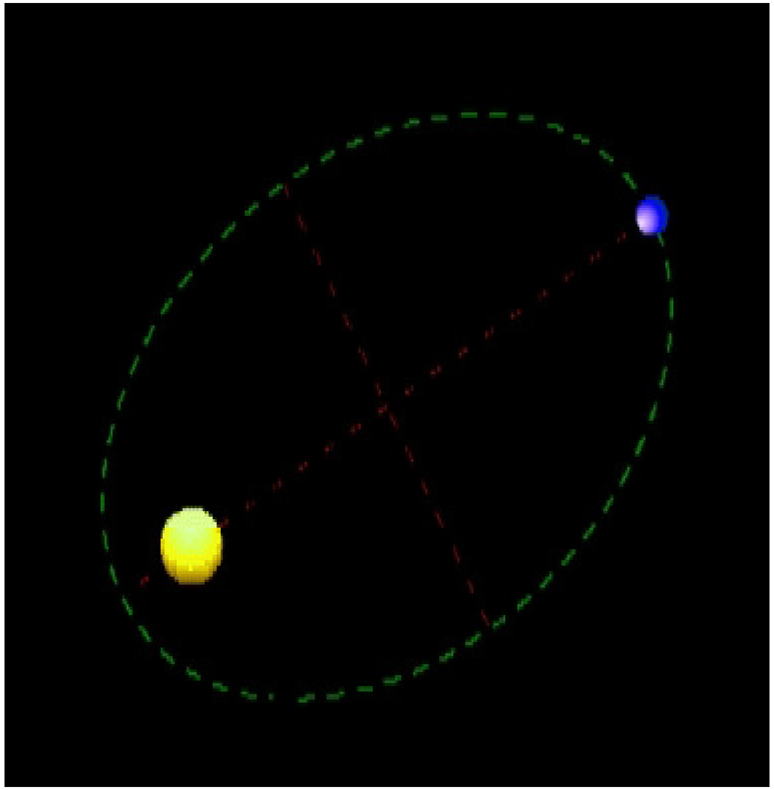
- using pre- and post-selected states (PPS) along with many existing proofs of the Kochen-Specker (KS) theorem, it is possible to localize the violation of noncontextuality to specific observables where it can be probed using weak measurements.

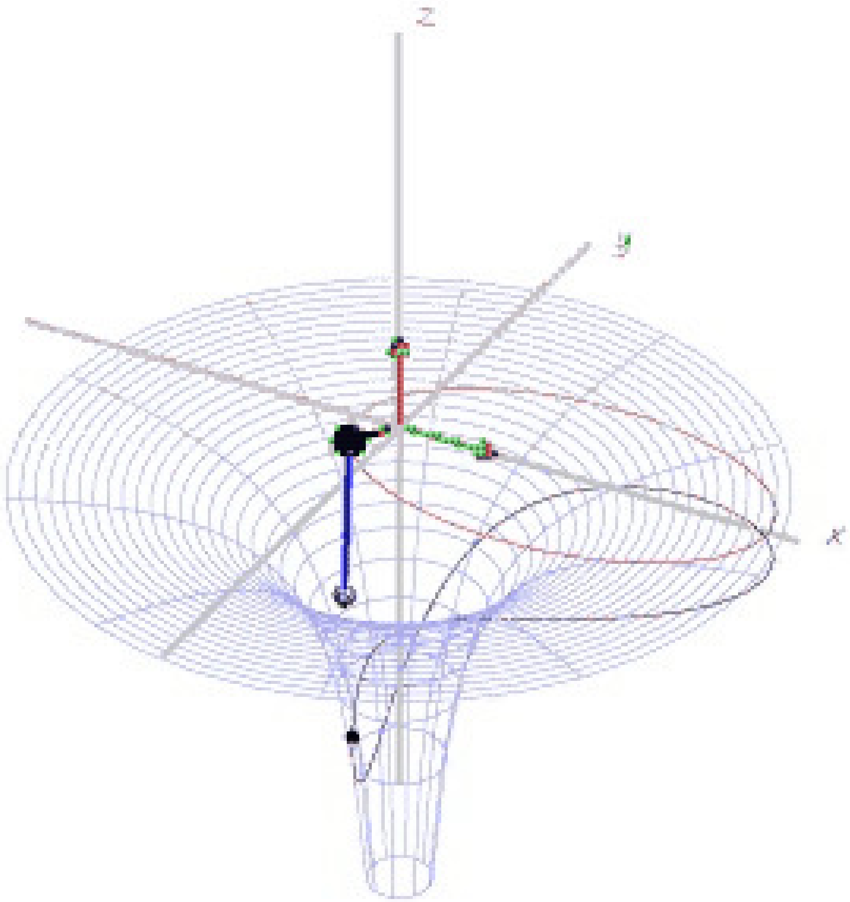
- Several important examples are discussed in detail, and a framework for a more general set of experimental tests based on known proofs of the KS theorem is given.

- The underlying ontological models that are used in these arguments are explored detail, and connections are made to PPS paradoxes such as the 3-box paradox, the quantum Cheshire Cat, and the quantum pigeonhole principle, as well as to the Mean King's problem.

"Kochen-Specker contextuality can be localized and observed through weak measurements," Mordecai Waegell , JT, arXiv:1505.00098

3.c Newton’s laws: local equations of motion



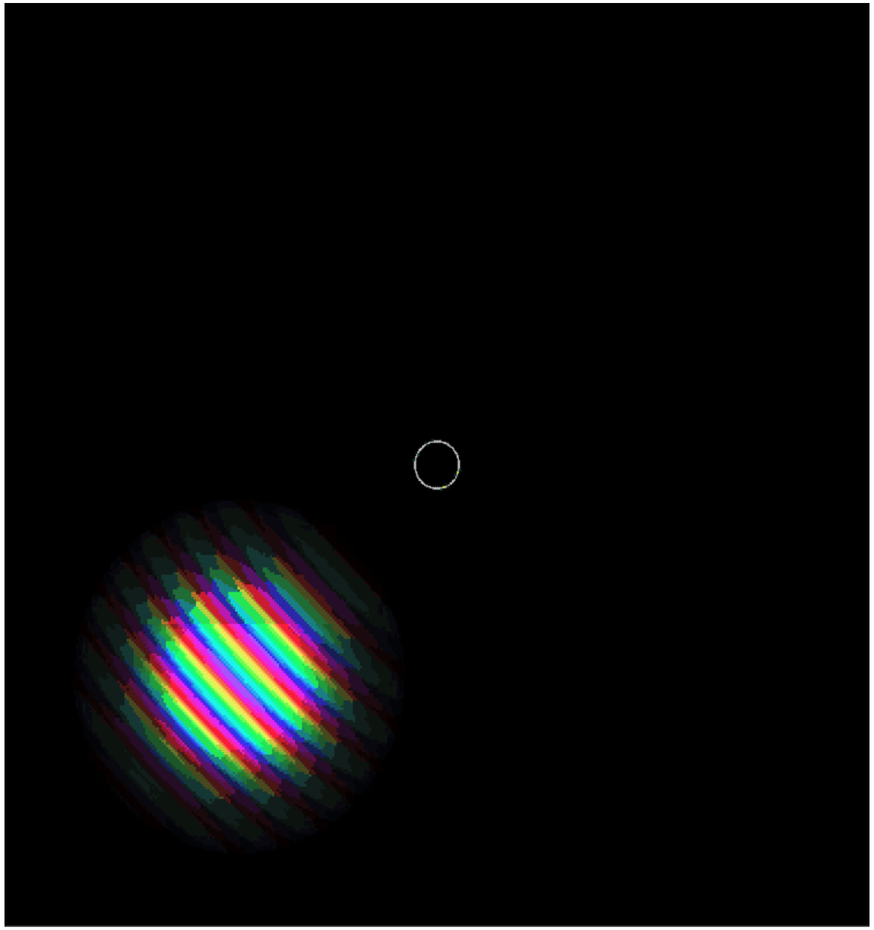


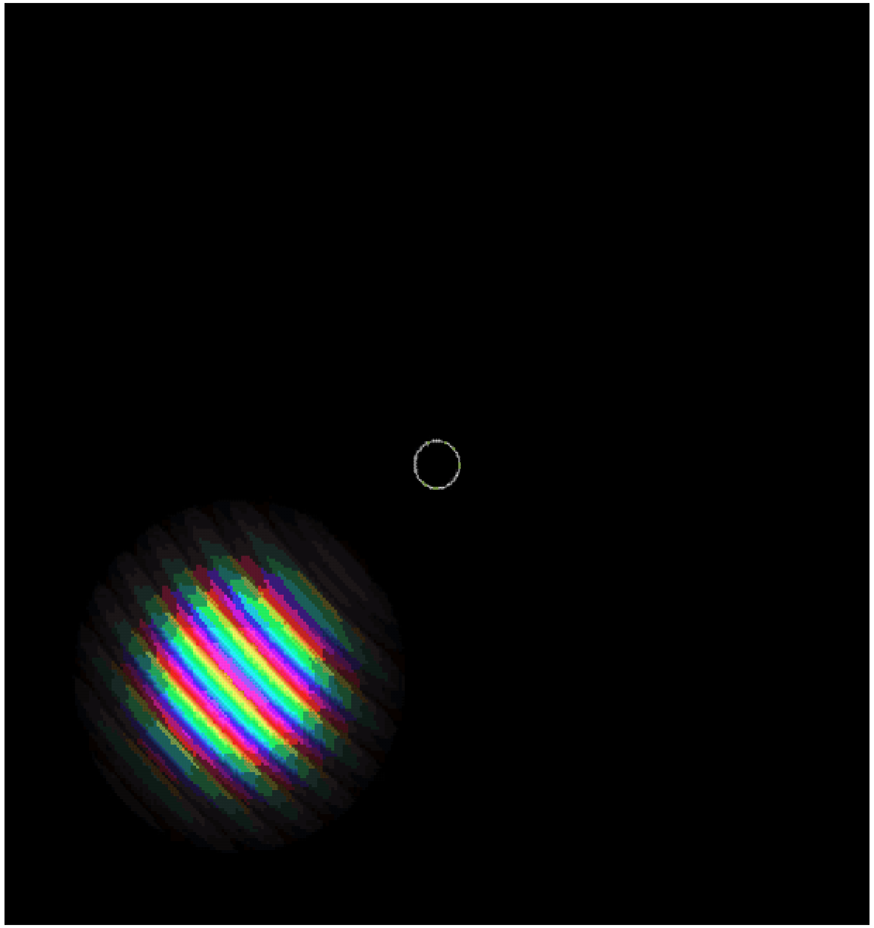
In classical physics, the force has to act in the same place where the particle is

3.c Nonlocality: kinematic vs. dynamic

Bell-inequality violations follow from the Hilbert-space structure of quantum mechanics; they are purely kinematic

Aharonov-Bohm effect demonstrates dynamical non-locality , i.e. in the quantum equations of motion

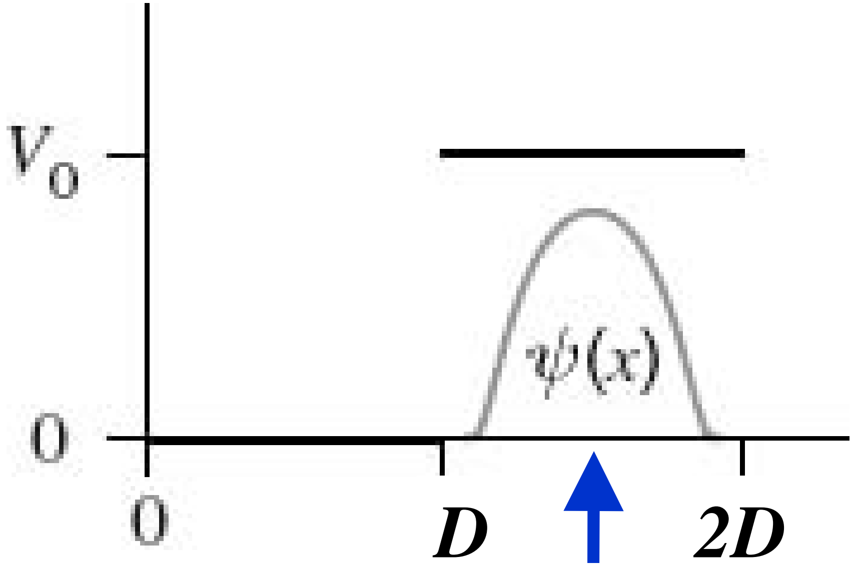




3.c Quantum interference, modular variables & weak measurements

- using $H = \frac{p^2}{2m} + V(x)$ and $e^{\frac{i}{\hbar}\hat{p}D} V(x) e^{-\frac{i}{\hbar}\hat{p}D} = V(x+D)$, we find *non-local* Heisenberg equations of motion for modular variables:

$$\frac{d}{dt} e^{\frac{i}{\hbar}\hat{p}D} = \frac{i}{\hbar}[H, e^{\frac{i}{\hbar}\hat{p}D}] = \frac{i}{\hbar}[V(x) - V(x+D)] e^{\frac{i}{\hbar}\hat{p}D}$$



$e^{\frac{i}{\hbar}\hat{p}D}$ changes even if $\frac{\partial V}{\partial x} = 0$

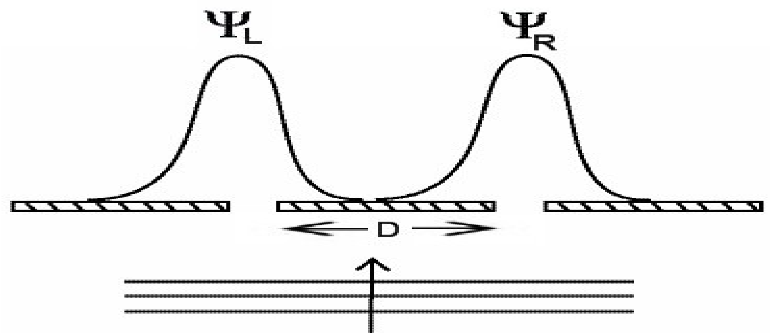
- no classical counterpart:

$$\frac{df(p)}{dt} = \{f(p), H\}_{PB}$$
$$= -\frac{\partial f}{\partial p}\frac{\partial H}{\partial x} + \underbrace{\frac{\partial f}{\partial x}}_{=0}\frac{\partial H}{\partial p} = 0$$

i.e. $f(p)$ changes only if $\frac{\partial V}{\partial x} \neq 0$ at the particle's location.

