Ignorance governs quantum experiments

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The neutron as a quantum object

Coherence Properties

Basics of Neutron Interferometry

Pre- and postselection

Dephasing - Decoherence

Weak Measurements

Unavoidable quantum losses

Résumé

The Neutron

Particle Properties

$m = 1.674928\,(1) \times 10^{-27}$ kg

$s = \frac{1}{2} h$

$\mu = -9.6491783(18) \times 10^{-27}$ J/T

$\tau = 887(2)$ s

$R = 0.7$ fm

$\alpha = 12.0\,(2.5) \times 10^{-4}$ fm^3

u - d - d - quark structure

m ... mass, s ... spin, μ ... magnetic moment, τ ... β-decay lifetime, R ... (magnetic) confinement radius, α ... electric polarizability; all other measured quantities like electric charge, magnetic monopole and electric dipole moment are compatible with zero

CONNECTION

de Broglie

$$\lambda_B = \frac{h}{m.v}$$

Schrödinger

$$H\psi(\vec{r},t) = i\hbar \frac{\delta\psi(\vec{r},t)}{\delta t}$$

&

boundary conditions

$-\mu B$ ________________

μB ________________

two level system

Wave Properties

$$\lambda_c = \frac{h}{m.c} = 1.319695\,(20) \times 10^{-15}\ \text{m}$$

For thermal neutrons
= 1.8 Å, 2200 m/s

$$\lambda_B = \frac{h}{m.v} = 1.8 \times 10^{-10}\ \text{m}$$

$$\Delta_c = \frac{1}{2\delta k} \cong 10^{-8}\ \text{m}$$

$$\Delta_p = v.\Delta t \cong 10^{-2}\ \text{m}$$

$$\Delta_d = v.\tau = 1.942(5) \times 10^{6}\ \text{m}$$

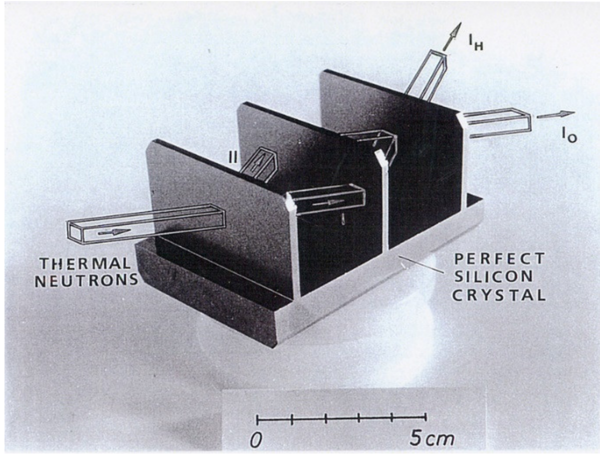
$$0 \le \chi \le 2\pi\ (4\pi)$$

λ_c ... Compton wavelength, λ_B ... deBroglie wavelength, Δ_c ... coherence length, Δ_p ... packet length, Δ_d ... decay length, δk.... momentum width, Δt ... chopper opening time, v ... group velocity, χ ... ... phase.

Neutron Interferometry

$$I_0 \propto \left|\psi_0^{I} + \psi_0^{II}\right|^2 \propto A + B\cos\chi$$

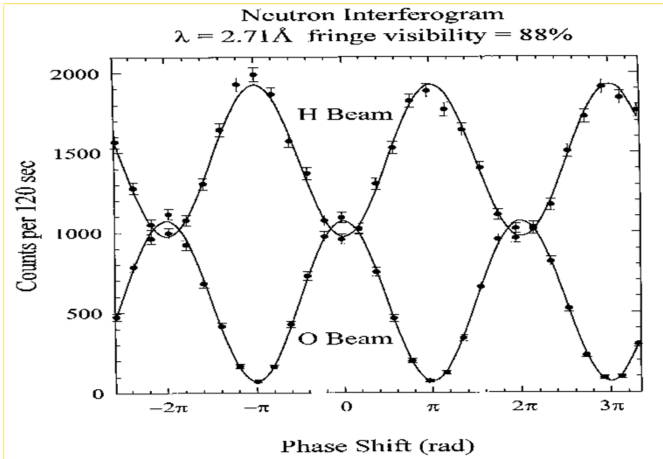
$$\chi = \oint \vec{k}\,d\vec{s} = (1-n)kD_{eff} \equiv -Nb_c\lambda D_{eff} = \Delta\cdot k = \Delta k\cdot D_{eff}$$



Self interference

(phase space density ~10^{-14})

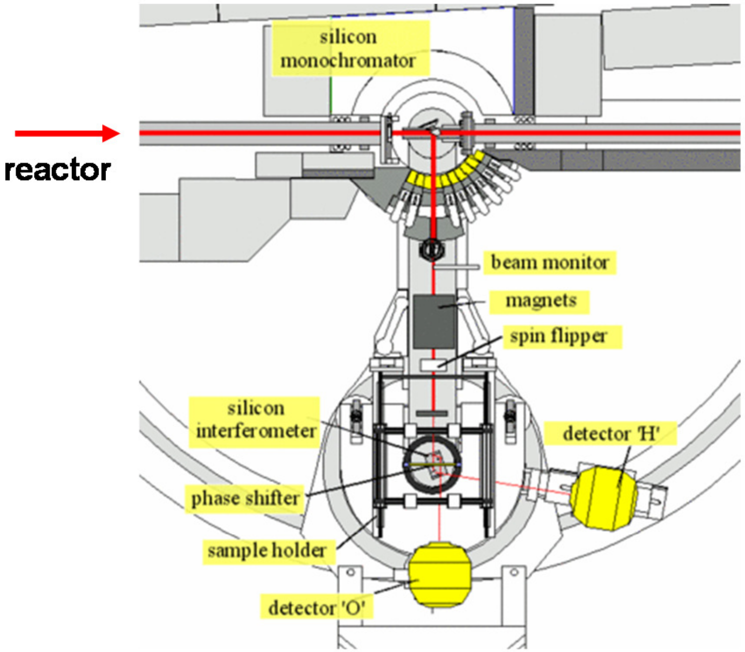
Efficiency of detectors, polarizers, flippers >99%