JT et al, New Jrnl Physics 12 (2010) 013023; see experiment Spence, Parks arXiv:1010.3289, ***Foundations of Physics***, 2011

Quantum interference, modular variables & weak measurements



- interference depends on relative phase α between different lumps: $\Psi_\alpha = \psi_\mathrm{L} + e^{i\alpha}\psi_\mathrm{R}$

- All moments of position and momentum are independent of the relative phase α. This characterizes all interference phenomenon:

  - Operators that *are* sensitive to the relative phase are exponentials of the position and momentum. These operators translate the different "lumps" so that they overlap:

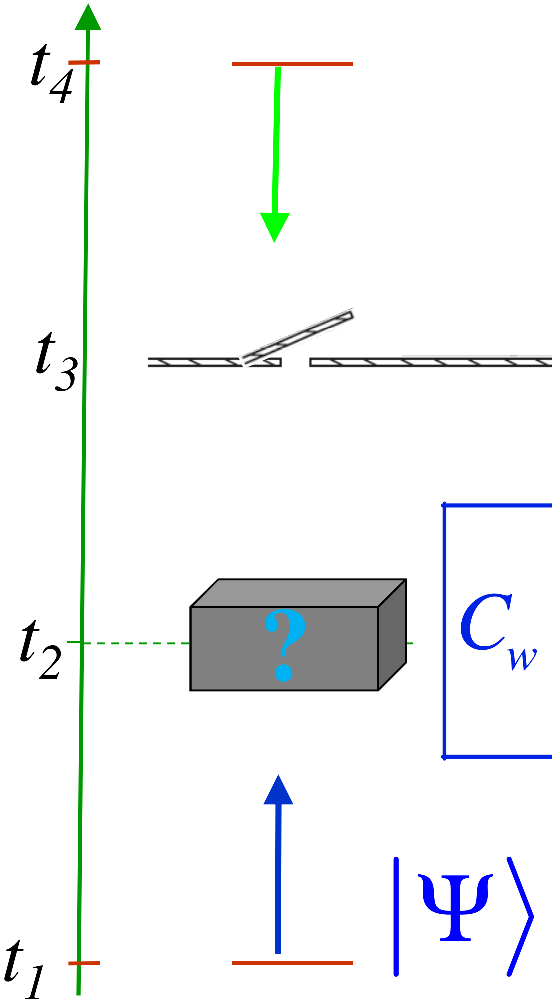
$$\exp\{-\tfrac{i}{\hbar}\hat{p}D\}\psi_\mathrm{R}(x) \to \psi_\mathrm{R}(x-D) \text{ overlaps } \psi_\mathrm{L}$$

$$\langle \Psi_\alpha \mid \exp\{i\hat{p}D/h\} \mid \Psi_\alpha\rangle = e^{-i\alpha}/2$$

- Replace p with $p - \frac{nh}{D}$, then $e^{\frac{i}{\hbar}\hat{p}D} \to e^{\frac{iD}{\hbar}\frac{nh}{D}} = e^{in2\pi} = 1$ - no change: $\langle e^{\frac{i}{\hbar}\hat{p}D}\rangle$ yields information about $p \bmod \frac{h}{D} \equiv p_\mathrm{mod}$ $(0 \le p_\mathrm{mod} \le \frac{h}{D})$

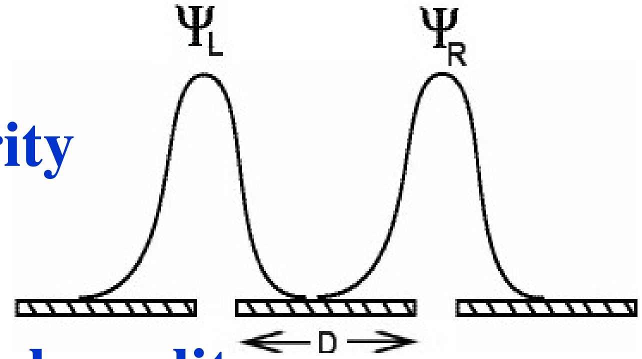
JT, Aharonov, Casher, Kaufherr, Nussino, *New Jrnl Physics* **12** (2010) 013023

3.c Having your cake & eating it-measuring nonlocal interactions w/o violating causality



Post-select an eigenstate of parity

$\frac{1}{\sqrt{2}}\{|\psi_L\rangle + |\psi_R\rangle\}$



Open or close slit

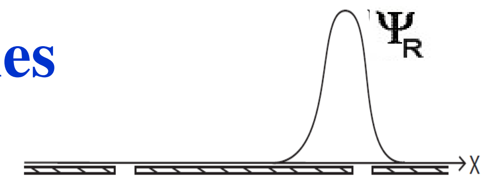
$$C_w \equiv \frac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}$$

Weakly measure modular variable exp(ipD//h)

(use parity for special case of $\psi_R + \psi_L$ and $\psi_R - \psi_L$)

$|\Psi\rangle$

Pre-select all particles at right slit $|\psi_R\rangle$



JT et al,New Jrnl Physics 12 (2010) 013023;
see experiment Spence, Parks arXiv:1010.3289, ***Foundations of Physics***, 2011

3.c

Schrödinger vs Heisenberg

- The Heisenberg picture leads us to a physical explanation for the different behavior of a *single* particle when the distant slit is open or closed

- instead of having a quantum wave that passes through all slits, we have a localized particle with *non-local* interactions with the other slits

Conclusions

- if your only tool is a hammer, then you tend to treat everything as if it were a nail
- To grasp the world more fully by grasping it gently

See Quantum.chapman.edu

Members Include:

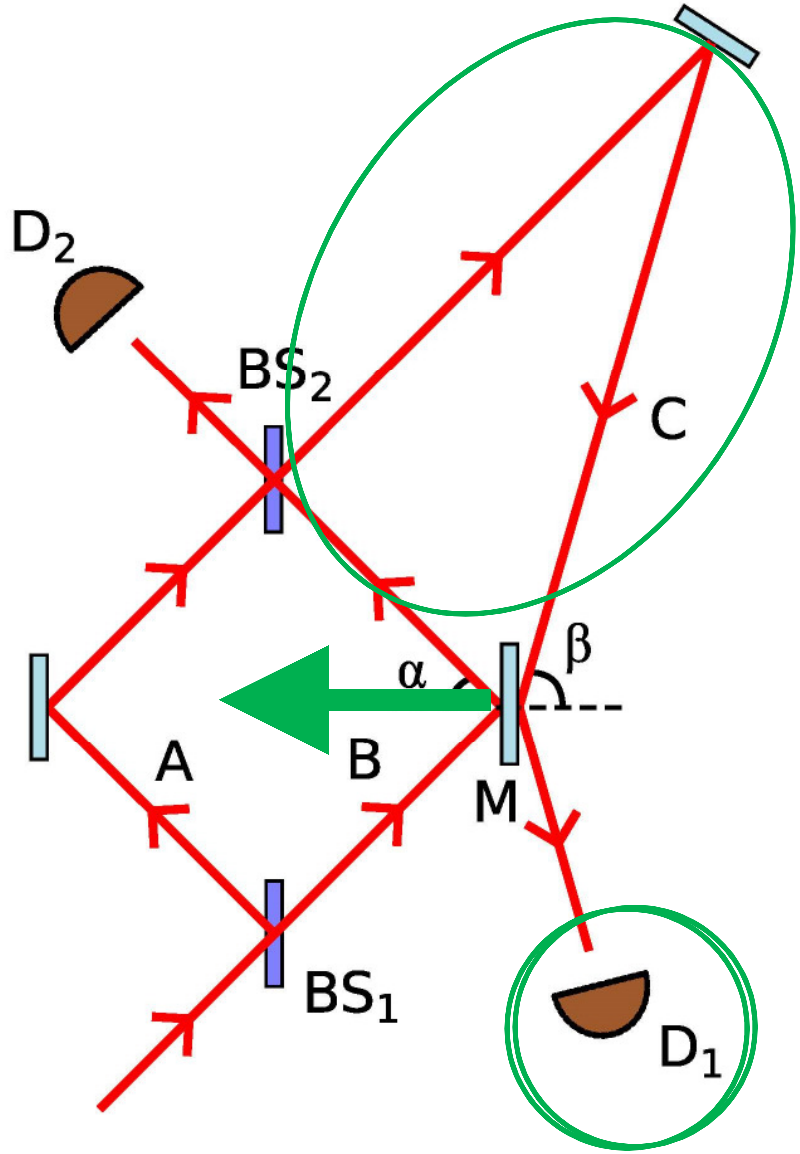
3.c Classical limit of quantum optics: not what it seems at first sight

- The reduction of quantum optics to wave optics has been considered to be relatively simple. It is not so.
- The classical limit of quantum optics is dramatically more involved and requires a fundamental revision of our intuitions.
- The revised intuitions can serve as a guide to finding novel quantum effects.

Y Aharonov, A Botero, S Nussinov, S Popescu, JT, L Vaidman, ***New Journal of Physics*** **15** 093006, 3 Sept 2013

3.c Classical limit of quantum optics: not what it seems at first sight

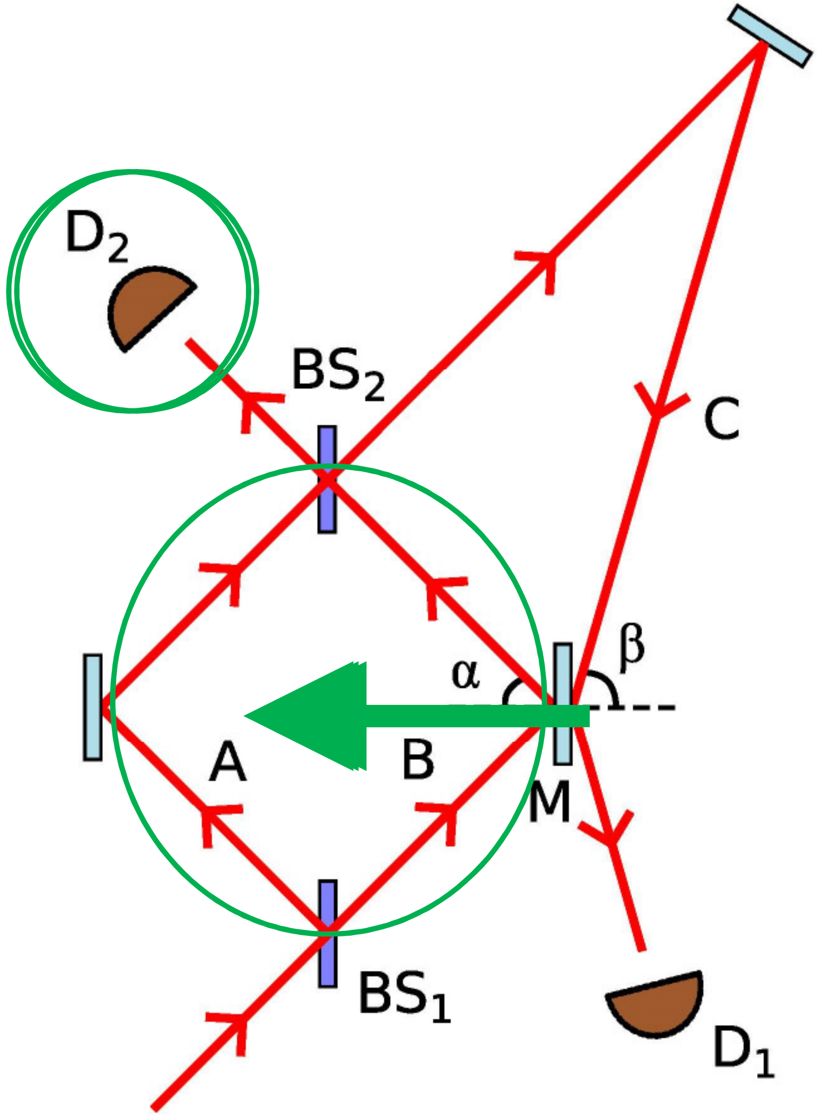
- Quantum and classical calculations lead to same result
- The issue however is with the story each theory has to tell.
- Although the external beam has a shallower incidence angle than the inside beam, its intensity is much higher and the momentum kick given by it is larger
- Classically, the external beam plays the central role - one would be tempted to assume that quantum mechanically the photons that constitute this beam are the ones responsible for the inward push.
- Remarkably, this is not so



Y Aharonov, A Botero, S Nussinov, S Popescu, JT, L Vaidman, ***New Journal of Physics*** **15** 093006, 3 Sept 2013

3.c Classical limit of quantum optics: not what it seems at first sight

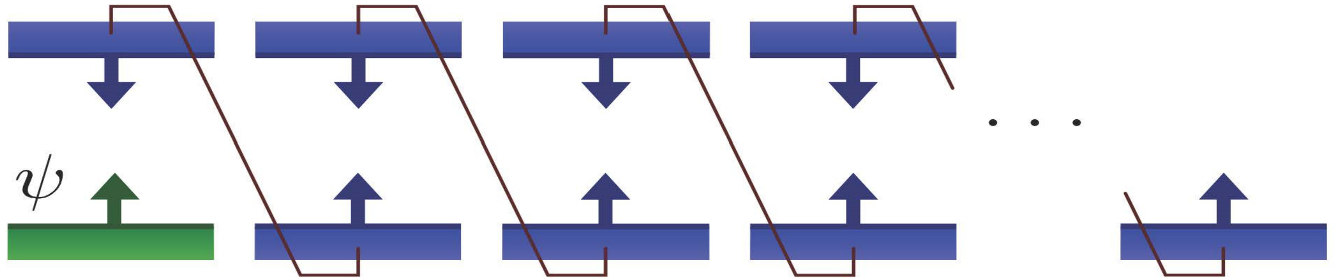
- Quantum and classical calculations lead to same result
- The issue however is with the story each theory has to tell.
- Although the external beam has a shallower incidence angle than the inside beam, its intensity is much higher and the momentum kick given by it is larger
- Classically, the external beam plays the central role - one would be tempted to assume that quantum mechanically the photons that constitute this beam are the ones responsible for the inward push.
- Remarkably, this is not so
- Photons at D_2!



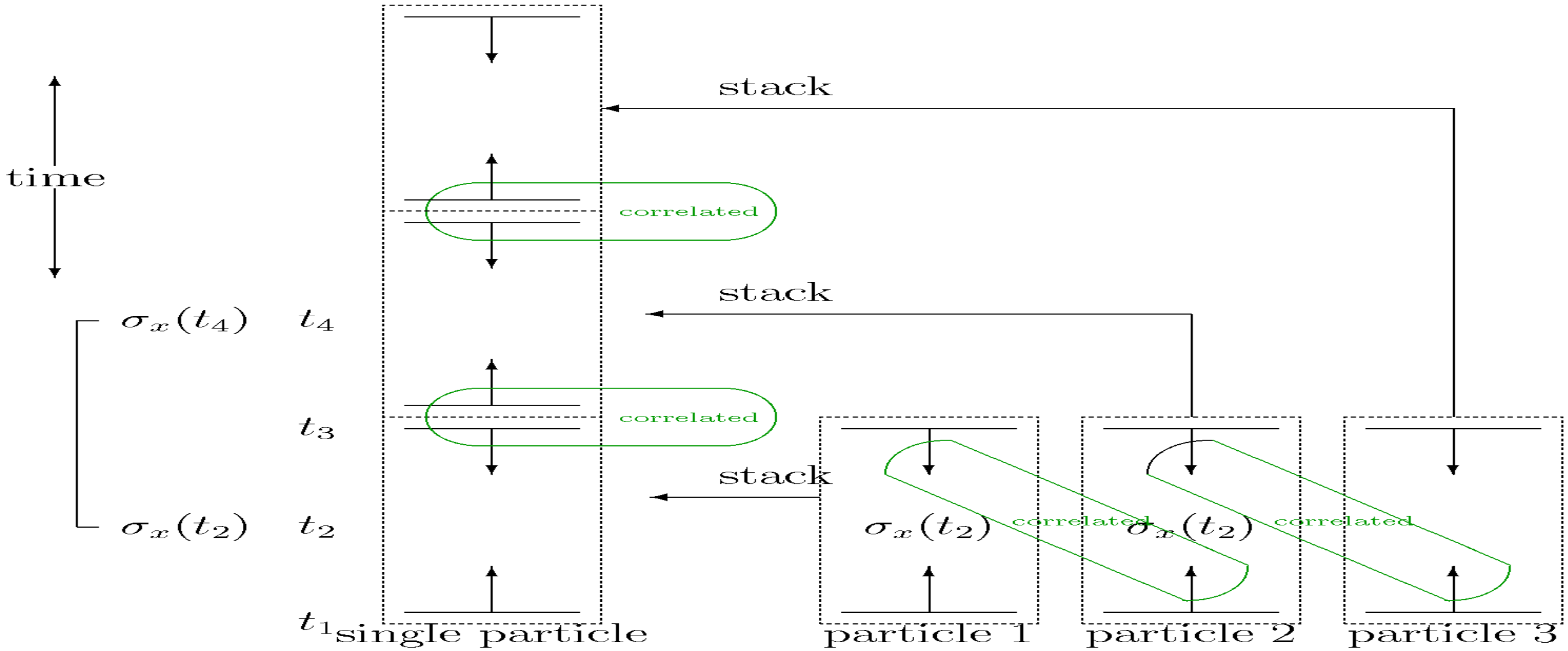
Y Aharonov, A Botero, S Nussinov, S Popescu, JT, L Vaidman, ***New Journal of Physics*** **15** 093006, 3 Sept 2013

4.a QM Generalization: Each moment of time a new universe

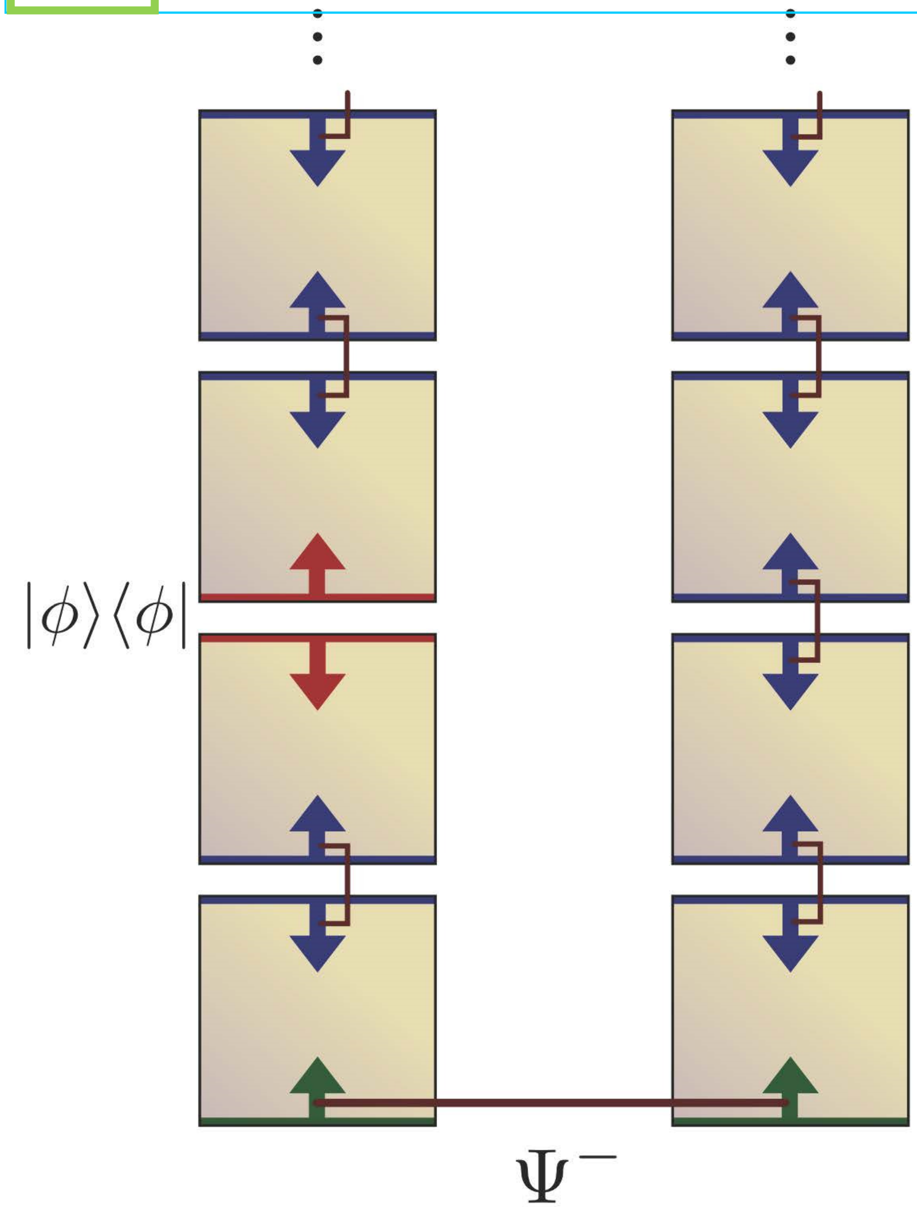
- New ability to obtain a post-selected state of one particle that is completely correlated to the pre-selected state of a second particle:



- stack N particles on top of another along the time axis:



4.a QM Generalization: Each moment of time a new universe



Two entangled spin 1/2 particles. Entanglement characterizes solely time t_0 where entanglement is produced. All other times are characterized by trivial time evolution, i.e. maximal entanglement between subsequent moments of time; there is however no entanglement between the particles associated to these times. Alice's measurement disentangles the time moments of her particle but have no effect on Bob's particle.