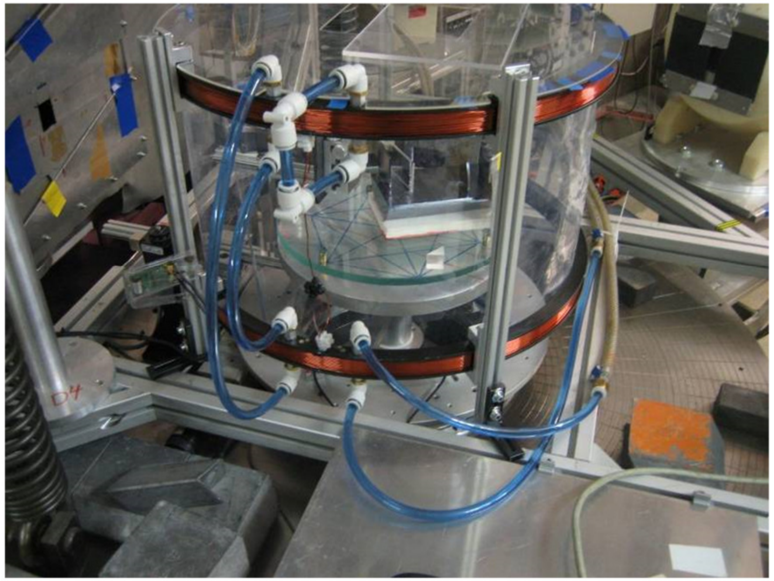


H. Rauch, W. Treimer, U. Bonse, Phys.Lett. A47 (1974) 369

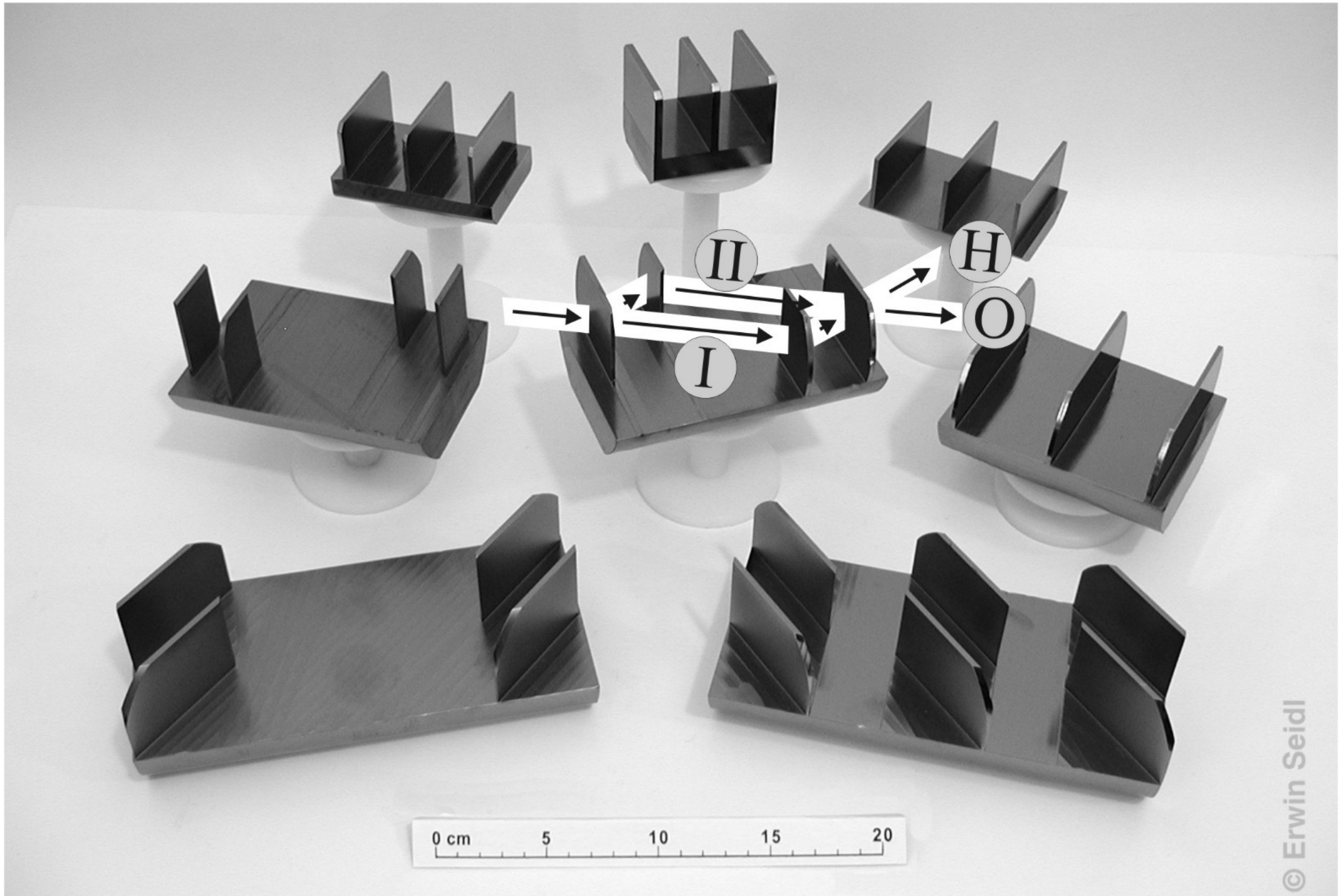
Interferometer set-up S18

Instutut Laue-Langevin, Grenoble



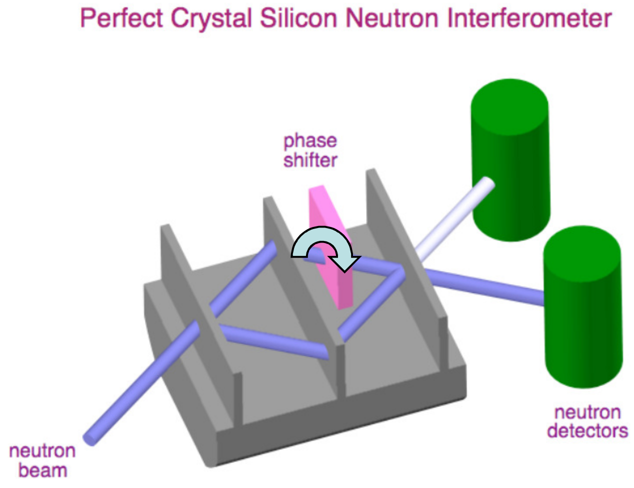


Interferometer family

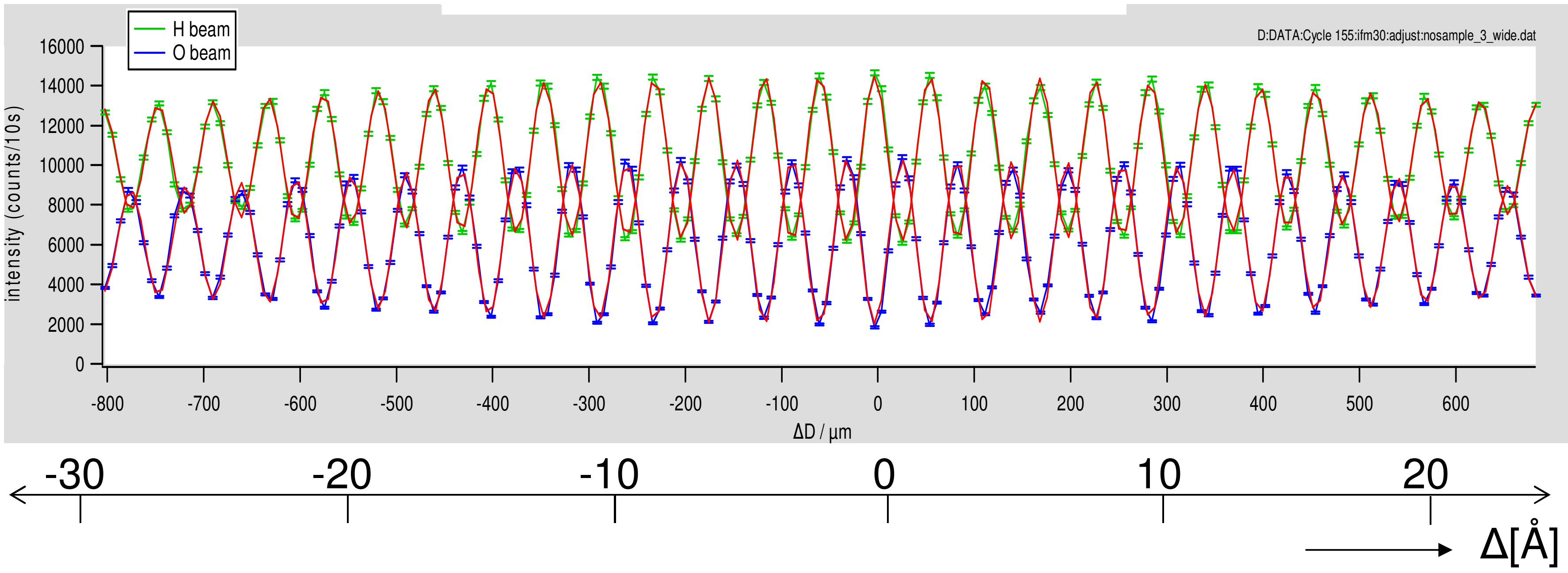


$$I_0 = c|\ trr + rrt\ |^2$$

High order interferences

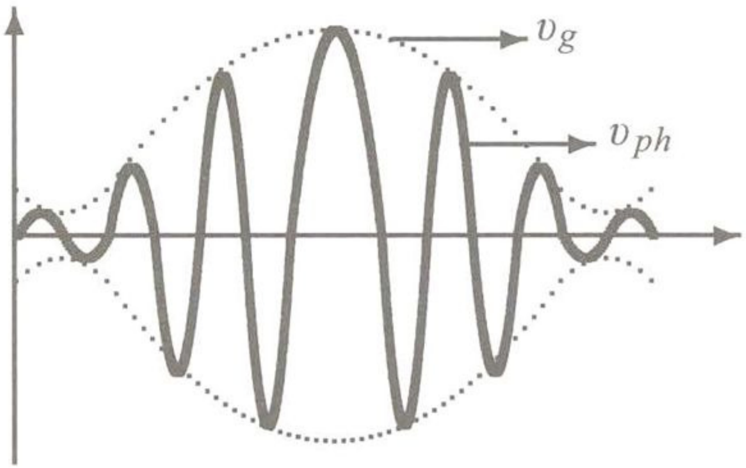


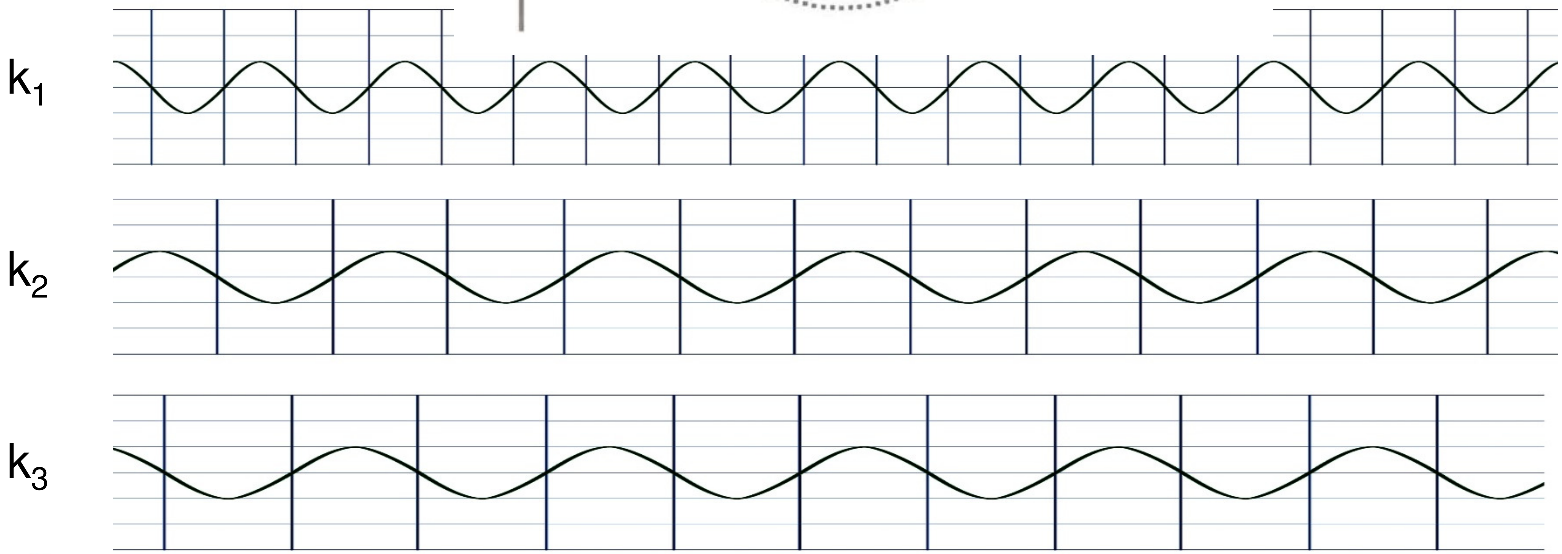
λ = 1.92(2) Å

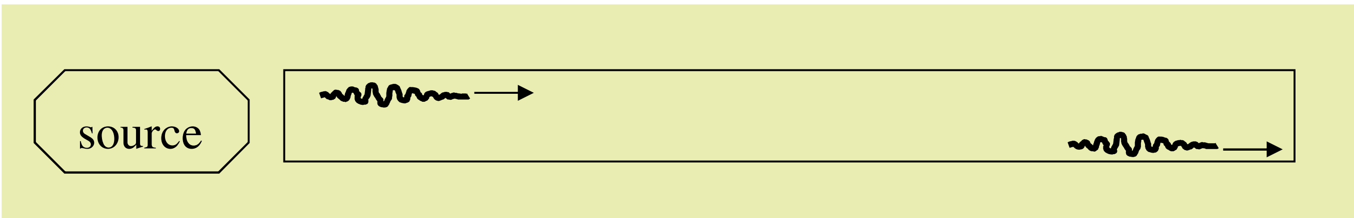


Wave-Packet

$$\Psi(x) \propto \int a(k)e^{ikx}dx$$







State presentations

Schrödinger Equation:

$$-\frac{\hbar^2}{2m}\Delta\psi(\vec{r},t) + V(\vec{r},t)\psi(\vec{r},t) = i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}$$

Partial waves fill the whole space

Wave Function (Eigenvalue solution in free space):

$$\Psi(\vec{r},t) = (2\pi)^{-3/2}\int\psi(\vec{k},t)e^{i(\vec{k}\cdot\vec{r}-\omega t)}d^3\vec{k}$$

and others (Wigner function etc.)

Spatial distribution:

$$\rho(\vec{r},t) = |\psi(\vec{r},t)|^2$$

Momentum distribution:

$$g(\vec{k},t) = |\psi(\vec{k},t)|^2$$

Coherence Function:

Stationary situation: $(\tau = 0)$:

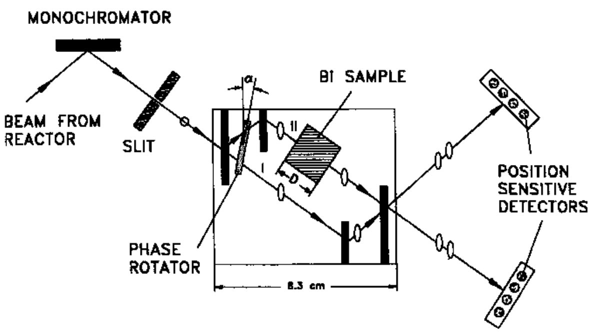
$$\tau = t - t'$$

$$\vec{\Delta} = \vec{r} - \vec{r}';$$

$$\Gamma(\vec{\Delta}) = <\psi(0)\,\psi(\vec{\Delta})> = (2\pi)^{3/2}\int g(\vec{k})e^{i\vec{k}\vec{r}}d^3k$$

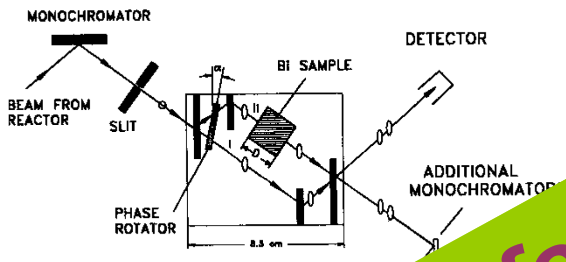
Post-selection methods

POSITION POSTSELECTION



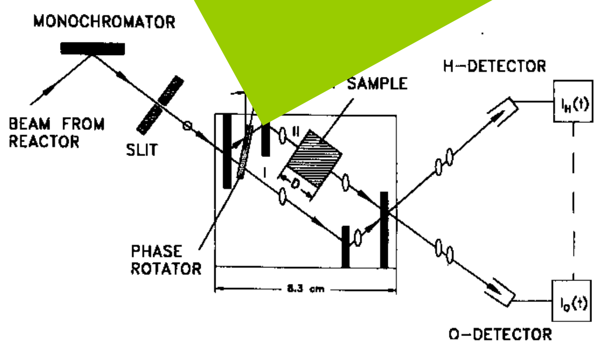
More information by means of position sensitive beam ima

MOMENTUM POSTSELECTION



by means of
sensitive beam
measurement

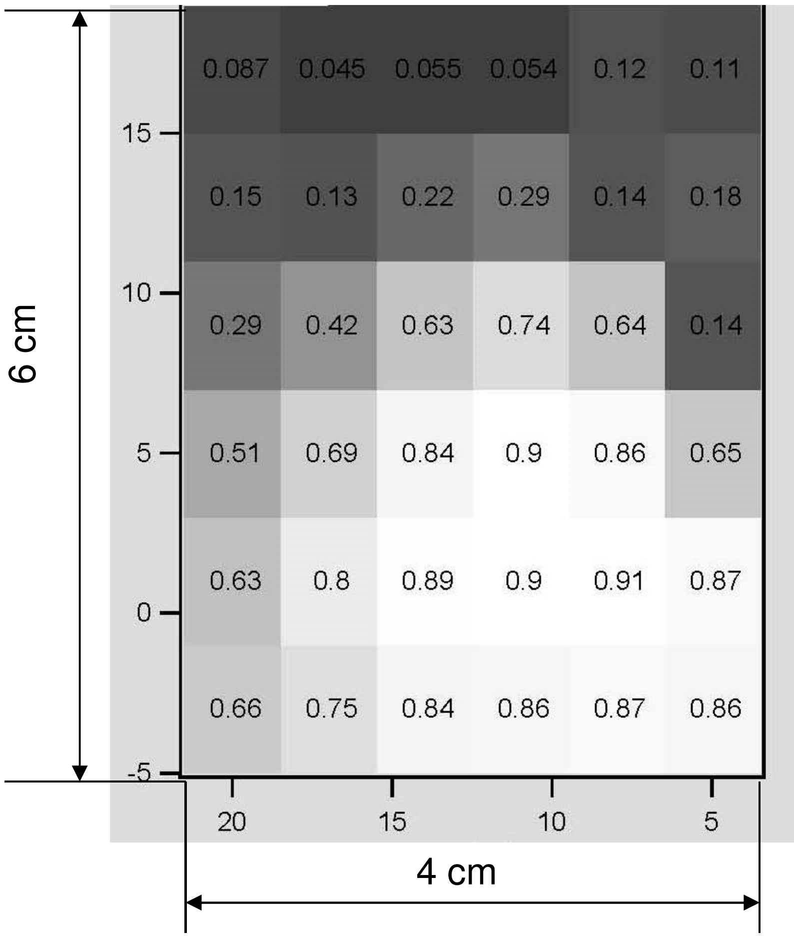
More information – less mystics
- due to post-selection !



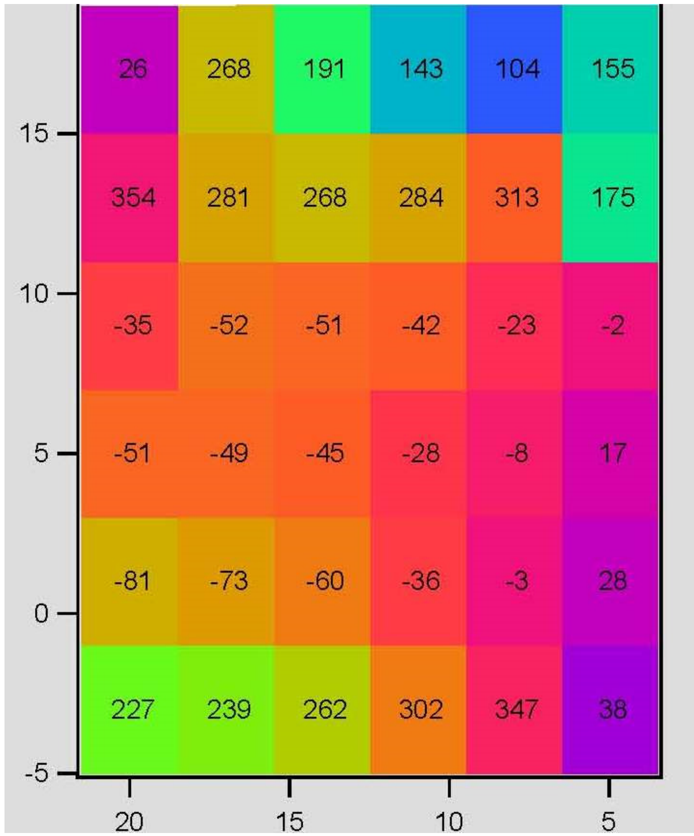
More information by means of time-resolved experiments