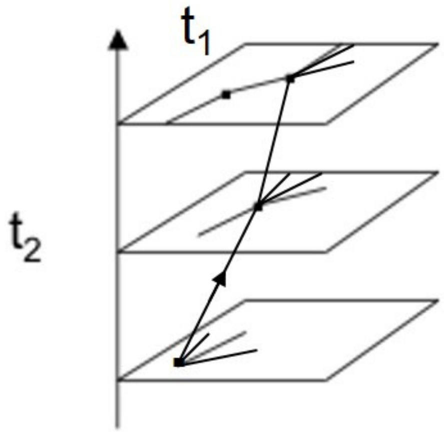
- **"Collapse" does not necessarily imply arrow of time at microscopic level**

Aharonov, JT, 2010, *Visions of Discovery*; Aharonov, Popescu, JT, arXiv:1305.1615

Aharonov, Popescu, JT, Vaidman, *Phys Rev A* 79, 052110 (May 1, 2009)

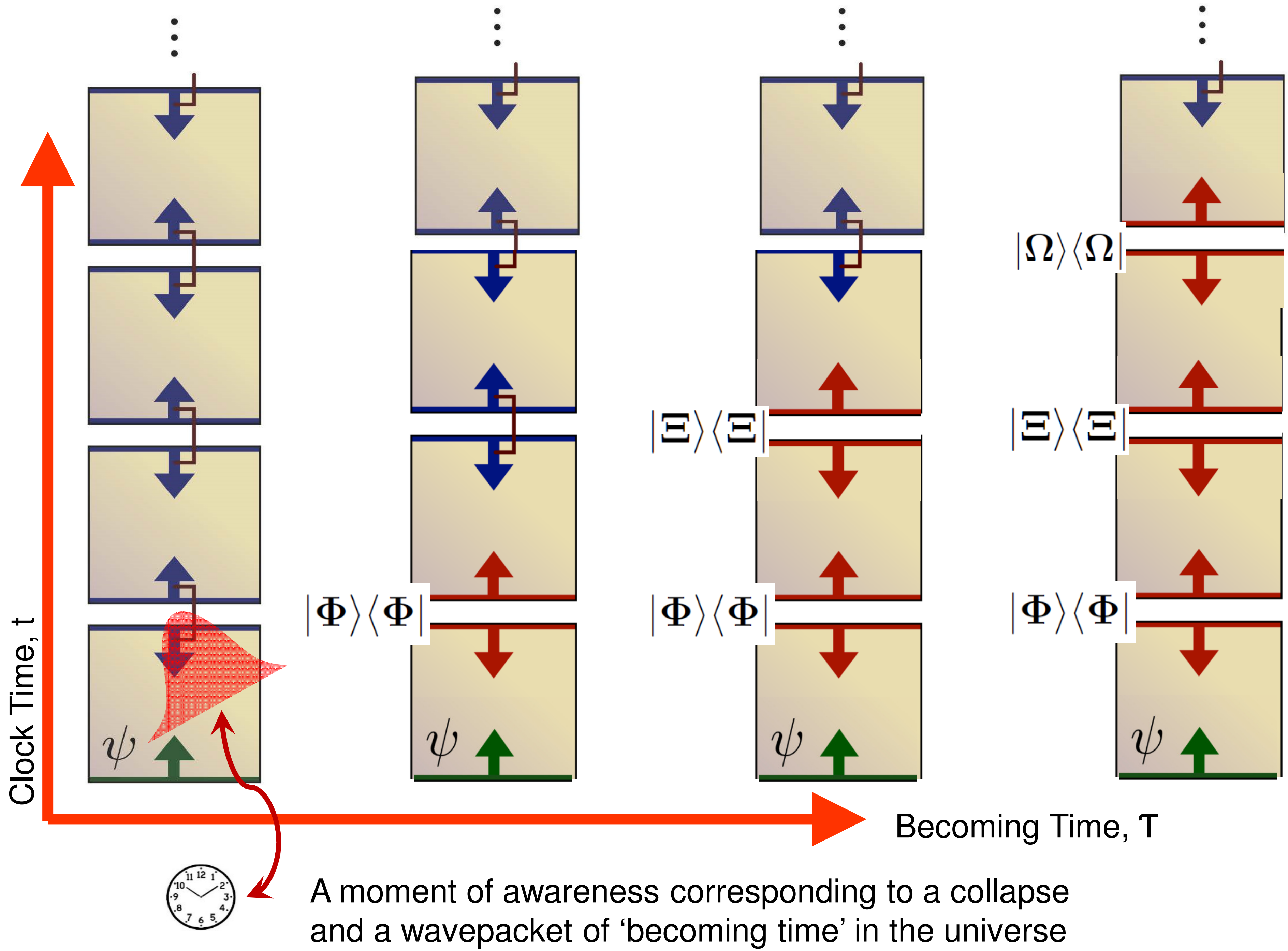
4.a Each moment of time is a new universe

“block universe on steroids”
David Albert



Anthony’s URB v ER

Aharonov, JT, 2010, *Visions of Discovery*; Aharonov, Popescu, JT, arXiv:1305.1615
Aharonov, Popescu, JT, Vaidman, *Phys Rev A* 79, 052110 (May 1, 2009)



Clock Time, t
Becoming Time, T
$|\Phi\rangle\langle\Phi|$
$|\Xi\rangle\langle\Xi|$
$|\Omega\rangle\langle\Omega|$
ψ
A moment of awareness corresponding to a collapse and a wavepacket of 'becoming time' in the universe

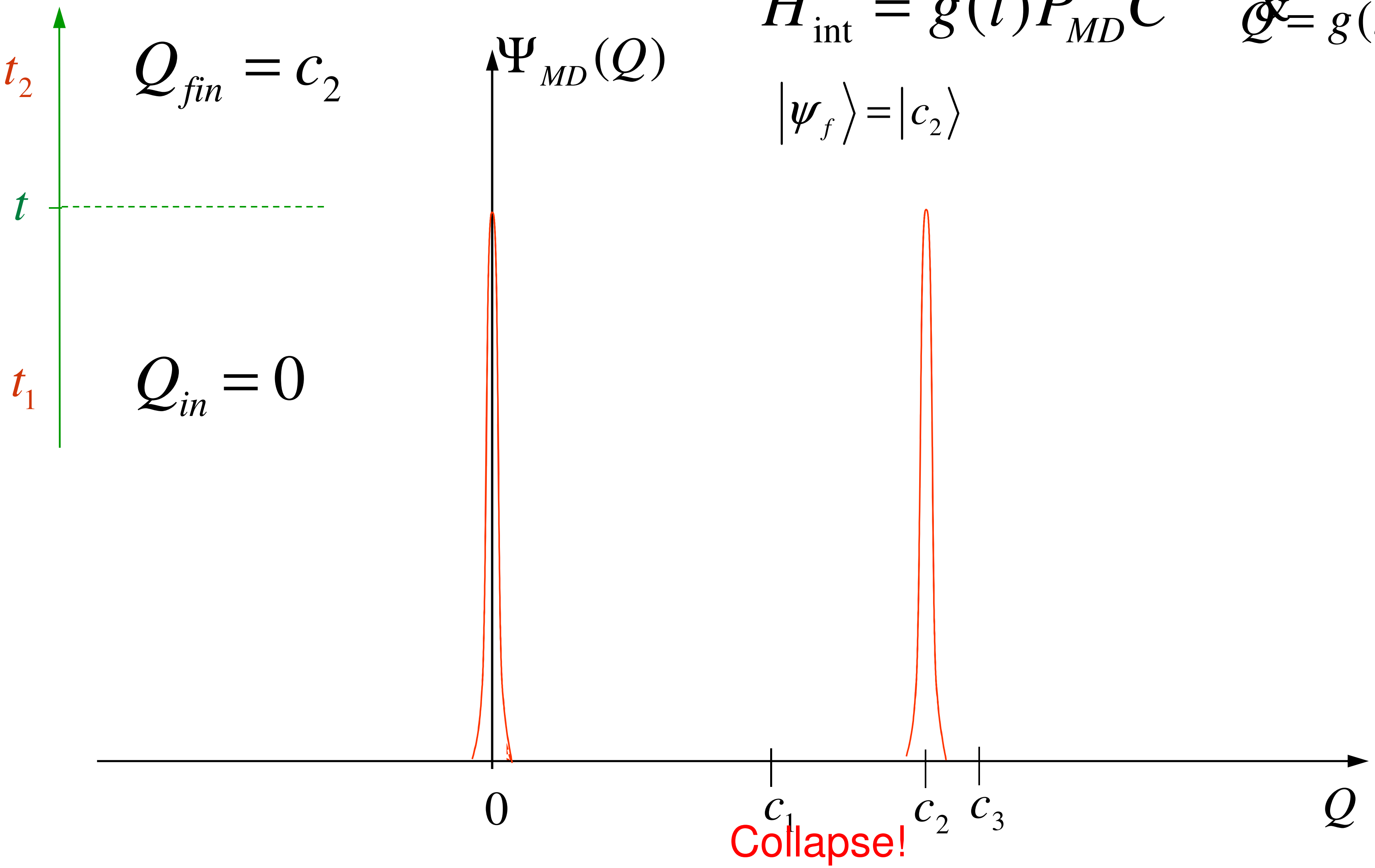
2 Measurement of arbitrary observable *C*

$$H_{\mathrm{int}} = g(t) P_{MD} C \qquad \dot{Q} = g(t) C$$

$$\left|\psi_f\right\rangle = \left|c_2\right\rangle$$

t_2: $Q_{fin} = c_2$

t_1: $Q_{in} = 0$



$$\sum_i \alpha_i \left|c_i\right\rangle \left|Q=0\right\rangle \rightarrow \sum_i \alpha_i \left|c_i\right\rangle \left|Q=c_i\right\rangle \rightarrow \left|c_2\right\rangle \left|Q=c_2\right\rangle$$

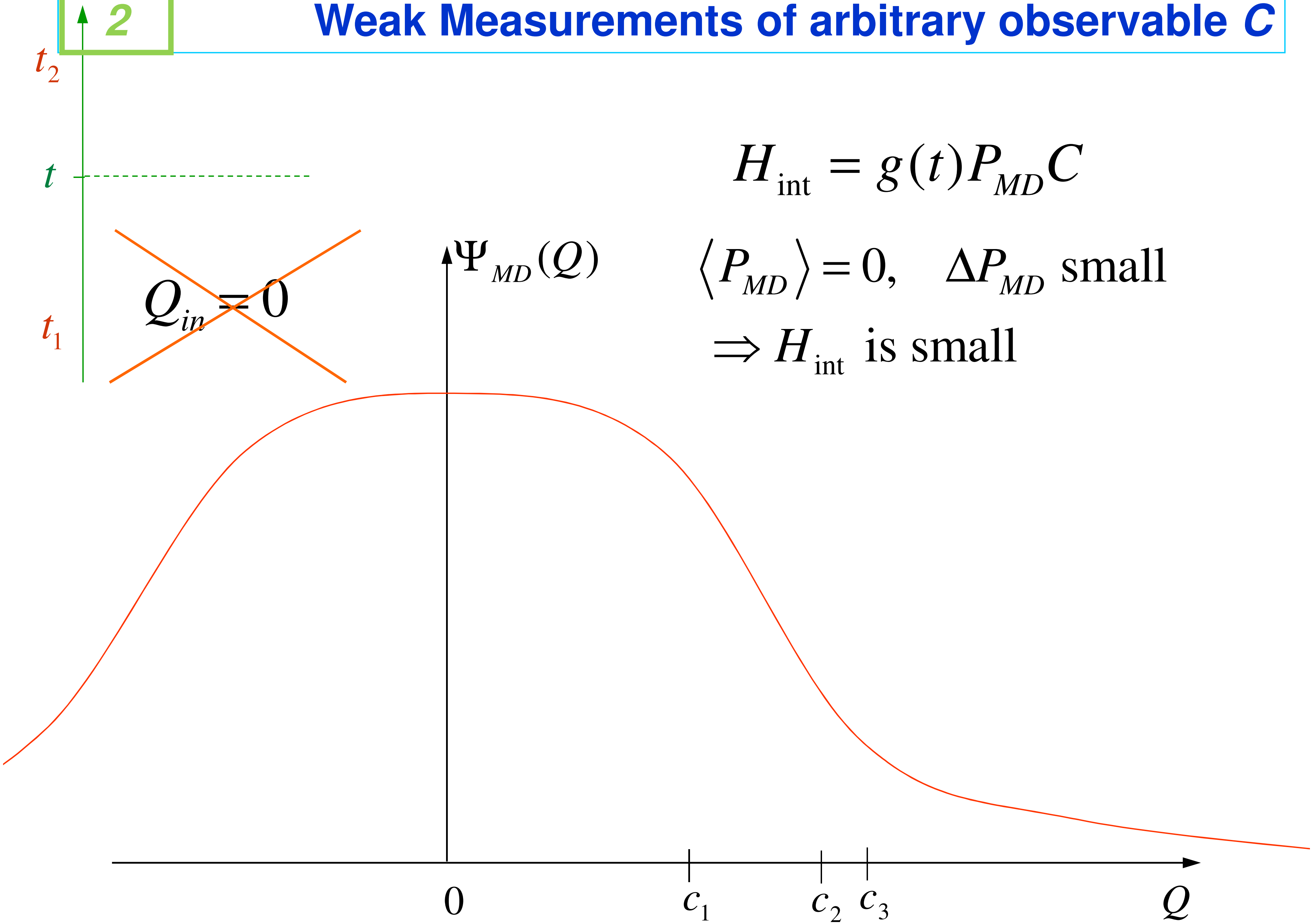
Weak Measurements of arbitrary observable *C*

2

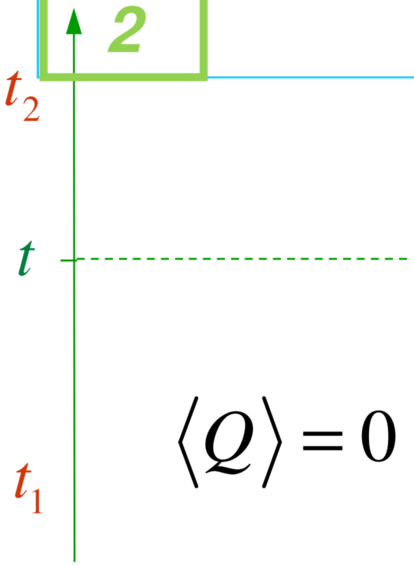
$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$



Weak Measurements of arbitrary observable *C*



$$\langle Q \rangle = 0$$

$$H_{\text{int}} = g(t) P_{MD} C$$

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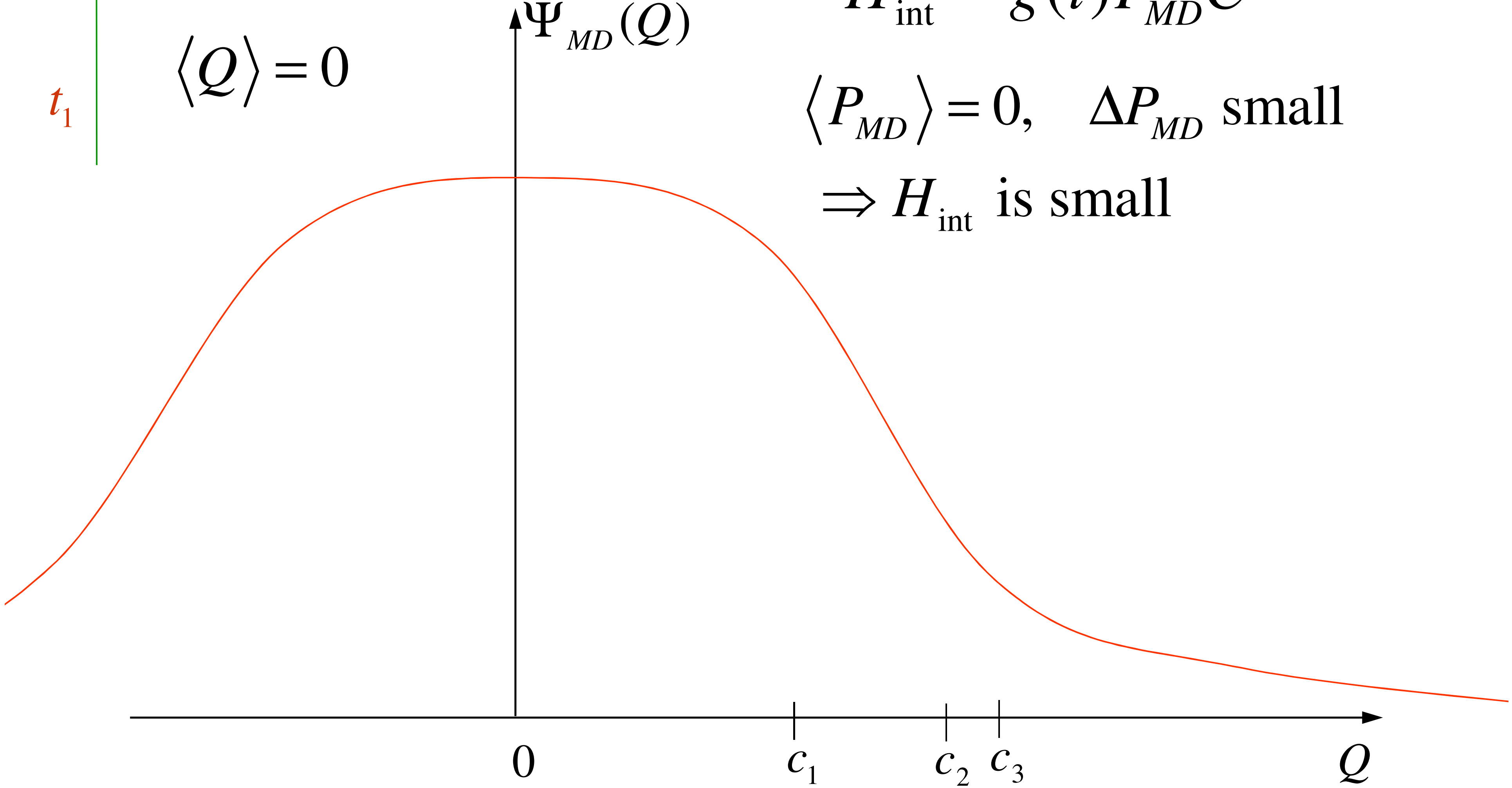
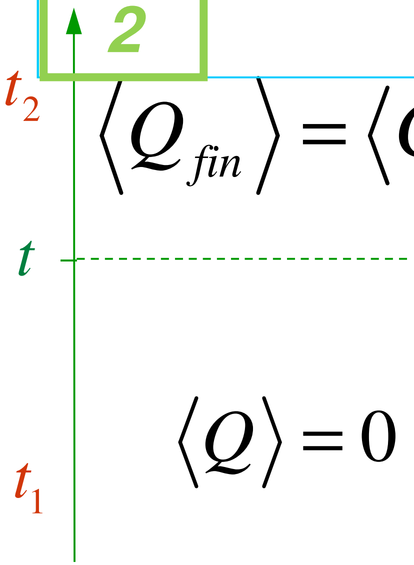


2 Weak Measurements of arbitrary observable *C*



$$\langle Q_{fin} \rangle = \langle C \rangle$$

$$\langle Q \rangle = 0$$

$$H_{\text{int}} = g(t) P_{MD} C$$

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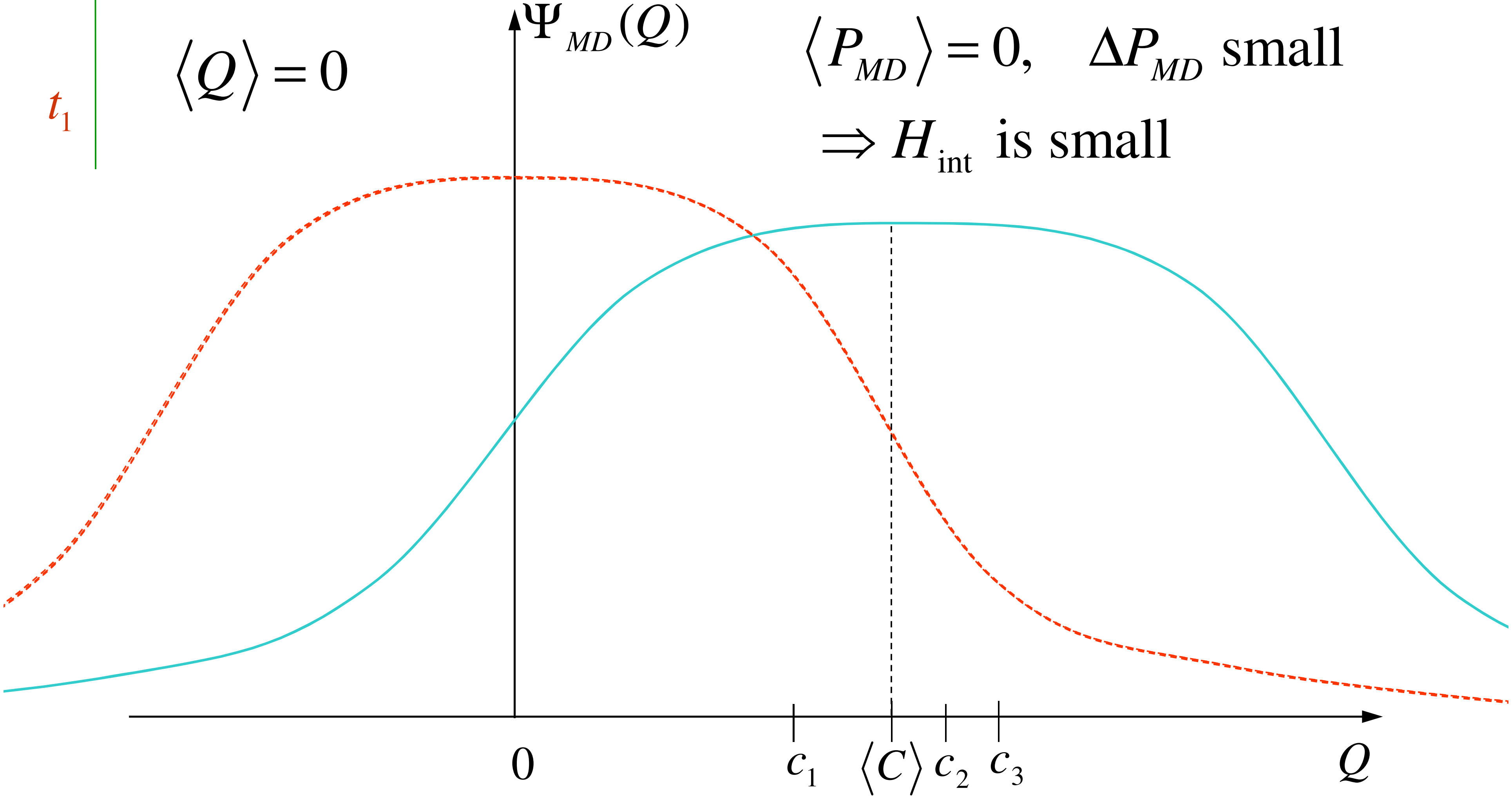
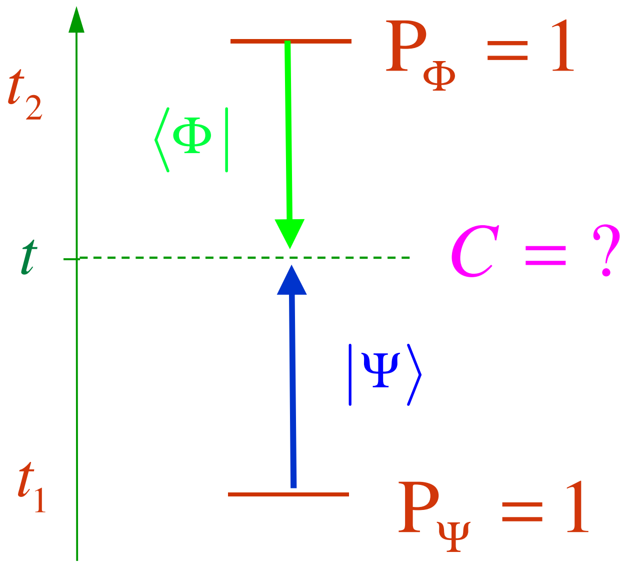


2 Weak Measurements of arbitrary observable *C*

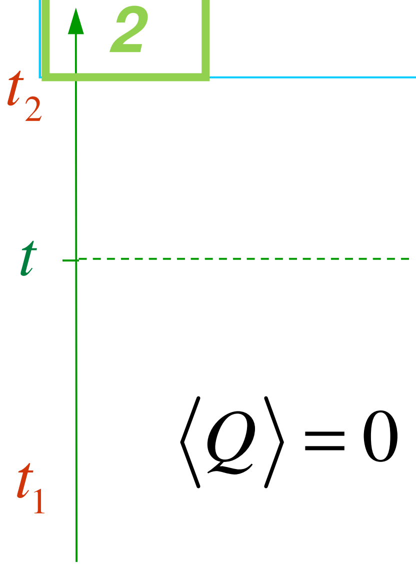
The outcomes of weak measurements are weak values

Weak value of a variable *C* of a pre- and post-selected system described at time *t* by the two-state vector $\langle\Phi|\ |\Psi\rangle$