Position Post-Selection

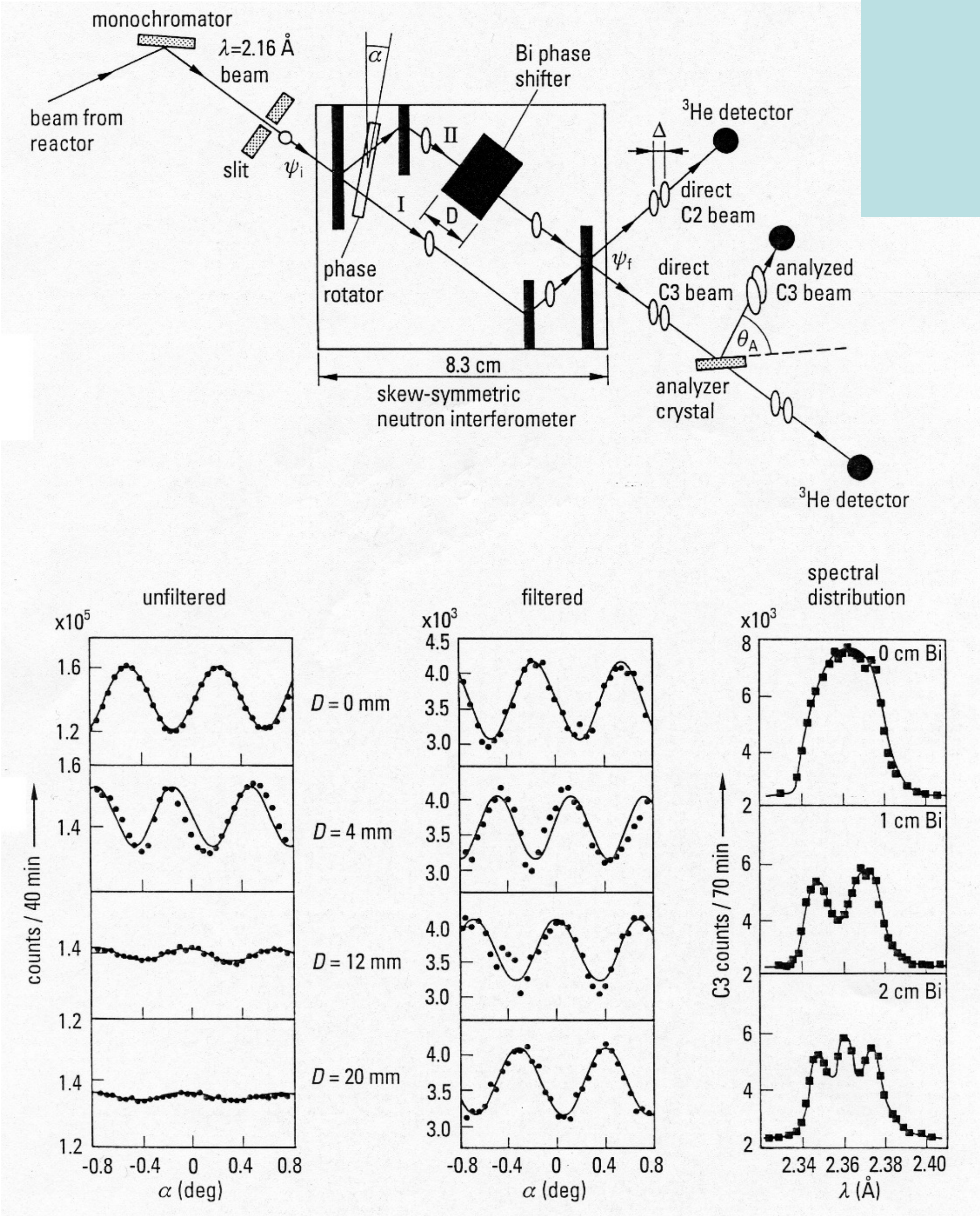
contrast



internal phase



$$I \propto A + B\cos(\chi + \phi)$$



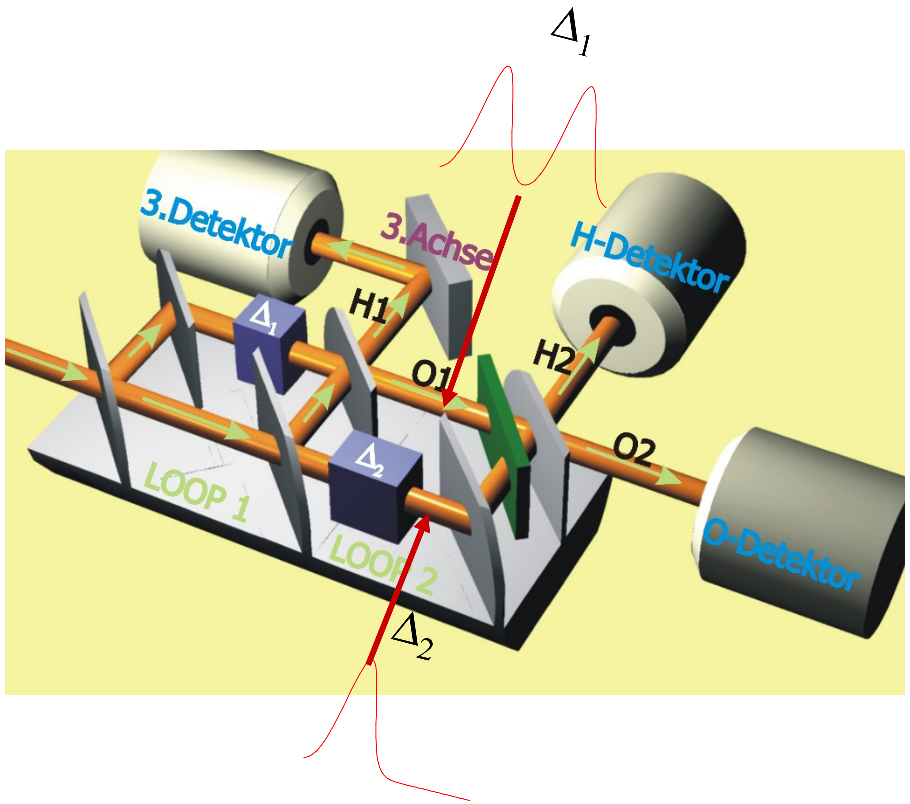
Momentum post-selection

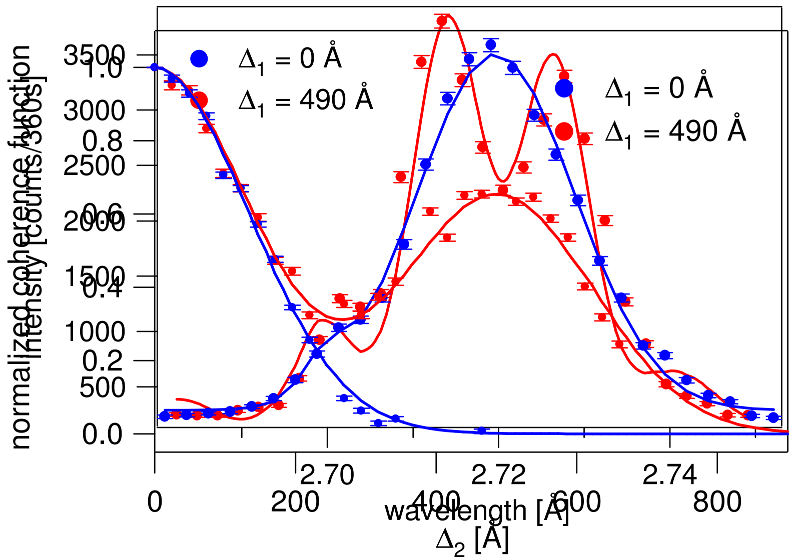
Verification of Schrödinger cat-like states

$$I_0(k) \propto g(k)\left[1-\cos\left(\chi_0 \frac{k}{k\partial}\right)\right]$$

D.L. Jacobson, S.A. Werner, H. Rauch, Phys.Rev A49 (1994) 3196

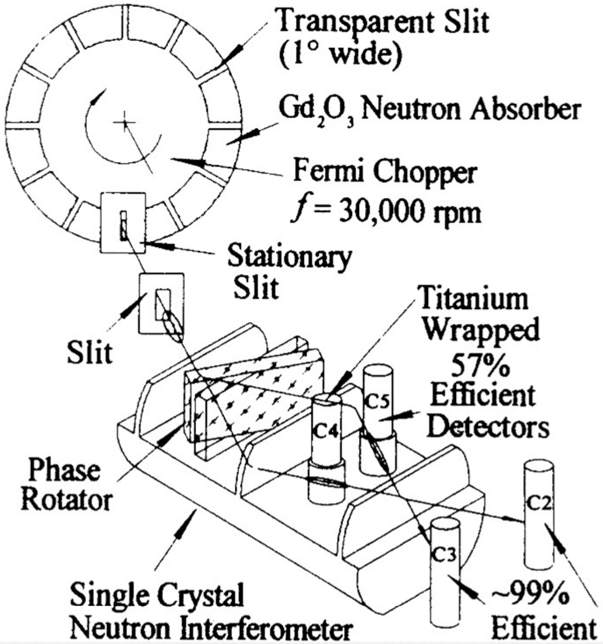
Wave Packet Structure



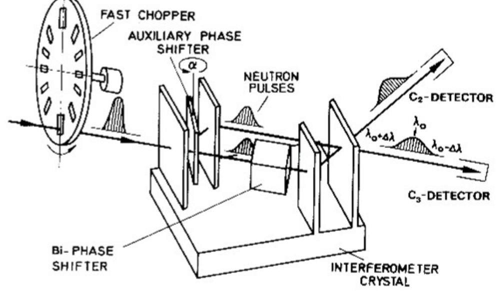


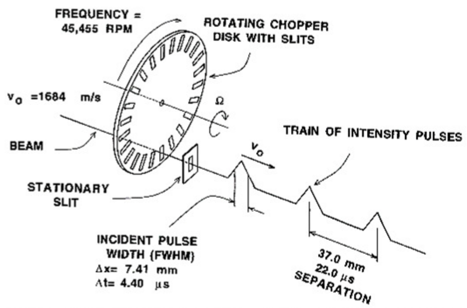
M.Baron, H.Rauch, M.Suda, J.Opt.B5 (2003) S341

Time-Post-selection

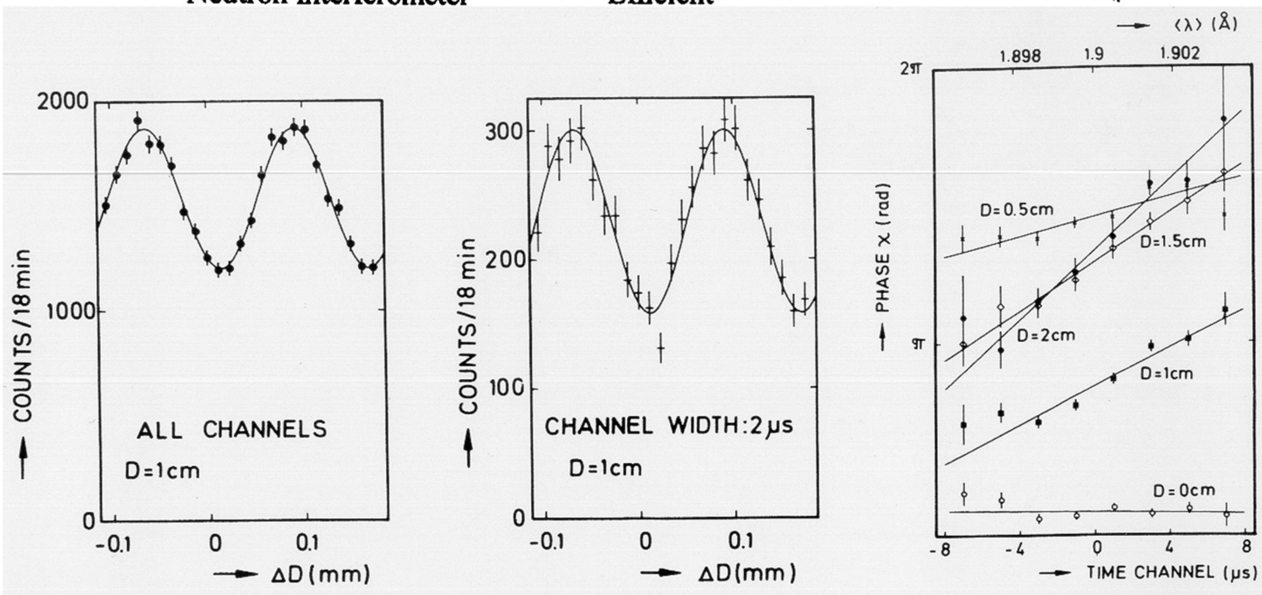


D.L. Jacobson, B.E. Allman, M. Zawisky, S.A. Werner, H. Rauch (1996) J.Jap.Phys.Soc. A65, 94



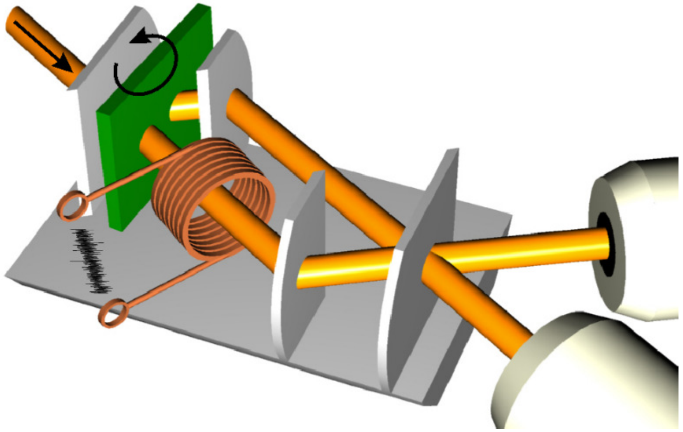


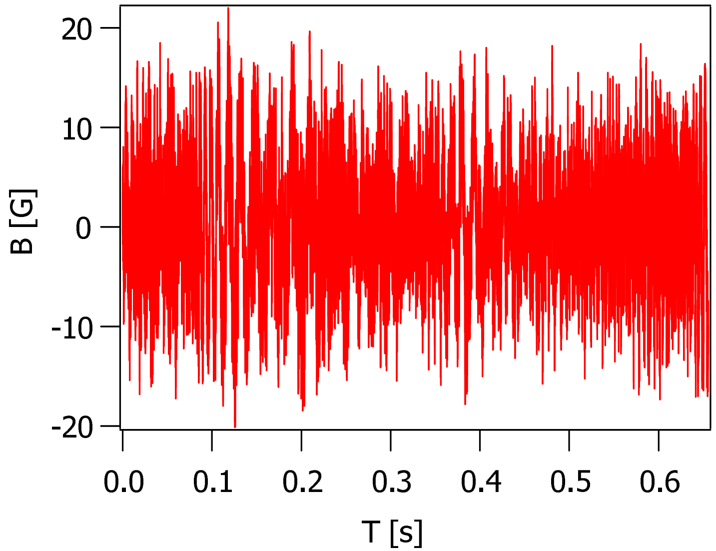
H. Rauch, H. Wölwitsch, R. Clothier, S.A. Werner (1992) Phys. Rev. A46, 49