$$\boxed{C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}}$$

2 Weak Measurements of *C with post-selection*

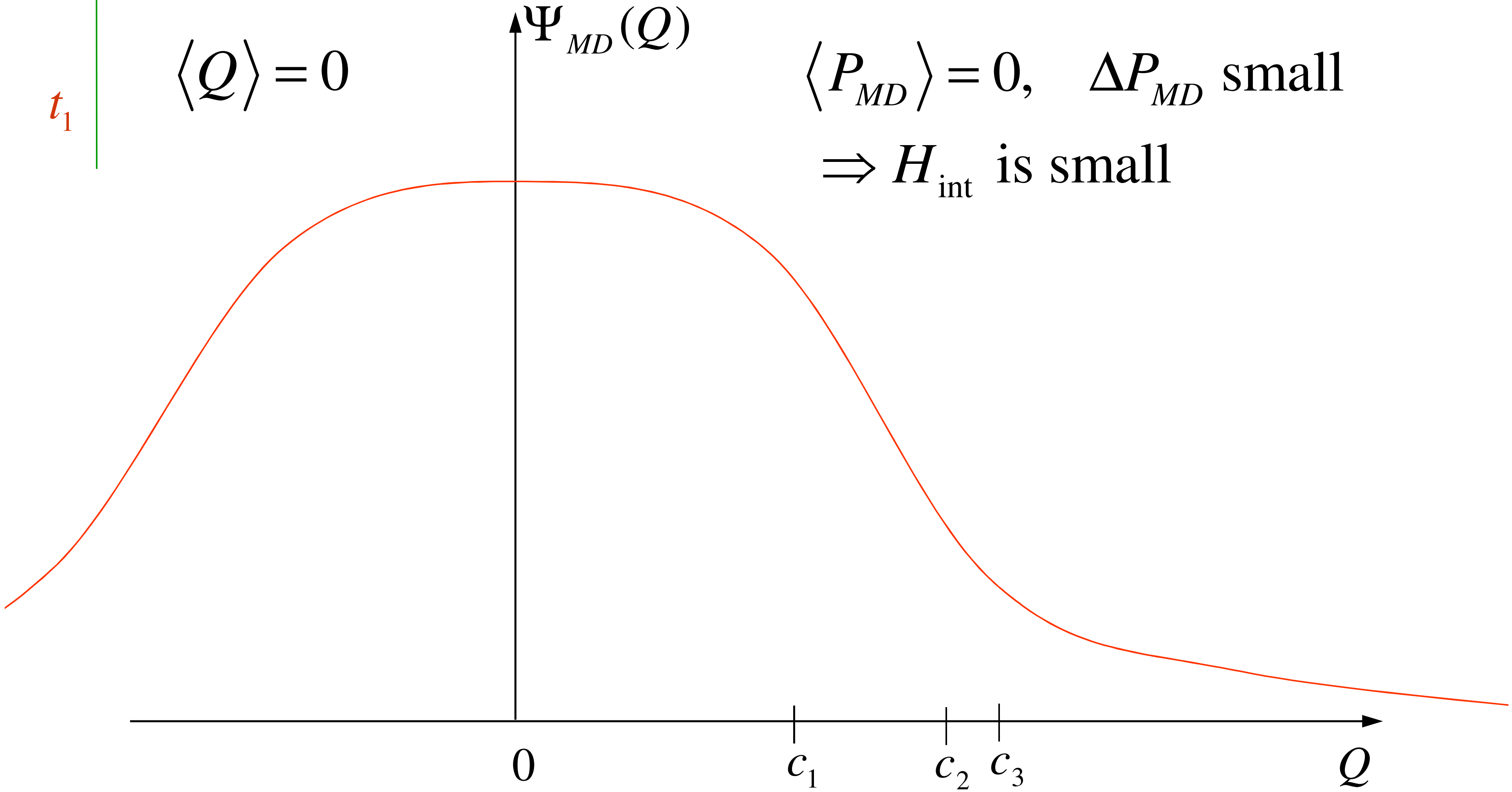


$$\langle Q \rangle = 0$$

$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$



Weak Measurements of *C with post-selection*

$$\left\langle Q_{fin} \right\rangle = C_w$$

t_2 —— $P_\Phi = 1$

t

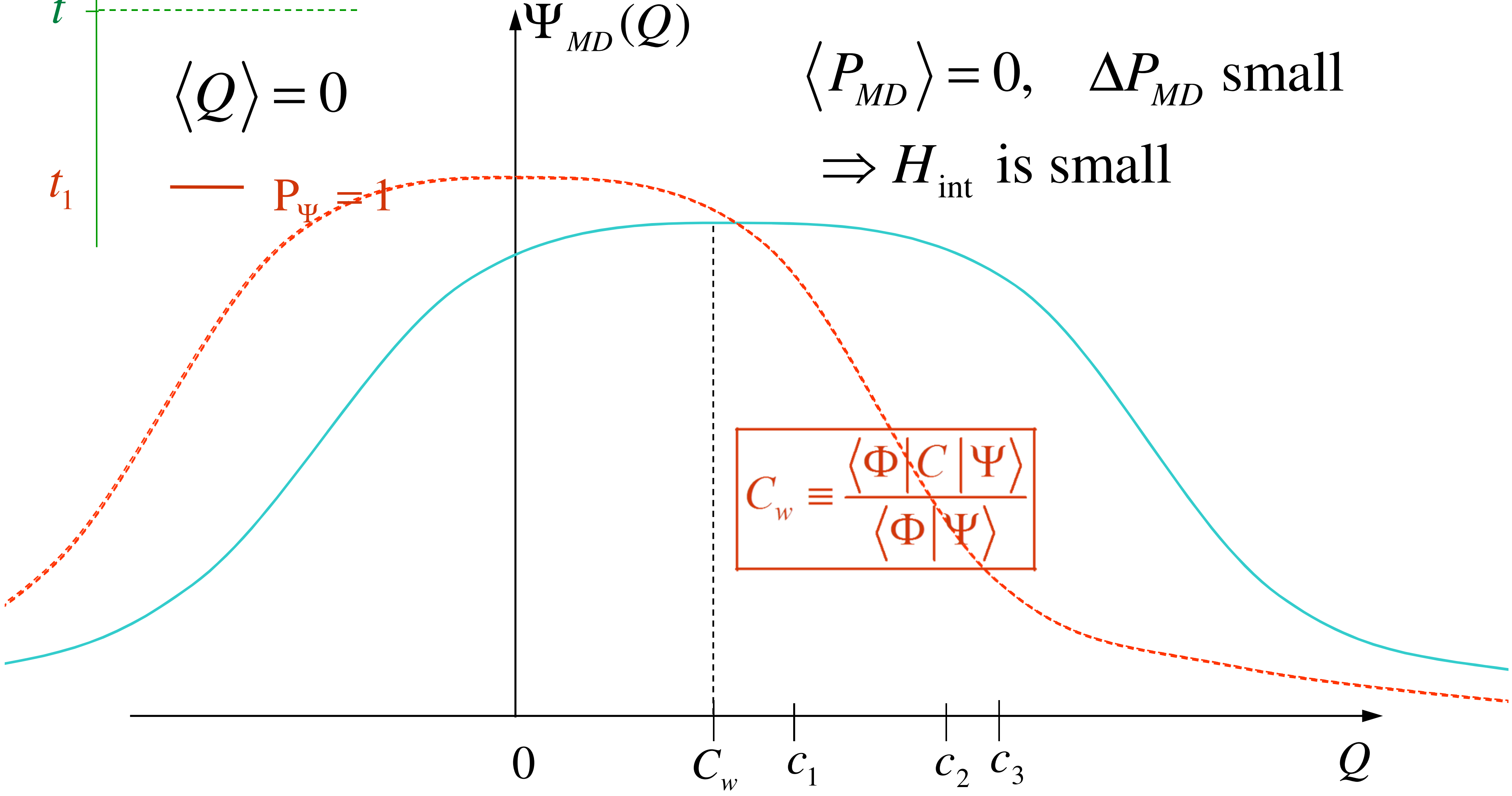
$$\langle Q \rangle = 0$$

t_1 —— $P_\Psi = 1$

$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

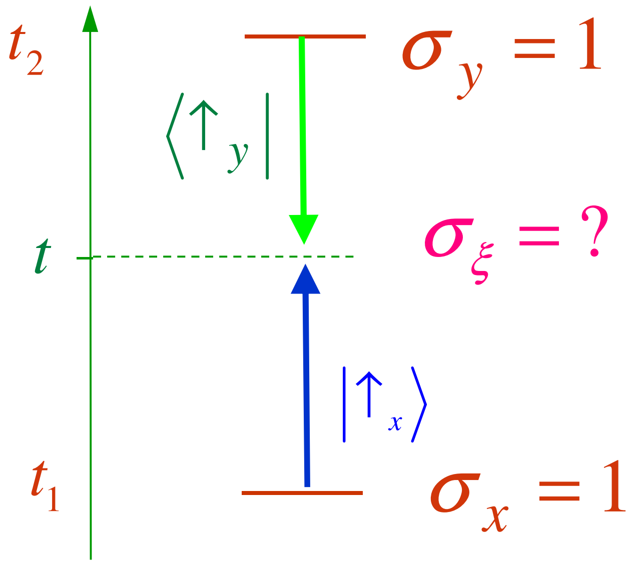
$$\Rightarrow H_{\text{int}} \text{ is small}$$



$$\boxed{C_w \equiv \frac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}}$$

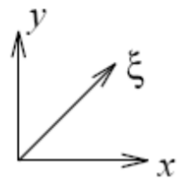
The outcomes of weak measurements are weak values

Weak value of a variable C of a pre- and post-selected system described at time t by the two-state vector $\langle\Phi|\ \ |\Psi\rangle$



$$\boxed{C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}}$$

$$\sigma_\xi \equiv \frac{\sigma_x + \sigma_y}{\sqrt{2}}$$



$$(\sigma_\xi)_w = \frac{\langle\uparrow_y|\sigma_\xi|\uparrow_x\rangle}{\langle\uparrow_y|\uparrow_x\rangle} = \frac{\langle\uparrow_y|\frac{\sigma_x+\sigma_y}{\sqrt{2}}|\uparrow_x\rangle}{\langle\uparrow_y|\uparrow_x\rangle} = \sqrt{2}$$

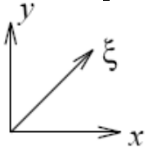
Weak measurements performed on a pre- and post-selected ensemble

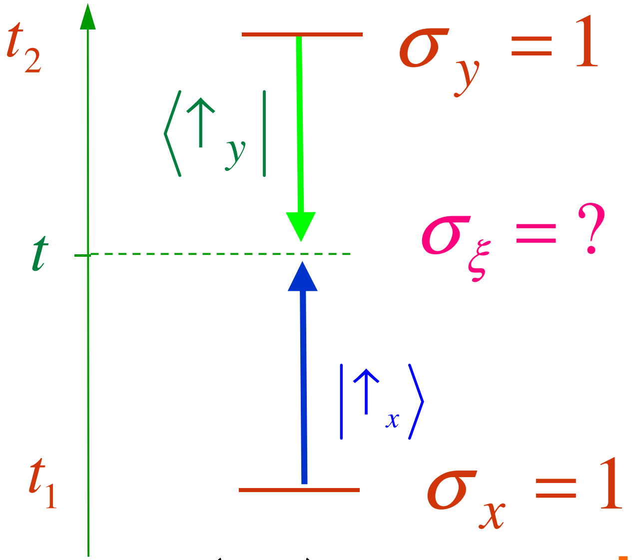
Weak Measurement of $\sigma_\xi = \dfrac{\sigma_x + \sigma_y}{\sqrt{2}}$

$$H_{\text{int}} = g(t) P_{MD} \sigma_\xi \qquad \Psi_{in}^{MD}(Q) = e^{-\frac{Q^2}{2\Delta^2}}$$

The particle pre-selected $\sigma_x = 1$

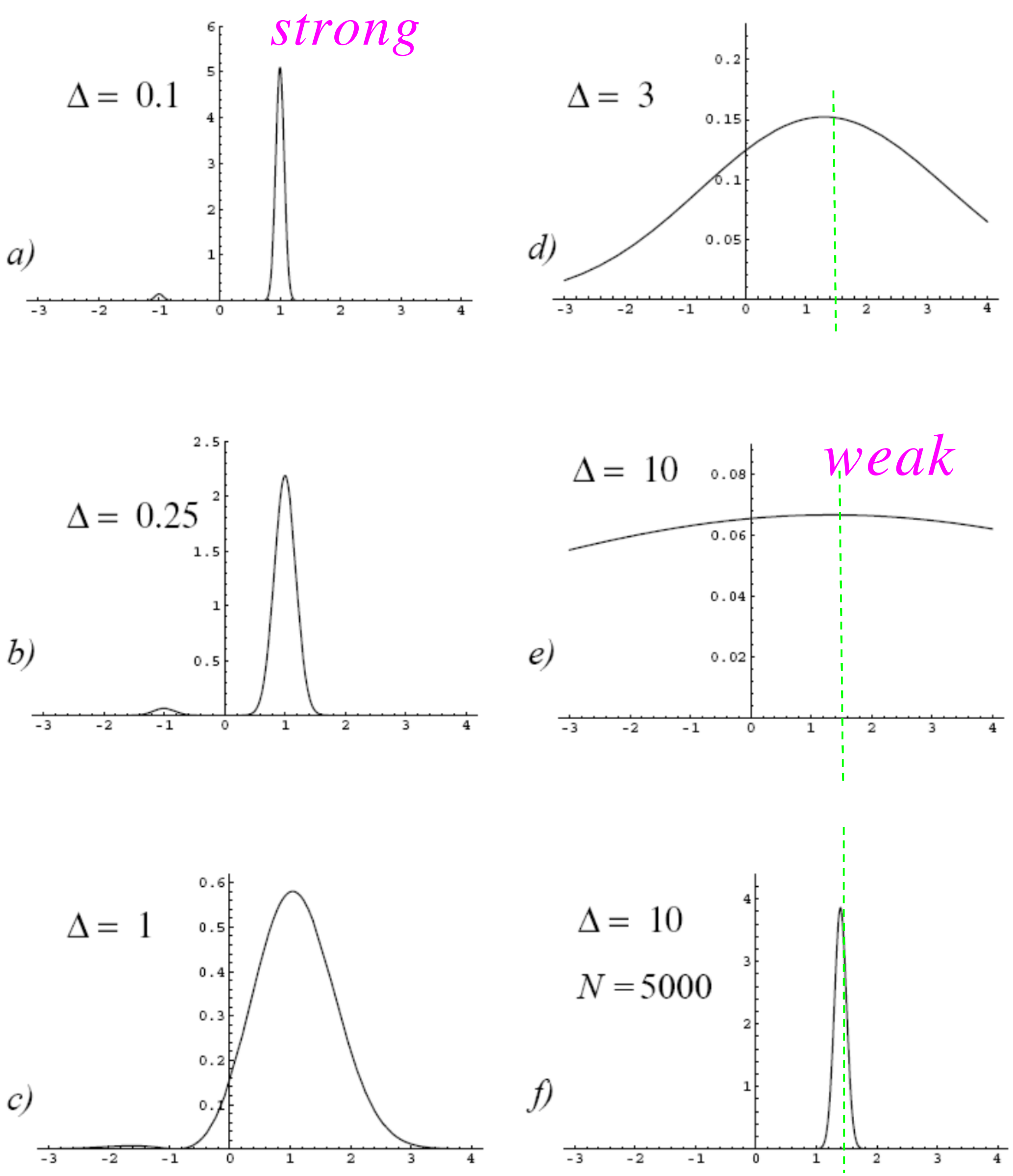
The particle post-selected $\sigma_y = 1$





$(\sigma_\xi)_w \; ; \; 1.4$!

Pointer probability distribution



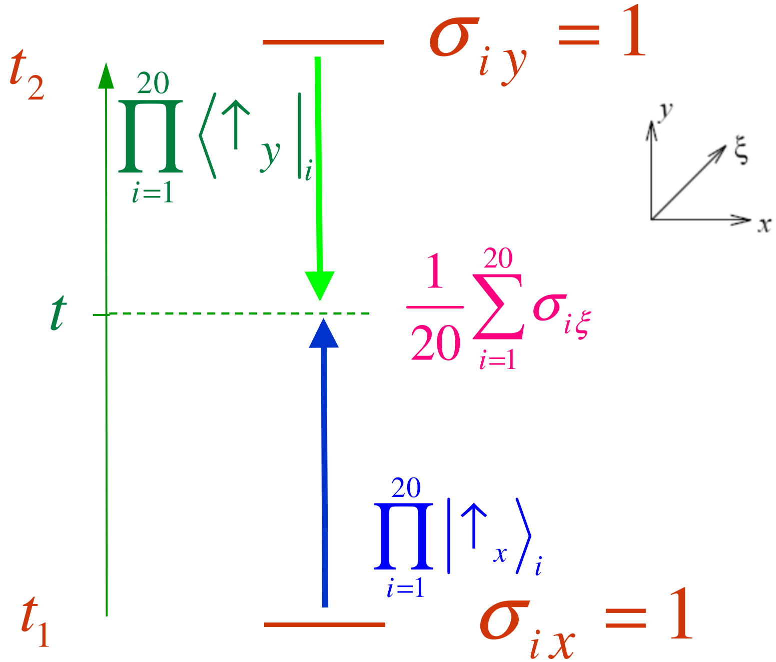
Robust weak measurement on a pre- and post-selected single system

The system of 20 particles

Weak Measurement of $\frac{1}{20}\sum_{i=1}^{20}\sigma_{i\xi}$

20 particles pre-selected $\sigma_x = 1$

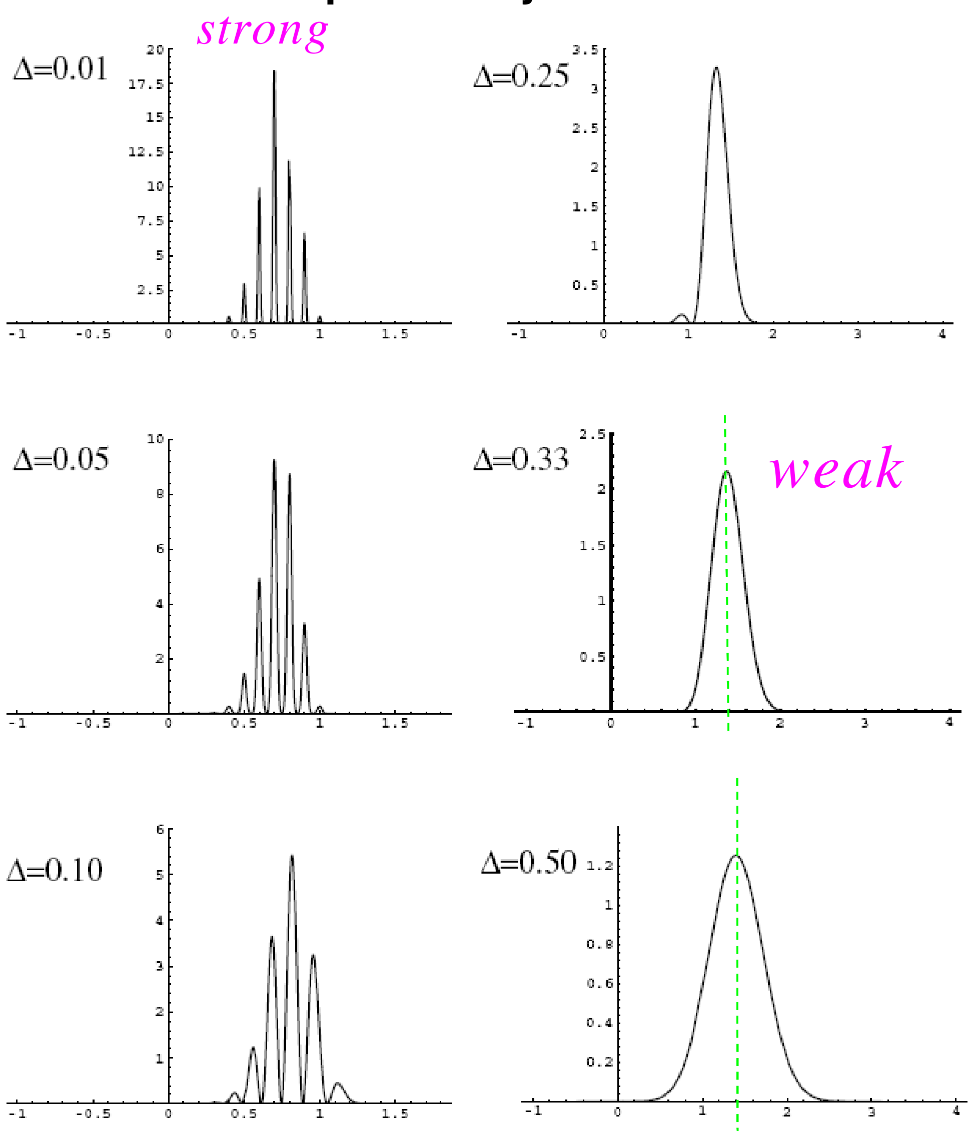
20 particles post-selected $\sigma_y = 1$



$$\left(\frac{1}{20}\sum_{i=1}^{20}\sigma_{i\xi}\right)_w \simeq 1.4$$

!