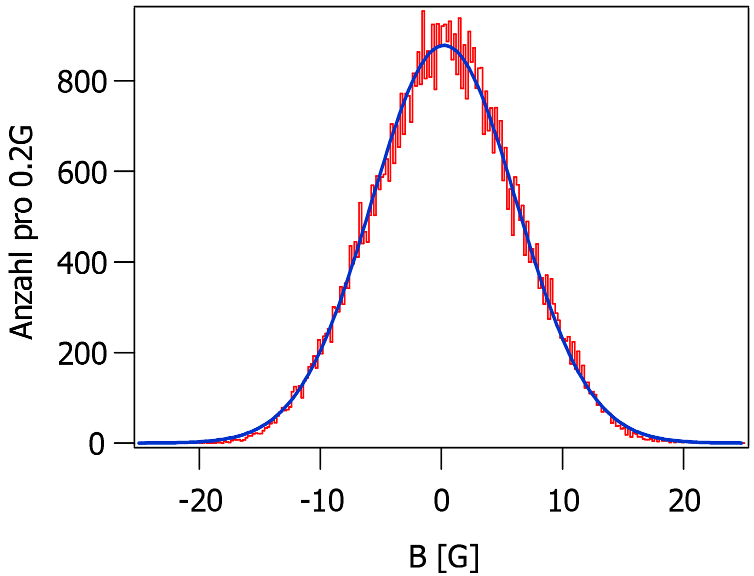


Dephasing - Decoherence

Magnetic noise fields

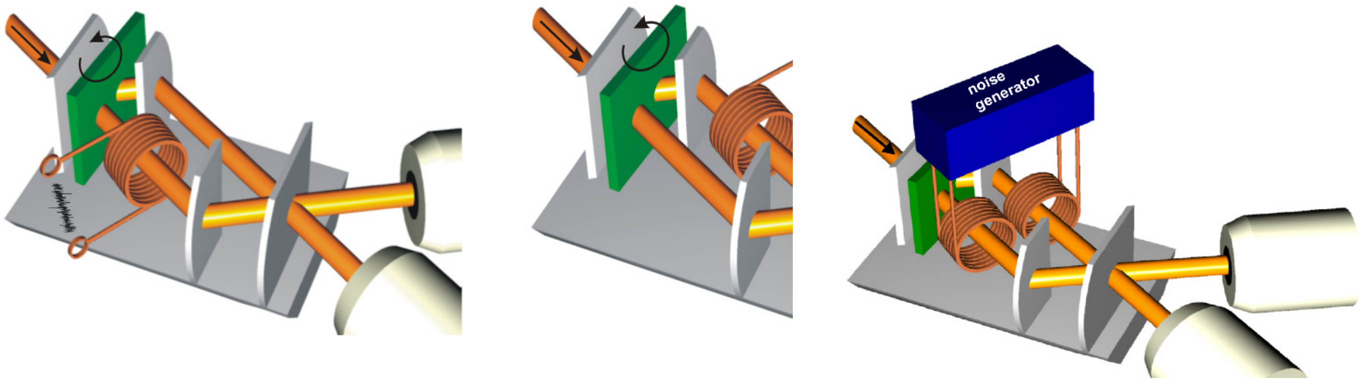


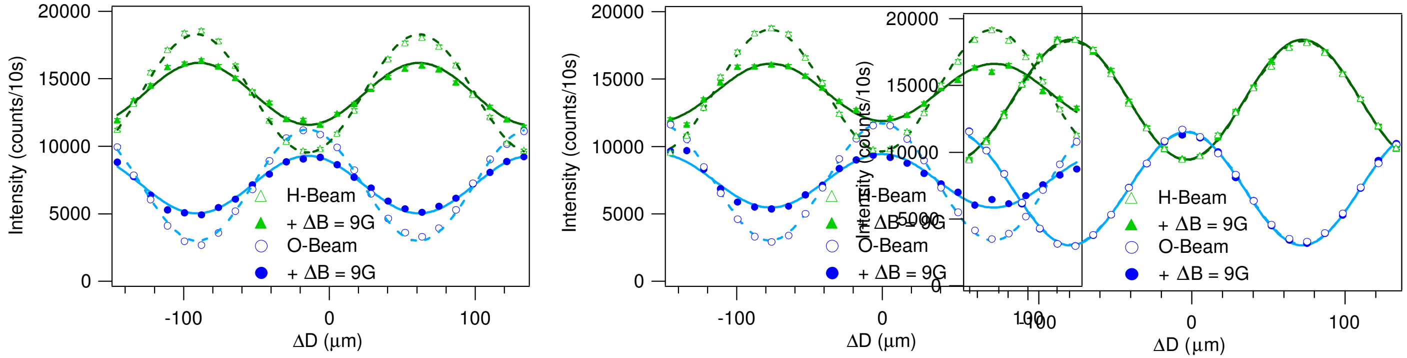




Dephasing at **low** order

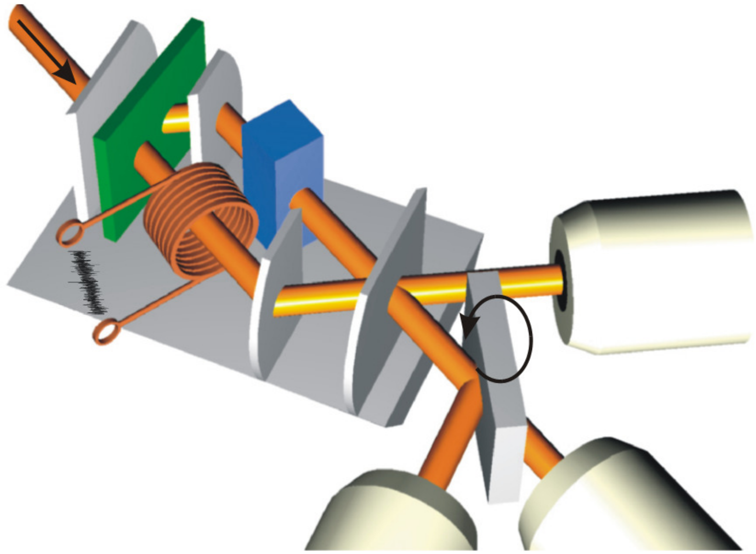
Magnetic noise fields

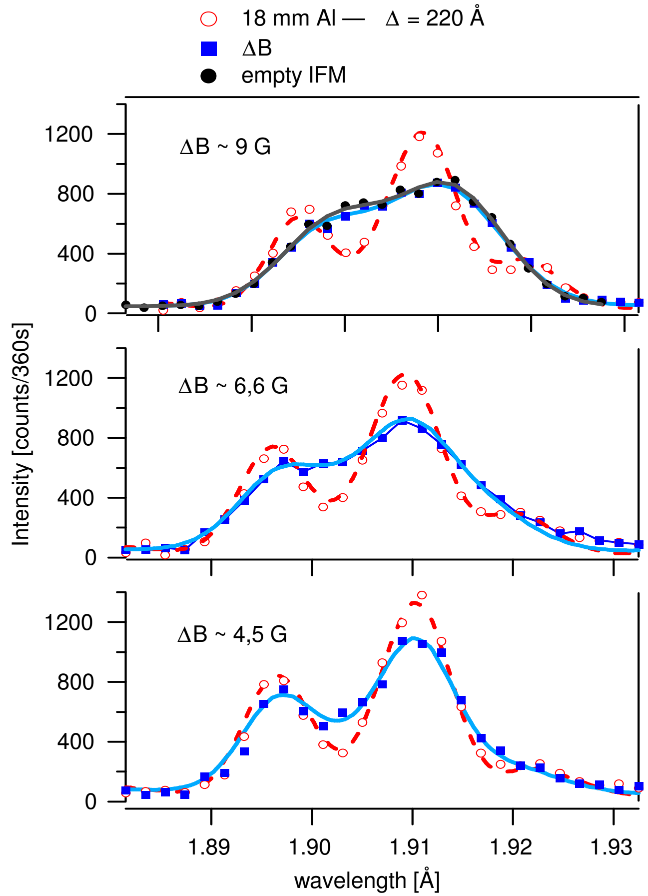




M.Baron, H.Rauch, M.Suda, J.Opt.B5 (2003) S244

Dephasing at high order

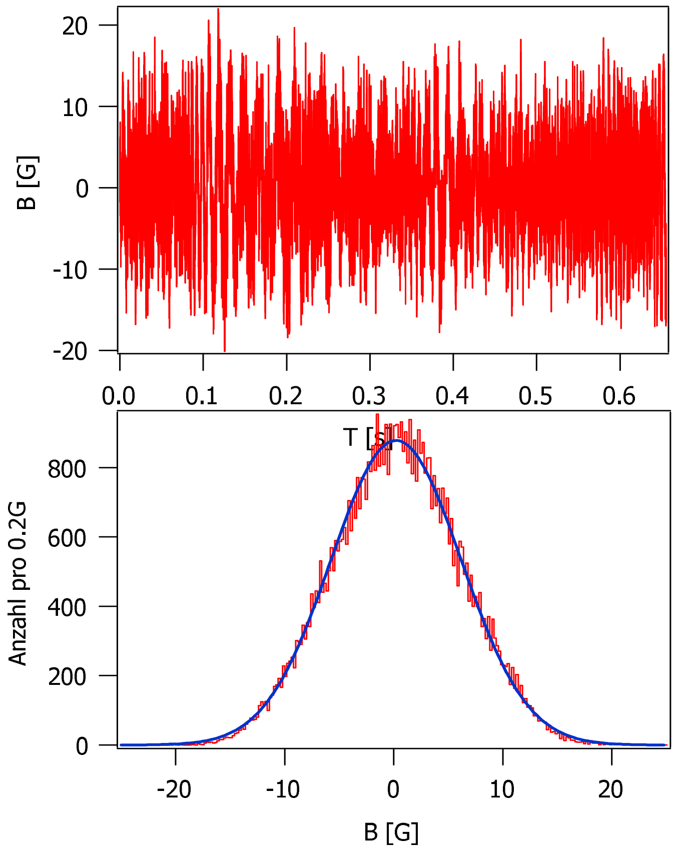




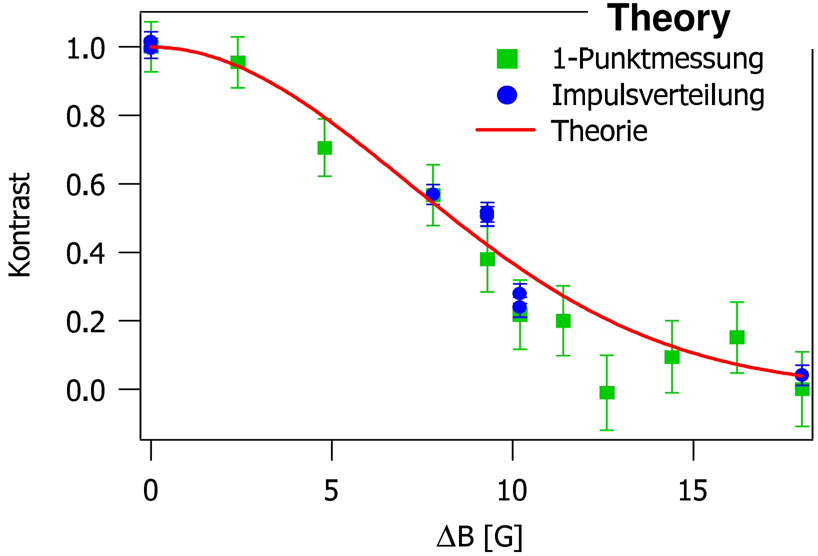
M.Baron, H.Rauch, M.Suda, J.Opt.B5 (2003) S244