Pointer probability distribution



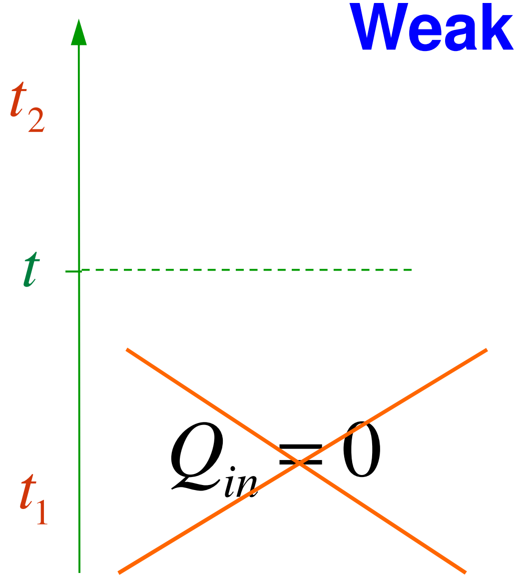
See also "Robust weak measurement " JT JOP, (2007)

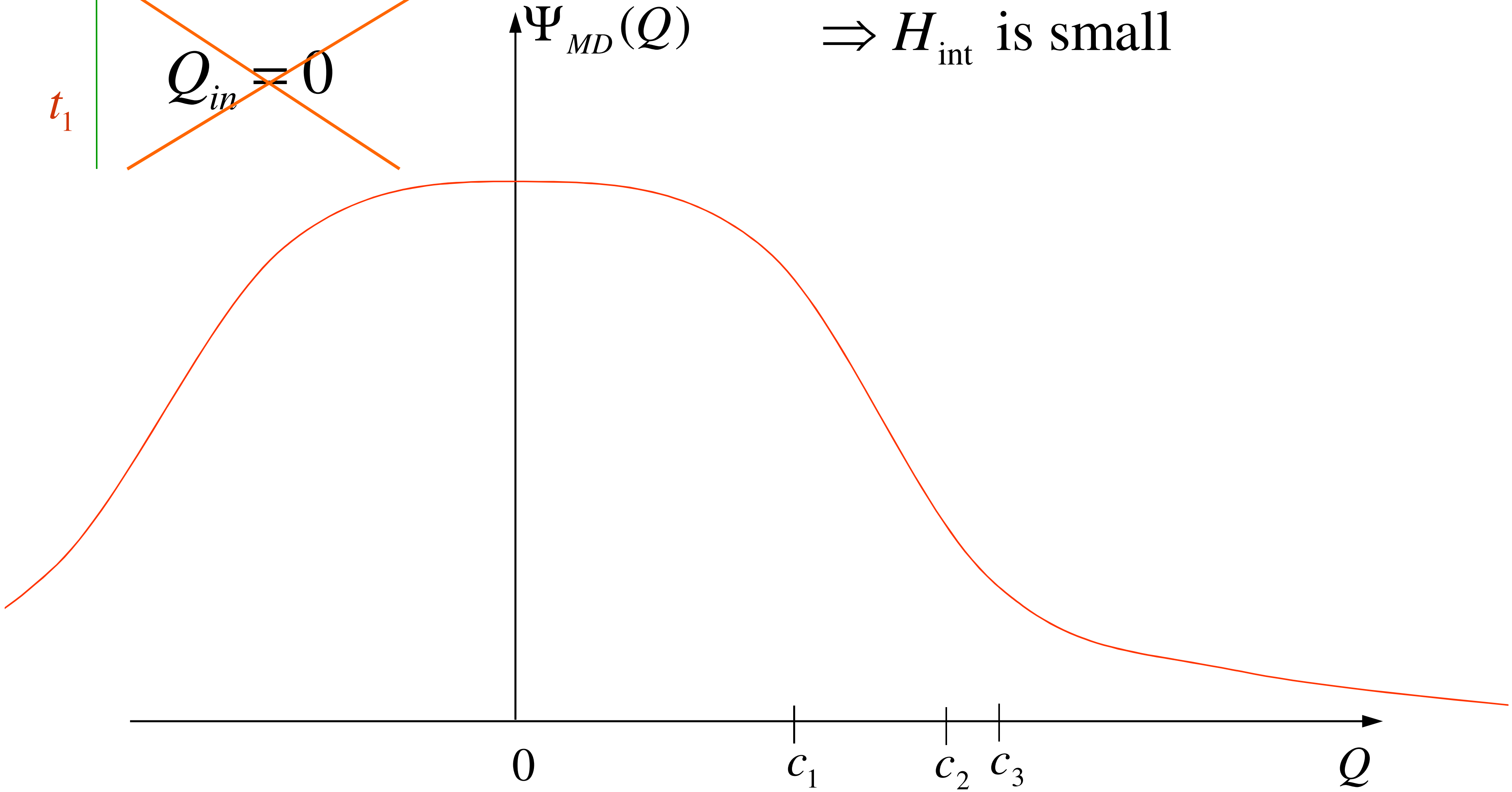
Weak quantum measurement of C

$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$





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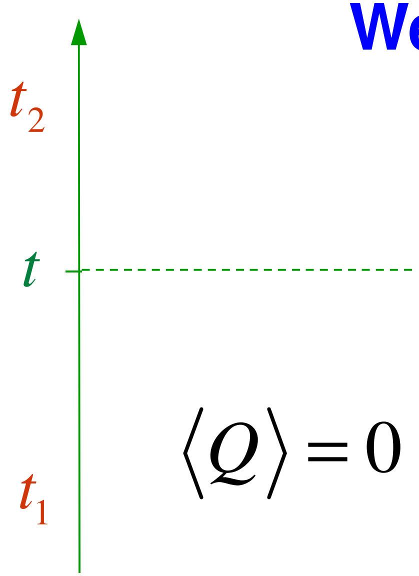
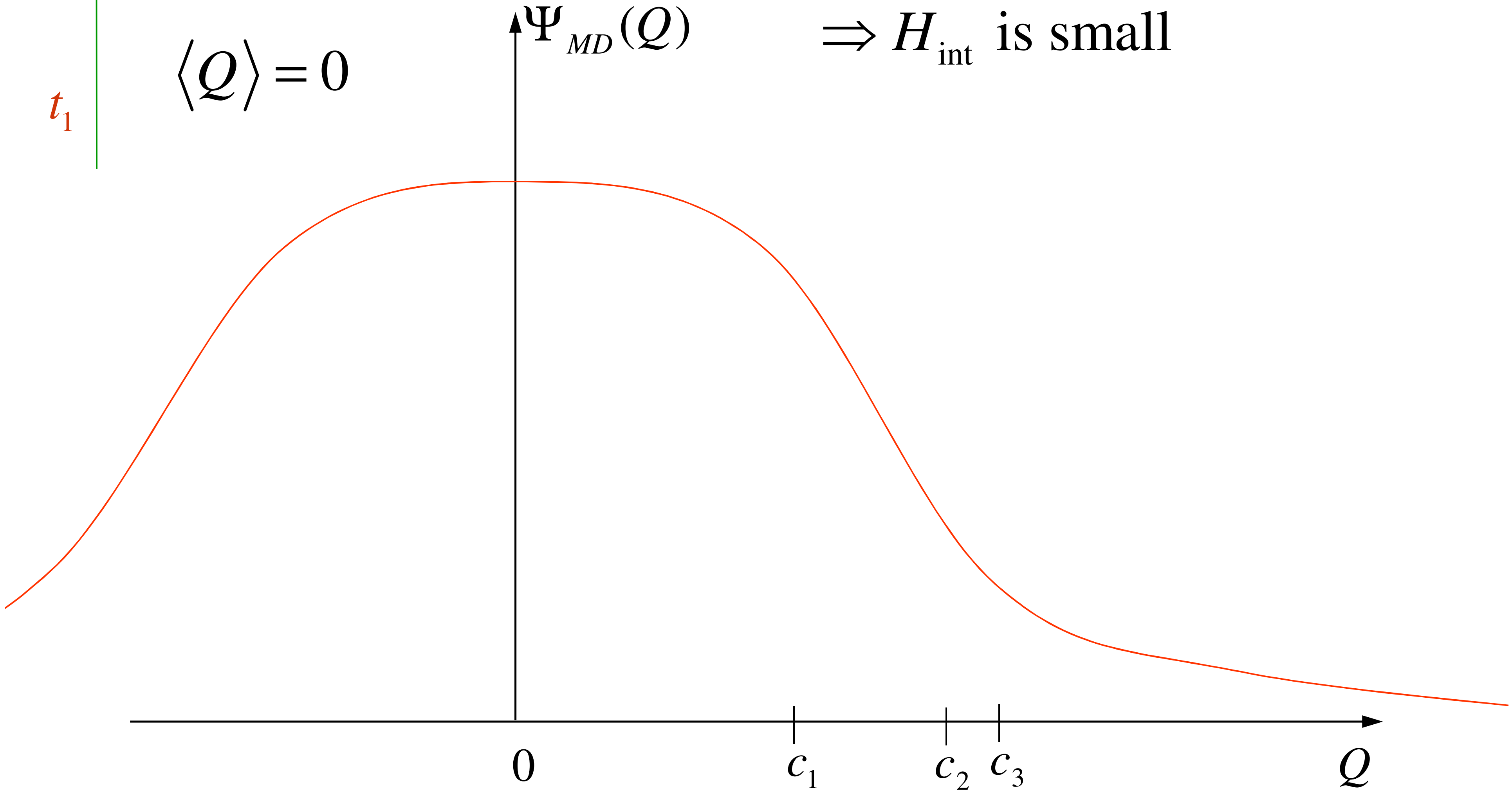


$$\langle Q \rangle = 0$$



Weak quantum measurement of C

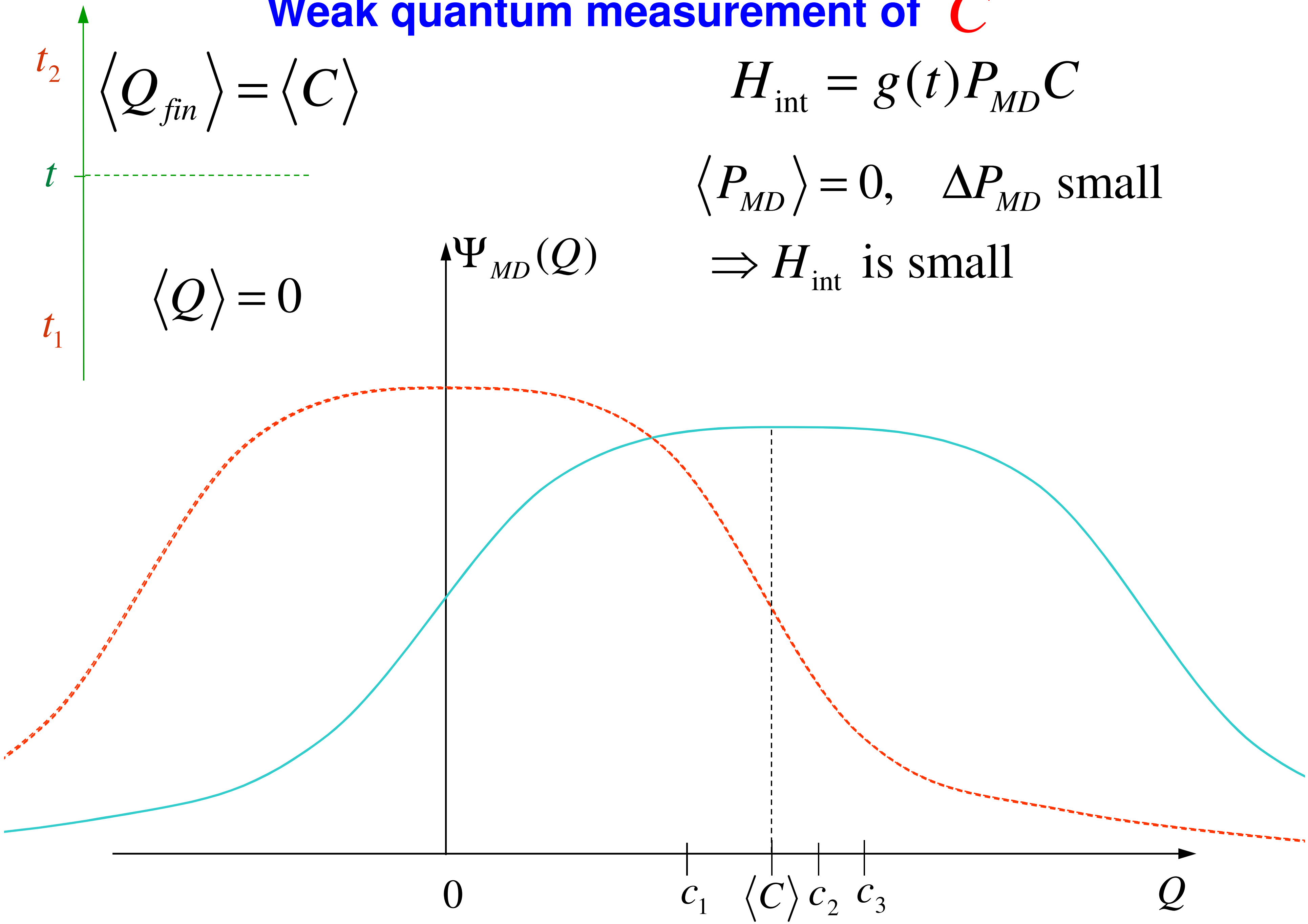
$$\left\langle Q_{fin} \right\rangle = \left\langle C \right\rangle$$

$$H_{\text{int}} = g(t) P_{MD} C$$

$$\left\langle P_{MD} \right\rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$

$$\left\langle Q \right\rangle = 0$$

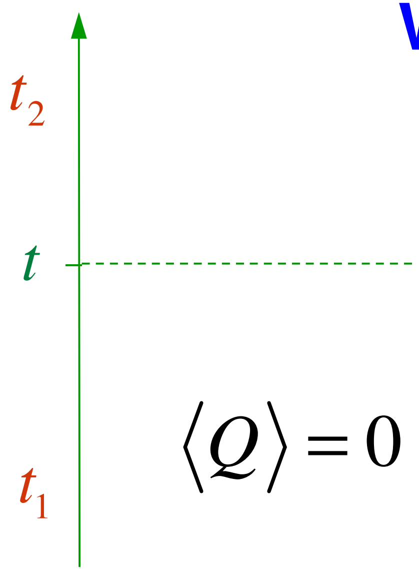


Weak measurement of C with post-selection

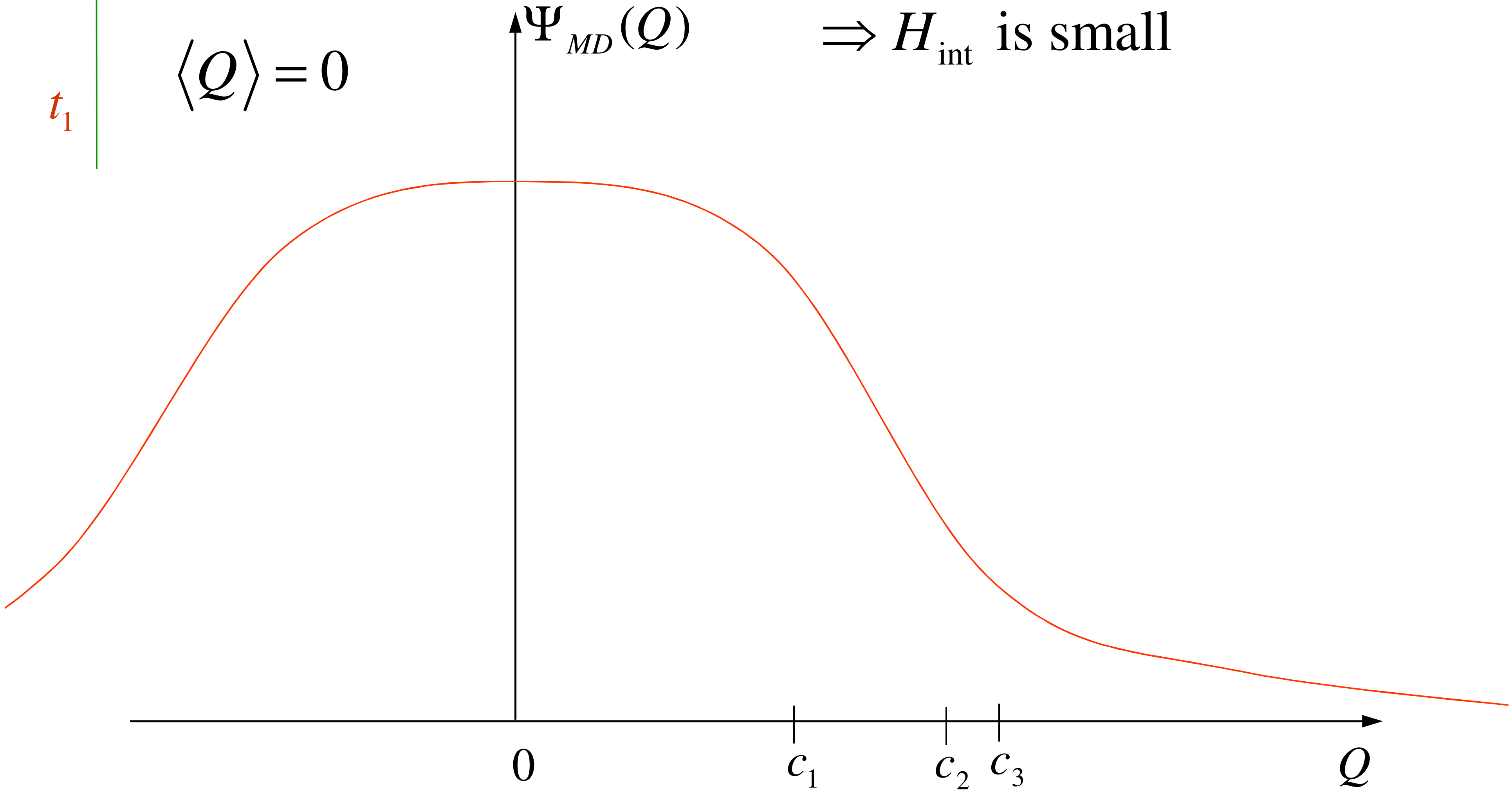
$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