Magnetic Noise Field

Dephasing

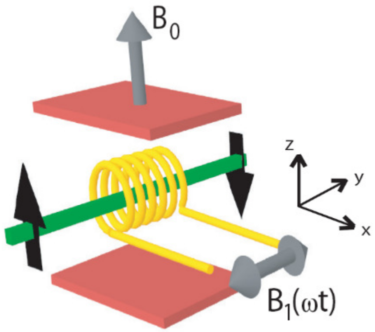


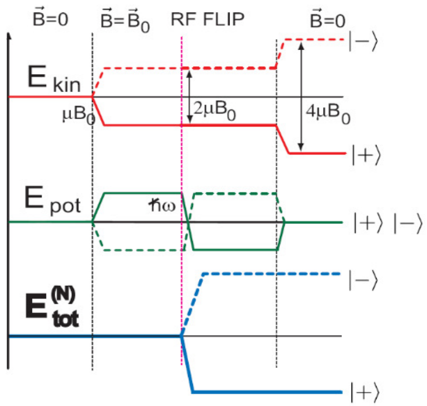
Contrast reduction
Momentum smearing
Theory

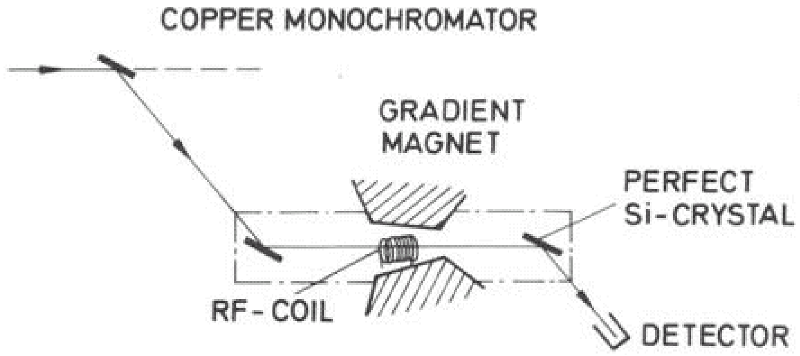


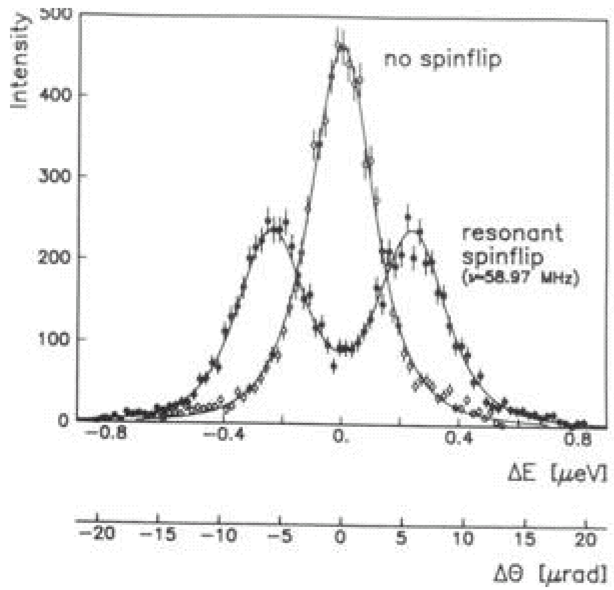
$C = C_0 \exp[-(\mu \Delta B D_{eff}/\hbar v)^2/2]$

Inelasticity: on resonance









$$\eta\omega_r = 2\mu B_0$$

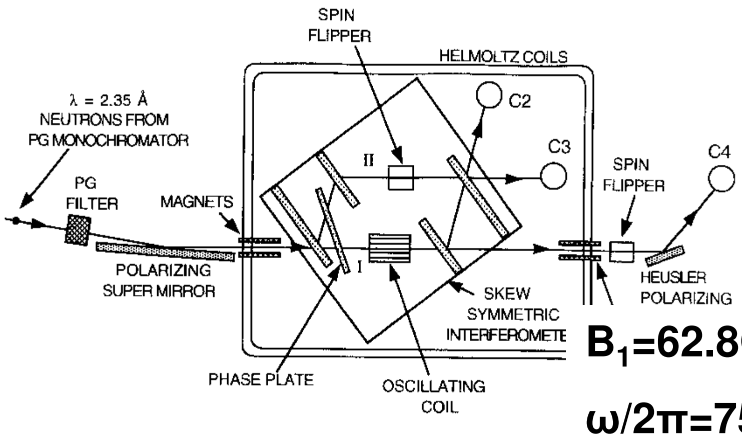
$$B_1 = \frac{\pi\eta v_n}{2\mu l}$$

i.e. single photon exchange

H.Weinfurter, G.Badurek, H.Rauch, D.Schwahn, Z.Phys. B72(1988)195

Off-resonance

(Multi photon exchange)



B_1=62.8G

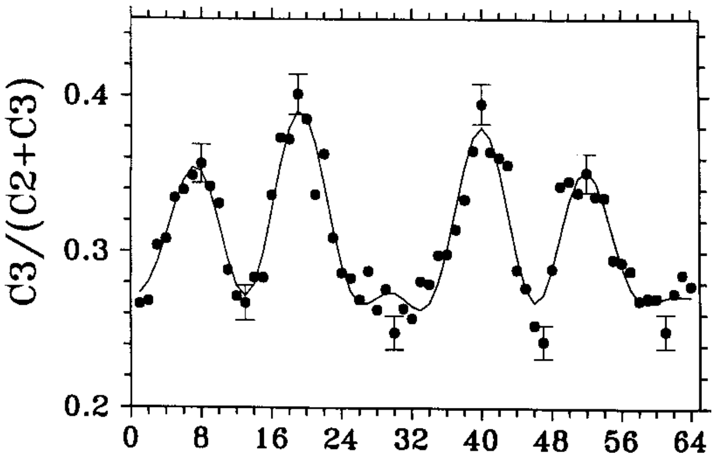
ω/2π=7534 Hz

$$\vec{B}(t) = \begin{pmatrix} 0 \\ 0 \\ B_0 + B_1 \cos \omega t \end{pmatrix}$$

$$\left|\psi_f\right\rangle = \sum_{j=-\infty}^{j=+\infty} \left[a_j \left|+z\right\rangle + b_j \left|-z\right\rangle\right] e^{i\omega_j t}$$

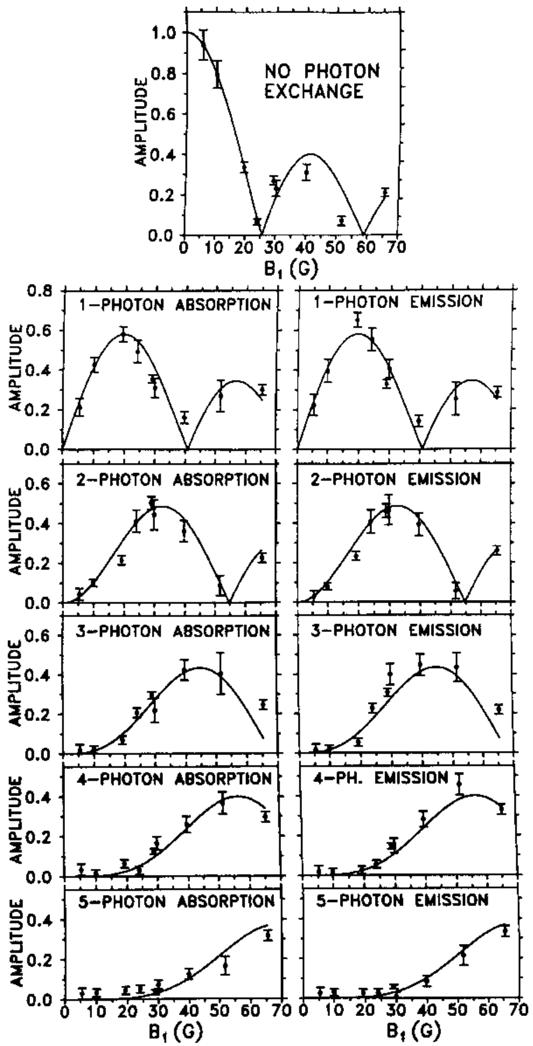
$$\omega_j = \omega_0 + j\,\omega$$

$$a_j = J_j(\alpha B_1) \qquad \alpha = \frac{\mu}{\eta\omega}\sin(\omega\tau/2)$$



$$I \propto \left|\left|+z\right\rangle + e^{i\chi}\left|\psi_f\right\rangle\right|^2 = 1 + \left|\sum_{j=-\infty}^{+\infty} \left(a_j\left|+z\right\rangle + b_j\left|-z\right\rangle e^{-ij\omega t}\right)\right|^2 + 2\sum_{j=-\infty}^{j=\infty}\left|a_j\right|\cos(\phi_j + \chi - j\omega t)$$

Multi-photon exchange: results



v = 7534 Hz → ΔE= $3.24.10^{-11}$eV

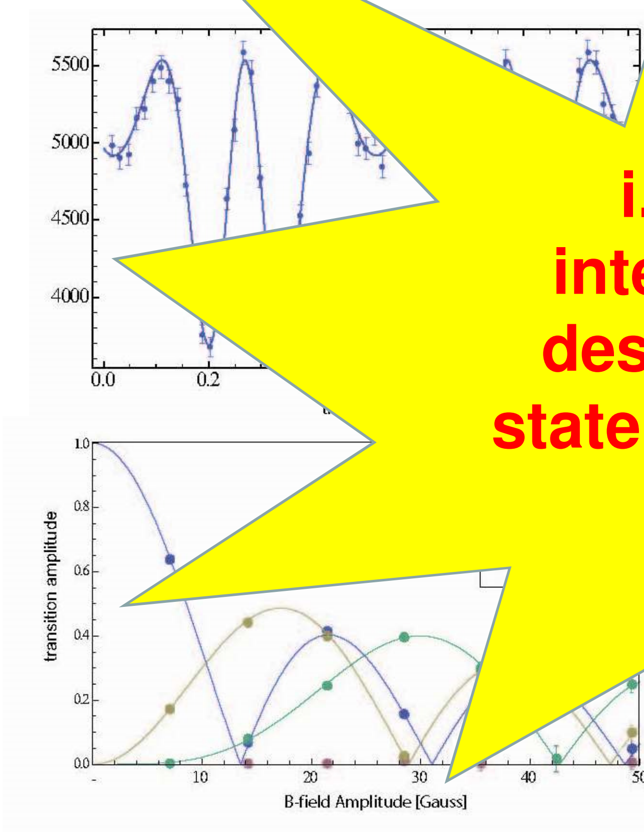
<< ΔE_{beam} = 10^{-4}eV

J. Summhammer, K.A. Hamacher, H. Kaiser, H. Weinfurter, D.L. Jacobson, S.A. Werner, Phys.Rev.Lett. 75 (1995) 3206

Multi-frequency photon exchange

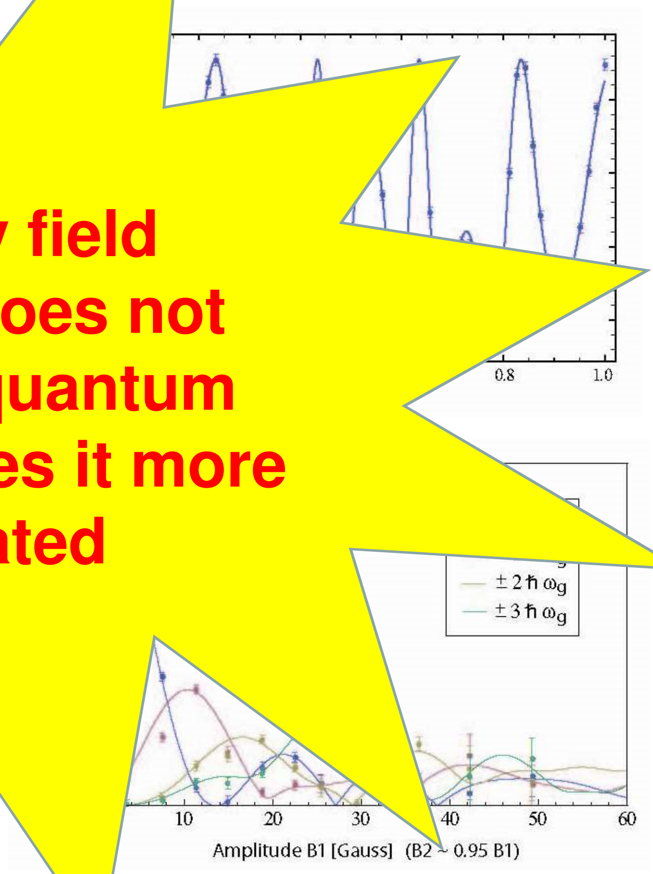
i.e. a noisy field interaction does not destroy the quantum state, but makes it more complicated

...e frequency



ω_1= 1 kHz, B_1= 40 G

two frequencies



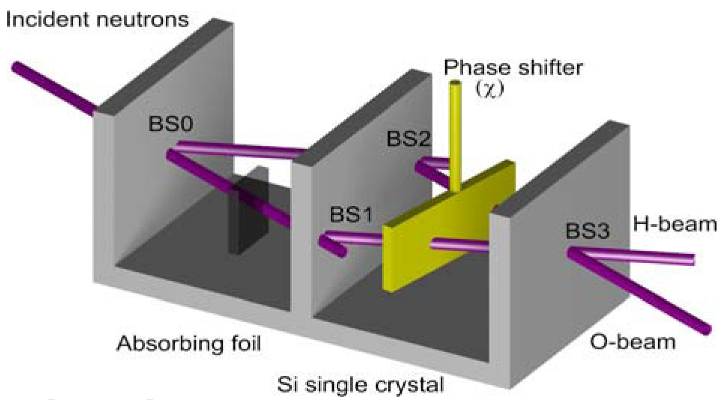
ω_1= 2 kHz; ω_2= 3 kHz;

B_1= 15 G; B_2= 14.2 G

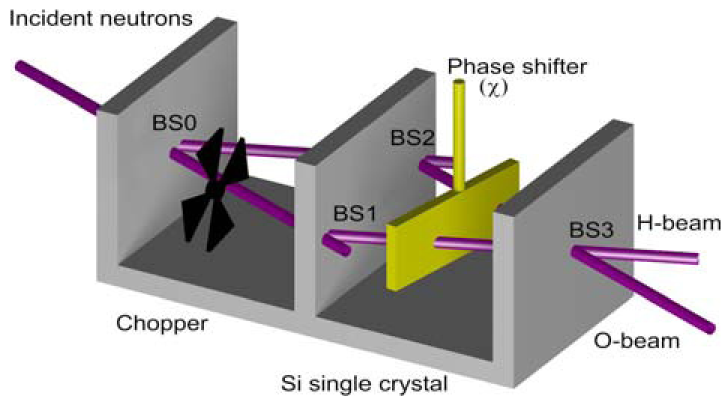
G. Sulyok, H. Lemmel, H. Rauch, Phys.Rev. A85 (2012) 033624

Weak Measurements

Absorbing phase shifter



stochastic



deterministic

$$\psi' = \psi\, e^{i(\chi' + i\chi'')} = \psi \sqrt{a}\, e^{i\chi'}$$

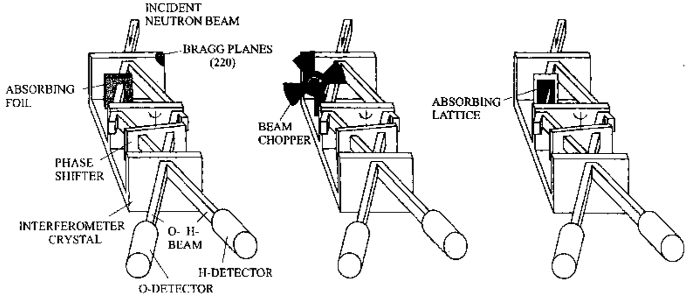
$$a = \frac{t_{open}}{t_{open} + t_{closed}}$$

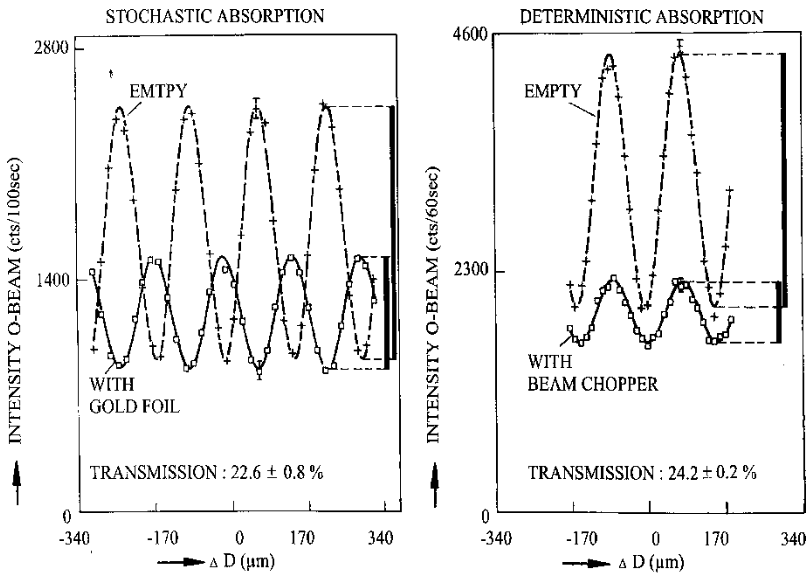
$$\chi'' = (\sigma_a + \sigma_{inc}) ND/2 \qquad a = \frac{I}{I_0} = e^{(\sigma_a + \sigma_{inc}) ND}$$

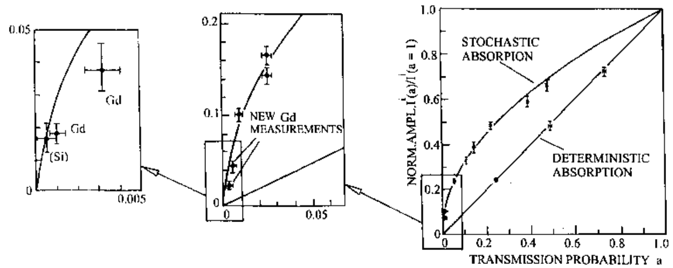
$$I_{det} \propto \left[(1-a)\left|\Psi_0^{II}\right|^2 + a\left|\Psi_0^I + \Psi_0^{II}\right|^2 \right]$$

$$I_{sto} = \left|\psi_0{}^I + \psi_0{}^{II}\right|^2 \propto \left|\psi_0{}^I\right|^2 \left((a+1) + 2\sqrt{a}\cos\chi'\right).$$

$$\propto \left|\Psi_0^I\right|^2 \left[(a+1) + 2a\cos\chi'\right]$$







J. Summhammer, H. Rauch, D. Tuppinger, Phys.Rev. A 36 (1987) 4447

Absorption results

Small a-case:

$$\chi'' = (\sigma_a + \sigma_{inc})ND/2$$

$$\chi'' \to \chi_0'' + \delta\chi''$$

$$\overline{e^{(\sigma_a+\sigma_{inc})ND}} = \sqrt{\bar{a}} = \overline{e^{-(\chi+\delta\chi'')}} = \sqrt{a_0}\,e^{(\delta\chi'')^2/2}$$