$$\Rightarrow H_{\text{int}} \text{ is small}$$



$$\langle Q \rangle = 0$$



Weak measurement of C with post-selection

$$\langle Q_{fin} \rangle = C_w$$

$$H_{\text{int}} = g(t) P_{MD} C$$

$$\langle P_{MD} \rangle = 0, \quad \Delta P_{MD} \text{ small}$$

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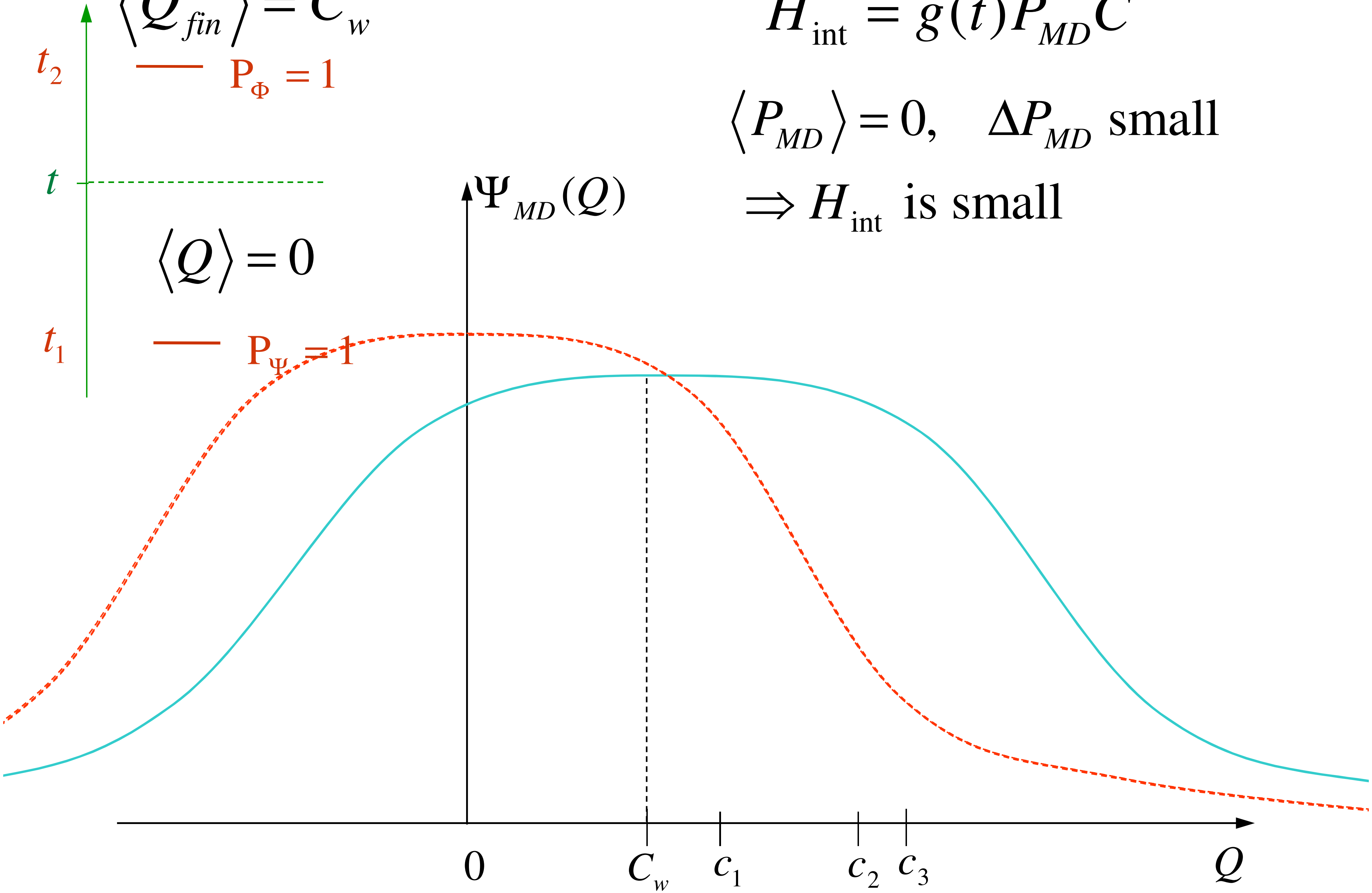
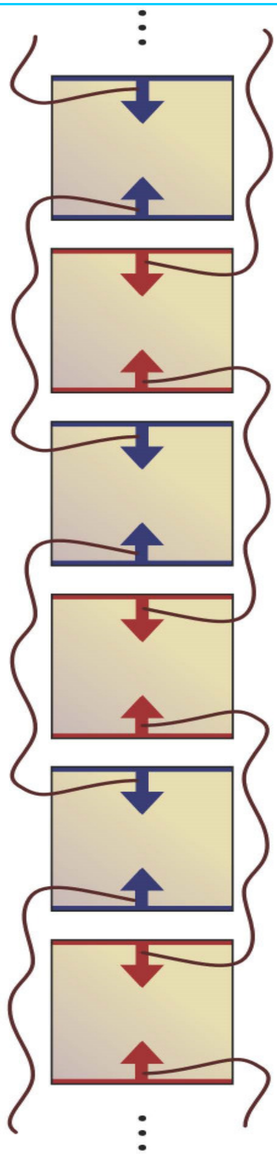


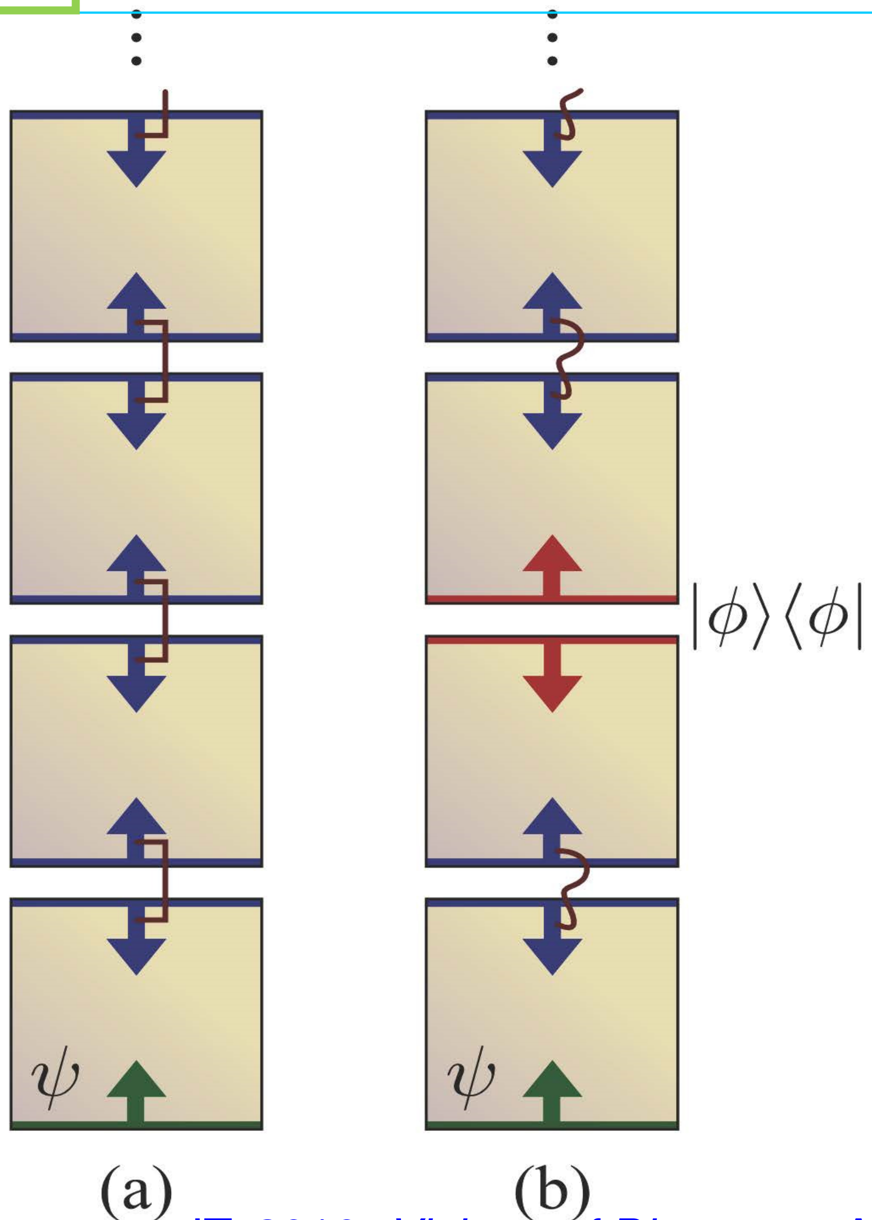
4.a QM Generalization: Each moment of time a new universe



Two independent "lives" lived in parallel by the same particle.

Aharonov, JT, 2010, *Visions of Discovery*; Aharonov, Popescu, JT, arXiv:1305.1615
Aharonov, Popescu, JT, Vaidman, *Phys Rev A* 79, 052110 (May 1, 2009)

4.a QM Generalization: Each moment of time a new universe



(a) Each moment of time is a little "brick." The Hilbert space at the future boundary of one time moment is maximally entangled - with complete correlation - with the Hilbert space at the past boundary of the next time moment (straight line). The state ψ is associated only to the moment t_0 where it was prepared.

(b) A more general time evolution. A measurement with a collapse on state ϕ disentangles the two subsequent moment of time. Non-trivial unitary time evolution at all other time is represented by maximal entanglement but between appropriately rotated bases (squiggled line).

Aharonov, JT, 2010, *Visions of Discovery*; Aharonov, Popescu, JT, arXiv:1305.1615
Aharonov, Popescu, JT, Vaidman, *Phys Rev A* 79, 052110 (May 1, 2009)

4.d

New forms of holism/emergence

although we may know the dynamics on a particular time-scale T, this doesn't mean that we know anything about the dynamics on a smaller time-scale:

$$e^{-iHT} = \{e^{\frac{-iHT}{N}}\}^N$$

consider a superposition of unitary evolutions (using $e^{-iHT} = \{e^{\frac{-iHT}{N}}\}^N$):

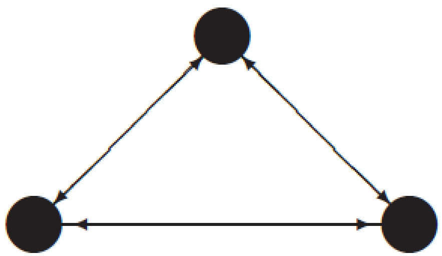
$$\int g(\nu)e^{-iH(\nu)t}d\nu \rightarrow \int g(\nu)\{1+iH(\nu)t\}d\nu \underbrace{\longrightarrow}_{if \int g(\nu)d\nu=1} 1+i\int g(\nu)H(\nu)td\nu \qquad (0.1)$$

This theory is the same as the usual theory but with an effective Hamiltonian

$$H_{eff} = \int g(\nu)H(\nu)td\nu \qquad (0.2)$$

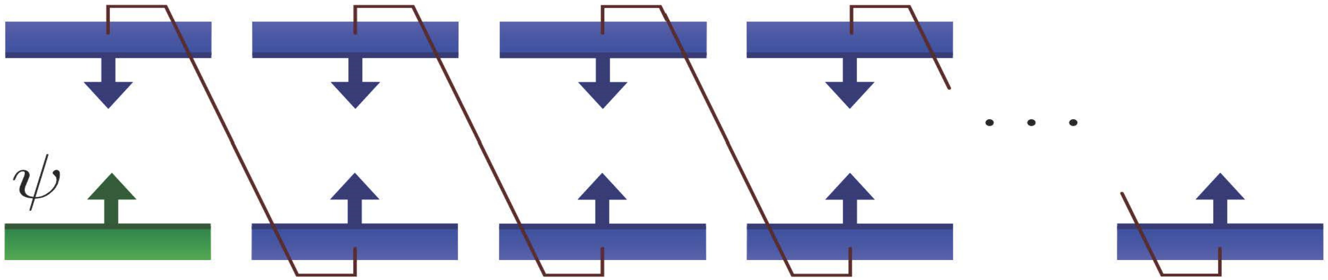
The finer grained Hamiltonian can be expressed as a superposition of evolutions $e^{\frac{-iHT}{N}} = \sum_n \alpha_n e^{\frac{-i\beta_n HT}{N}}$, i.e. the Hamiltonian can be represented as a superposition of different laws given by pre-and-post-selection

Top-down approach?

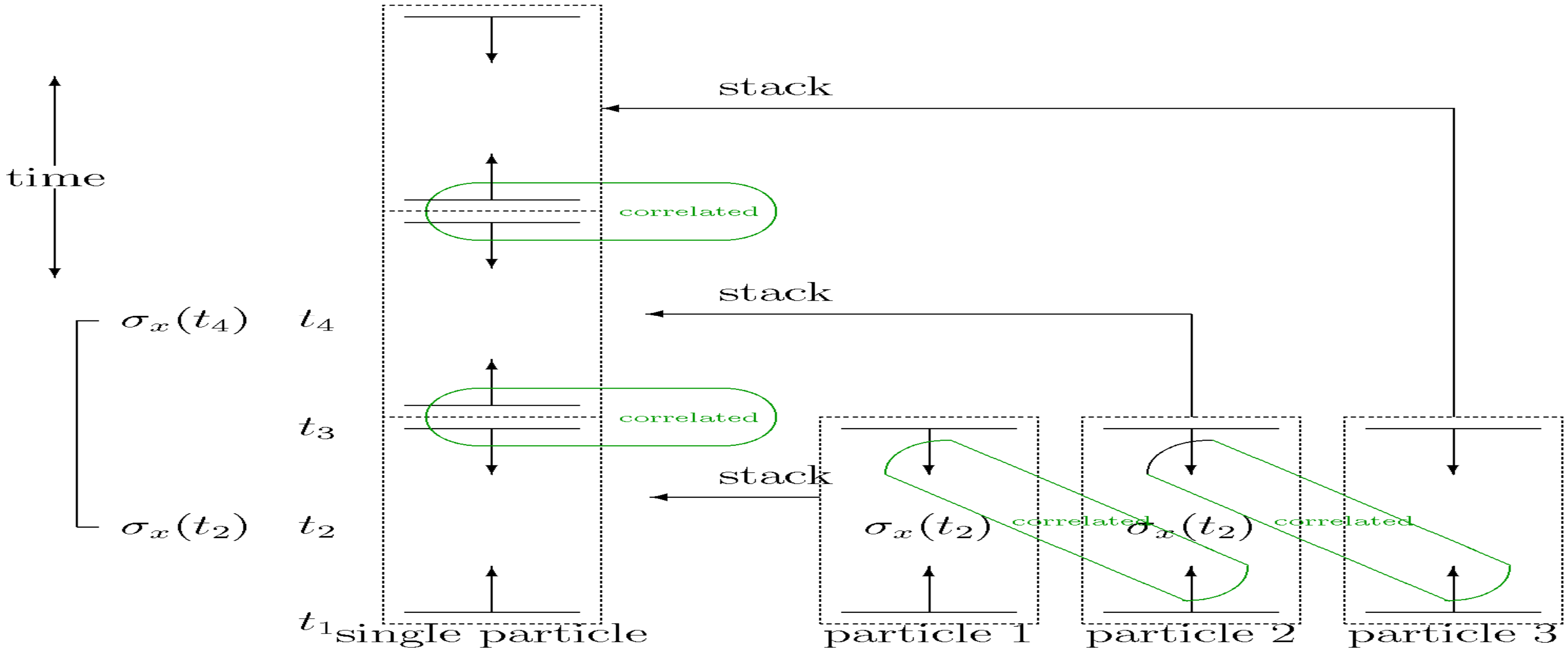


4.a QM Generalization: Each moment of time a new universe

- New ability to obtain a post-selected state of one particle that is completely correlated to the pre-selected state of a second particle:

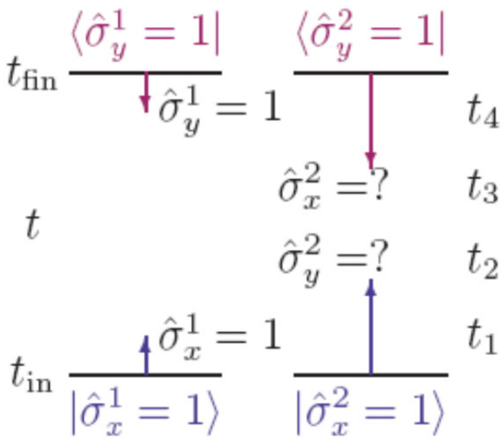


- stack N particles on top of another along the time axis:

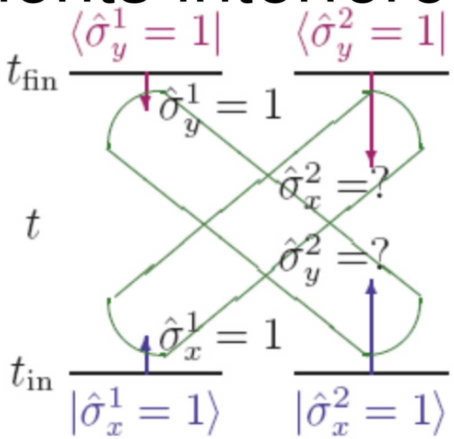


Ascertaining the results of products of the 9 observables
(Tollaksen, Jrnl of Phys A, 40 (2007) 9033**)**

- If we measure the operators corresponding to the first 2 observables of row 3 $\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_x^2\hat{\sigma}_y^1$ in the PPS



- then the measurements interfere with each other



Diagonal PPS – generic feature

Ascertaining the results of products of the 9 observables
(Tollaksen, Jrnl of Phys A, 40 (2007) 9033**)**

- **Given one PPS, the subset of observables circled (and the products of those circled observables) can be assigned eigenvalues in a way that satisfies the product rule**

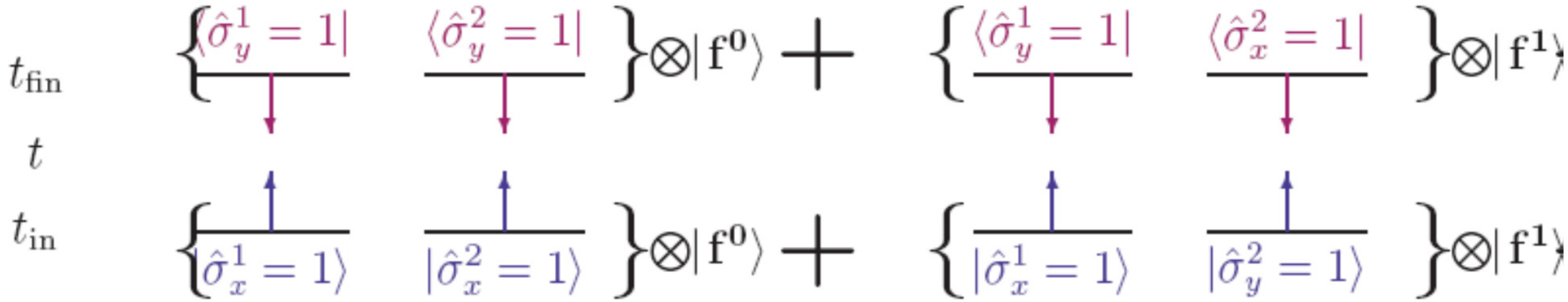
$$
\begin{array}{ccc}
\hat{\sigma}_x^1 & \hat{\sigma}_x^2 & \hat{\sigma}_x^1\hat{\sigma}_x^2 \\
\hat{\sigma}_y^1 & \hat{\sigma}_y^2 & \hat{\sigma}_y^1\hat{\sigma}_y^2 \\
\hat{\sigma}_x^1\hat{\sigma}_y^2 & \hat{\sigma}_x^2\hat{\sigma}_y^1 & \hat{\sigma}_z^1\hat{\sigma}_z^2
\end{array}
\qquad\qquad
\begin{array}{ccc}
\hat{\sigma}_x^1 & \hat{\sigma}_x^2 & \hat{\sigma}_x^1\hat{\sigma}_x^2 \\
\hat{\sigma}_y^1 & \hat{\sigma}_y^2 & \hat{\sigma}_y^1\hat{\sigma}_y^2 \\
\hat{\sigma}_x^1\hat{\sigma}_y^2 & \hat{\sigma}_x^2\hat{\sigma}_y^1 & \hat{\sigma}_z^1\hat{\sigma}_z^2
\end{array}
$$

- **But, the product of the other observables can only be ascertained (given this particular PPS) using information from both the pre- and post-selected vector in a diagonal sense, and will thus violate the product rule.**

Physical reason for restrictions on these assignments

(Tollaksen, Jrnl of Phys A, 40 (2007) 9033**)**

- All sets of boundary conditions are needed
- However, when the first observable is ascertained, then it will depend on both the pre- and the post-selection measurement (i.e. it will be diagonal-PPS) and will collapse the entire configuration onto a subset of the PPSs, thereby disturbing the terms of the multiple-time state.



- Mermin's statement: ``Alice's other two `results' have nothing to do with any properties of the particle or the results of any measurement actually performed."

If $C = c_i$ w/ prob 1 then $C_w = c_i$

This theorem circumvent the need to consider measurements that are temporal successors in the PPS paradox as counterfactual alternatives in the proof of contextuality

Theorem: Logical-PPS-paradoxes imply ``quantum contextuality" through weak values

- **Quantum Contextuality: For any initial quantum state which exhibits a breakdown of non-contextuality in the associated HVT for a certain set of operators (i.e. for which ABL assigns definite values of 0/1), one can find at least one post-selected state which will show how the function composition rule (i.e. sum and product rules) is violated.**
- **Applied this analysis to Mermin, EPR, GHZ: in each case, eccentric WVs outside EV spectrum**
- **Post-selections suggests a physical picture for why the assignments cannot be made. In addition, the existence of strange WVs demonstrate a new way that QM copes with contextuality.**

Summary

- Used PPS to probe contextuality-can be tested experimentally. As Mermin states, this is not “theorizing about `hidden variables'. It is a rock solid quantum mechanical effort to answer a perfectly legitimate quantum mechanical question."
- weak values go outside the spectrum with contextuality
  - Mermin contextuality: $\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_x^2\hat{\sigma}_y^1 = -1$ even though separately $\hat{\sigma}_x^1\hat{\sigma}_y^2 = +1$ and $\hat{\sigma}_x^2\hat{\sigma}_y^1 = +1$
  - these 3 outcomes can be measured weakly without contradiction because the product of WVs is not equal to the WV of the product
- With the assumption that a WM does not considerably modify the hidden variable, then this strengthens a hidden variables proof of contextuality

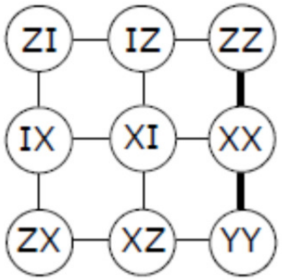
Weak Values and ContextualityM Waegell, JT "Contextuality, Pigeonholes, Cheshire Cats, Mean Kings, and Weak Values,"arXiv: 1505.00098

The Kochen-Specker Theorem

The Kochen-Specker (KS) Theorem posits that quantum mechanics is inconsistent with Noncontextual Hidden Variable Theories (NCHVTs) of reality.

(1) By a hidden variable model, we mean that for each observable that can be measured, a predetermined outcome exists that is simply realized by the measurement.
(2) By noncontextual, we mean that the predetermined outcome for each observable should be independent of what other compatible (commuting) observables may be measured at the same time.

The simplest proof of the KS Theorem is the well-known Peres-Mermin square for two qubits. Any state can be prepared, and we will choose to measure all three mutually commuting observables on a randomly chosen line of the square.



(3) According to (1) and (2), in order to predict all possible outcomes we must assign a single eigenvalue +1 or -1 to each of these 9 observables.
(4) According to QM (and experiment), the product of the three values along a given (thick) line of the square must be (-1) +1.

It is impossible to find an assignement of eigenvalues that satisfies both (3) and (4), which proves the KS theorem.

To see this, consider the product of all six lines of the square (three horizontal and three vertical). According to (3) this product is +1, since every value appears in two lines, and therefore becomes squared in the product, while according to (4) this product is -1, since their are five thin lines (+1) and one thick line (-1).

In order to experimentally verify that (4) is satisfied and (3) is violated, we now consider the sum of all six lines, with a negative sign for the thick line.

Then, when we perform the experiment, quantum mechanics predicts that the sum is 6, which will be reduced by experimental imperfections.

If however, we insist that (3) must be satisfied and (4) violated, then we see at least that one of the terms must get a sign flip, and as a result the sum can never exceed 4.

Therefore any experiment in which the measured value of this sum significantly exceeds 4 experimentally rules out NCHVTs.

The important point to take away from this is that if (3) were satisfied and (4) violated, then the product rule would be violated in experiments with nonvanishing probability.

No pure state in the vector space formalism of quantum mechanics can violate the product rule, and thus there are no projectors onto these outcomes, nor POVMs describing a measurement that contains them.

This has an important consequence for the Ahoronov-Bergmann-Lebowitz (ABL) reformulation of quantum mechanics, which implicitly assumes that both the prepared state and the measurement-outcome state belong to the vector space formalism, and therefore neither one can violate the product rule. Thus the situation where (3) is satisfied cannot even be expressed within the ABL reformulation of quantum mechanics.

The next important question concerns how we interpret the thesis of the Kochen-Specher theorem. If the NCHVTs are ruled out, what explanations are we left with? There are several possible ways to answer this question.

We should now emphasize that there was a crucial assumption in our derivation of the KS theorem, and this was the assumption of free random choice. It is only because we choose which measurement to make at random that it is reasonable to require the existence of a predetermined outcome for all 9 observables that could be measured.

The KS theorem then shows us that if such predictions exist, they must be explicitly contextual, which is to say that (2) is violated, and a given observable is free to have different predicted values in each context

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A New Type of Hidden Variable Model

The other, usually overlooked, explanations are those which call random free choice into question. These have collectively been called the *superdeterminism loophole*.

We propose a new type of hidden variable model here that satisfies (1), (2), and (3) by placing a restriction on free random choice. Specifically the product rule (4) must be violated in this model, but it is impossible to choose to measure the context (line) where this violation occurs.

This model can also be intepreted in a way that preserves free random choice, by allowing nature to play a sort of `Shell Game' in which the assignments to the entire set depend on which measurement is ultimately chosen. This interpetation is actually a sort of retrocausal contextuality, in that the finally chosen measurement context influences eigenvalues that are preseumed to exist in the past. It is worth mentioning that this retocausalitry could never violate the no-signalling principle.

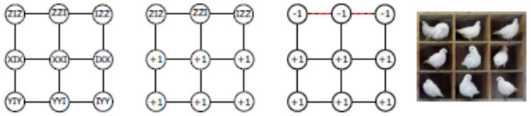
For our purpose here, all that matters is that in either interpretation, the product rule is violated, but the violation is never observed. We can thus remain agnostic about which interpretation we apply to our model.

Importantly, since neither the prepared, nor the finally measured state violate the product rule, this model is perfectly consistent with the pre- and post-selected (PPS) states of the ABL reformulation.

For the example that follows, we use a different 3-qubit KS set (still a square). We pre-select and post-select the product states,

$$|\Psi\rangle = |X=+1\rangle^{\otimes 3} \quad \& \quad |\Phi\rangle = |Y=+1\rangle^{\otimes 3}$$

The ABL formula then predicts unit probability to obtain the outcomes in the top of each vertical line by an intermediate strong measurement, and these values subsequently violate the product rule for the top horizontal line. In this PPS-paradox the violation also occurs in a product basis, and thus all three quibts can be considered independently to obtain this result.



Taking Z to be the classical basis with no superposition, this has also been called the ***quantum pigeonhole effect***.

In that language, there are 3 pigeons (the qubits) and 2 boxes (the Hilbert space of each qubit). The eigenstates of the Z basis are the classical states (e.g. 2 pigeons in the left box and 1 in the right, or all 3 in the left, etc...) . The violation of the product rule shows that all classical states are forbidden, and it is thus impossible to specify where the 3 pigeons are during the intermediate time.

Furthermore, returning to the 2-qubit Peres-Mermin square, we can obtain the ***quantum cheshire cat*** in exactly the same way - pre-selecting a Bell state along the bottom horizontal line, and a product state along the middle horizontal line, again causing a violation of the product rule in the classical basis.

Specifying a PPS state also defines the weak values of all observables at once, regardless of commutation properties. The weak values are identical to the eigenvalues for all observables with either the pre-selected and post-selected state as an eigenstate. Furthermore, the weak values are defined in measurement contexts where the product rule may be violated, and this information can be probed using weak measurements. In special cases where the ABL reformulation allows us to localize the violation to specific contexts, some weak values must be anomalous, and this signature of contextuality can be experimentally observed.

The ABL formula can be expressed very simply in terms of the weak values of the projectors in the measurement basis.

$$P(|\Pi_i^{r_i} = 1/|\Psi\rangle, |\Phi\rangle, \vec{B}) = \frac{|(\Pi_i^{r_i})_w|^2}{\sum_k^{d'} |(\Pi_k^{r_k})_w|^2}$$

It is then easy to obtain the *ABL rule* and *Reverse ABL rule* for projectors: if the ABL formula predicts unit (zero) probability to obtain a projector by a strong measurement during the intermediate time, then the weak value of that projector is 1 (0). And the reverse; if the weak value of a projector is 1 (0) then the probability to obtain that projector by an intermediate strong measurement is 1 (0)*.

The ABL rules thus allow us to include these forced values in the eigenvalue assignment of our hidden variable model, and we can justify using these values by weakly measuring the weak value.

Finally, weak values have several natural features that make them promising as elements of reality.

(5) Weak values are noncontextual, in the sense that every observable has its own weak value, independent of what other observables it commutes with.

(6) Weak values sum linearly by definition and thus obey the sum rule.

* the measurement basis B must be the 2-element POVM that contains only this projector and its orthogonal complement, such that the two span the entire Hilbert space.

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Hidden Variable and Weak Values of Projectors

We now consider the hidden variables and weak values of the projectors in our previous example, and violations of the sum rule (the eigenvalues of projectors in any basis must sum to 1). Each of the 6 lines of the square defines an eigenbasis of 4 rank-2 projectors, and we index these 1 through 24, and show the 24 orthogonal bases they form.

The top 3 rows are the eigenbases of the horizontal lines, the next 3 the vertical lines. This set is saturated, showing all orthogonal pairs.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24
1	2	13	16
1	3	17	20
1	4	21	24
2	3	22	23
2	4	18	19
3	4	14	15
5	6	15	16
5	7	19	20
5	8	23	24
6	7	21	22
6	8	17	18
7	8	13	14
9	10	14	16
9	11	18	20
9	12	22	24
10	11	21	23
10	12	17	19
11	12	13	15

The weak values of the 24 projectors, superposed in their bases. Note the anomalous weak value in the conflict basis and flat ABL probability.

-0.5	0.5	0.5	0.5
1	0	0	0
1	0	0	0
1	0	0	0
1	0	0	0
1	0	0	0
-0.5	0.5	1	0
-0.5	0.5	1	0
-0.5	0.5	1	0
0.5	0.5	0	0
0.5	0.5	0	0
0.5	0.5	0	0
1	0	0	0
1	0	0	0
1	0	0	0
0	0	1	0
0	0	1	0
0	0	1	0
1	0	0	0
1	0	0	0
1	0	0	0
0	0	1	0
0	0	1	0
0	0	1	0

The hidden variable assignments in our model, with underlines for forced values, and the conflict bases in bold.

0	**0**	**0**	**0**
1	0	0	0
1	0	0	0
1	0	0	0
1	0	0	0
1	0	0	0
0	0	1	0
0	0	1	0
0	0	1	0
0	**0**	**0**	**0**
0	**0**	**0**	**0**
0	**0**	**0**	**0**
1	0	0	0
1	0	0	0
1	0	0	0
0	0	1	0
0	0	1	0
0	0	1	0
1	0	0	0
1	0	0	0
1	0	0	0
0	0	1	0
0	0	1	0
0	0	1	0

These three tables show the steps of determing the hidden variables and thus locating the conflict bases in our model. Projectors highlighted in green have an eigenvalue 1, while those in red have eignevalues 0.In the first step, projector 5 is pre-selected and projector 9 is post-selected, so they are green, and orthogonal projectors are red. In the second step, projectors 13, 17, and 21 are forced (underlined green) by the ABL rule, and in the third step projectors orthogonal to these are forced (underlined red), revealing the conflict bases.

1	2	3	4
5	6	7	8
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13	14	15	16
17	18	19	20
21	22	23	24
1	2	13	16
1	3	17	20
1	4	21	24
2	3	22	23
2	4	18	19
3	4	14	15
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9	12	22	24
10	11	21	23
10	12	17	19
11	12	13	15

1	2	3	4
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1	3	17	20
1	4	21	24
2	3	22	23
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6	8	17	18
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9	10	14	16
9	11	18	20
9	12	22	24
10	11	21	23
10	12	17	19
11	12	13	15

1	**2**	**3**	**4**
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24
1	2	13	16
1	3	17	20
1	4	21	24
2	**3**	**22**	**23**
2	**4**	**18**	**19**
3	**4**	**14**	**15**
5	6	15	16
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7	8	13	14
9	10	14	16
9	11	18	20
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10	11	21	23
10	12	17	19
11	12	13	15

Foiling the Mean King

As a final correllary, we use weak values to show that the Mean King will always fail if the suitor makes measurements from a KS set.

To begin, recall that in order to succeed, the King must have a PPS state for which the ABL probability is 0 or 1 for every projecto the suitor can measure. To see why no such PPS state exists, consider the following:

According to (5), weak values are noncontextual in the same sense as the eigenvalues of (2), and according to (6) they must obey the sum rule.

Furthermore, any complete noncontextual assignment of 0s and 1s to a KS set of projectors must violate the sum rule in some bases. The weak values cannot violate the sum rule, and therefore the weak values of the projectors in a KS set cannot all be 0 and 1 for any PPS.

It then follows that for any PPS, the ABL formula cannot predict probabiity 0 or 1 for all projectors in the KS set, and therefore the suitor can beat the Mean King at his game.

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