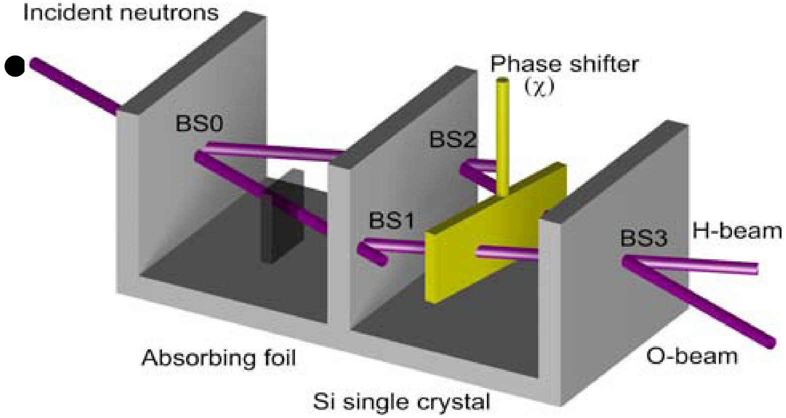
$$\bar{a} = a_0 e^{(\delta\chi'')^2}$$

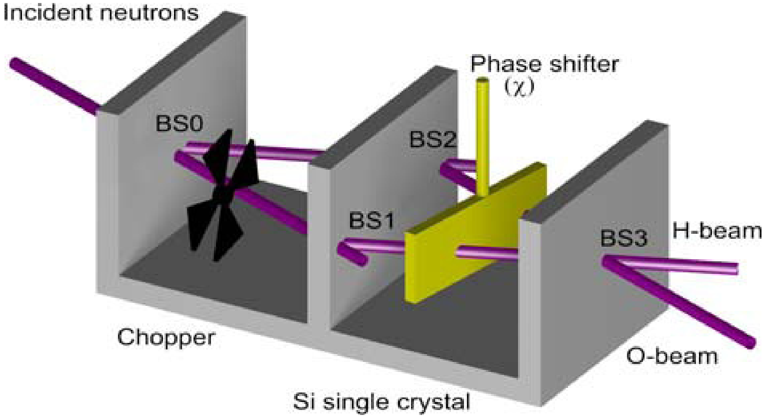
$$\sqrt{\bar{a}} < \sqrt{a_0}$$

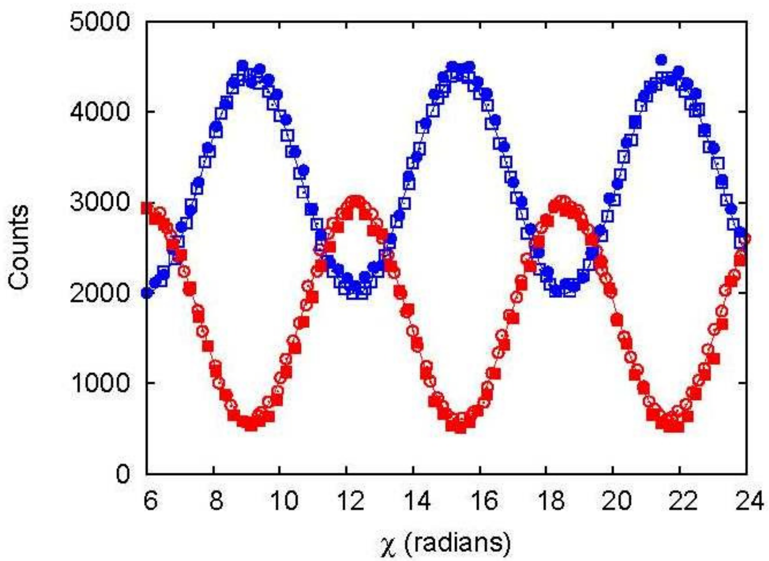
$P^2+V^2 > 1$

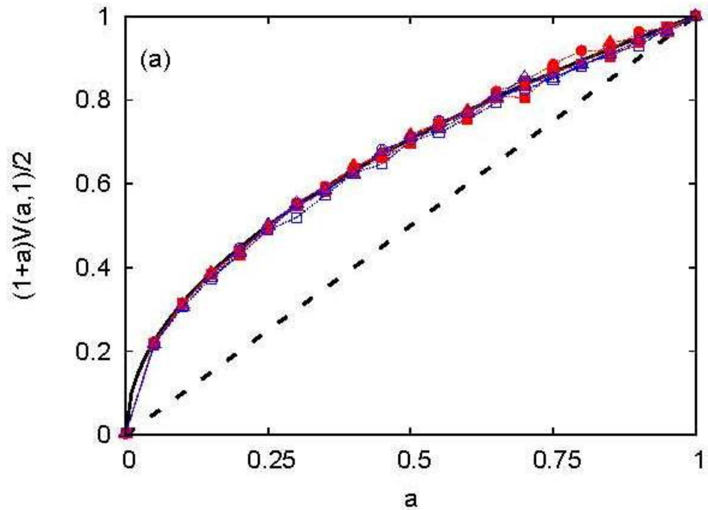
(Greenberger-Englert relation)

Event by event simulation



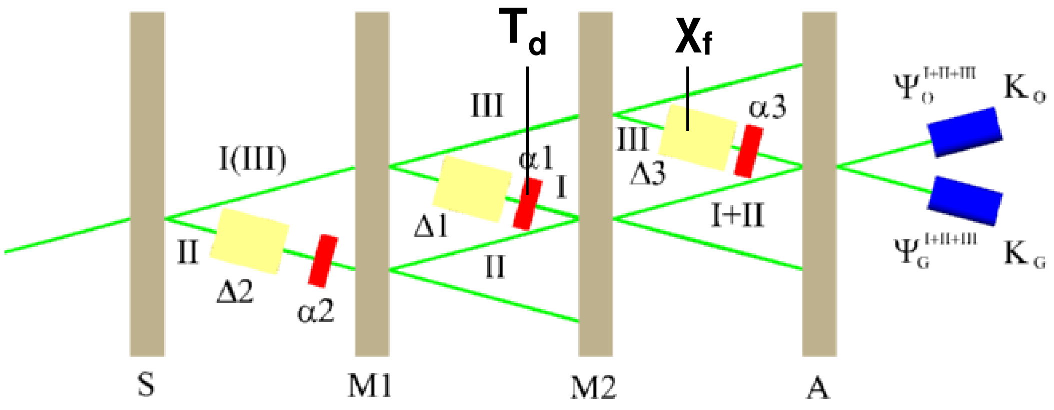






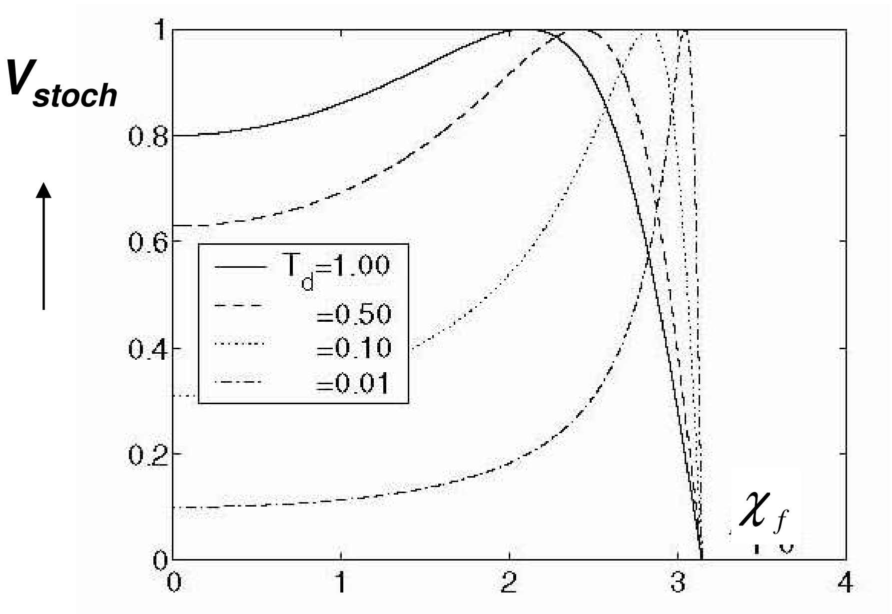
H. De Raedt, F. Jin, K. Michielsen, Quantum Matter 1 (2012) 20

Double Loop Visibility



$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$V_{sto2\Delta_f} = \frac{4\sqrt{T_d}\cos(\chi_f/2)}{4\cos^2(\chi_f/2) + T_d}$$

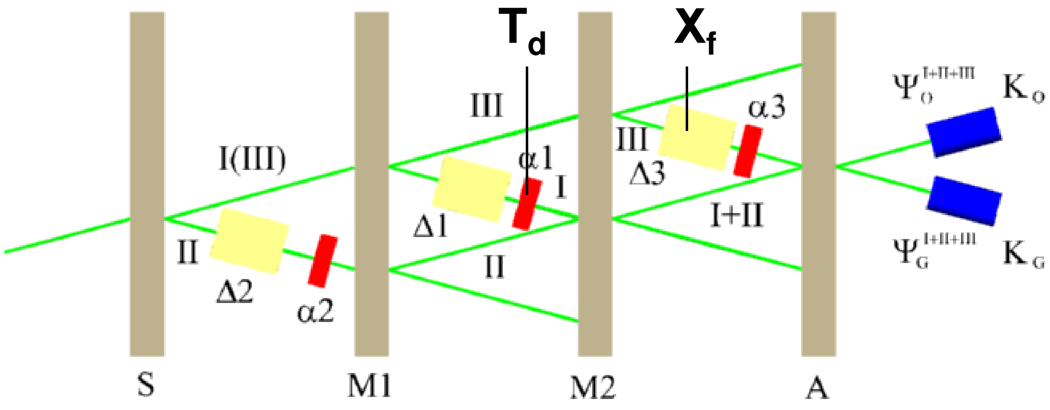


$$V_{sto2\Delta_f} = 1 \quad if \quad T_d = 4\cos^2(\chi_f/2)$$

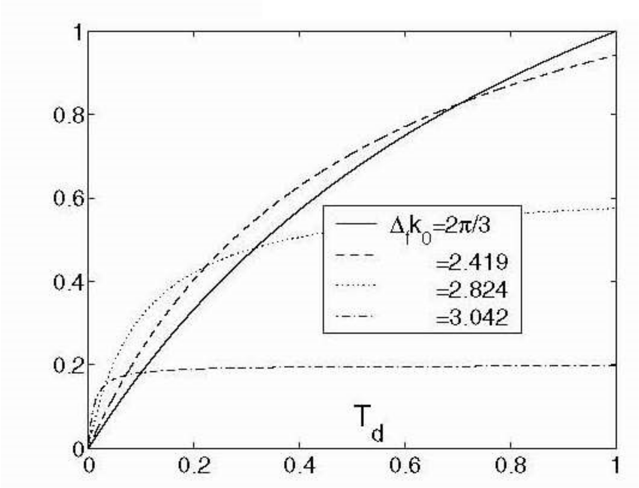
$\Rightarrow$ homodyne detection

M. Suda, H. Rauch, M. Peev,
J.Opt.B:Quantum Semiclass.Opt. 6(2004)345

Stimulated Coherence

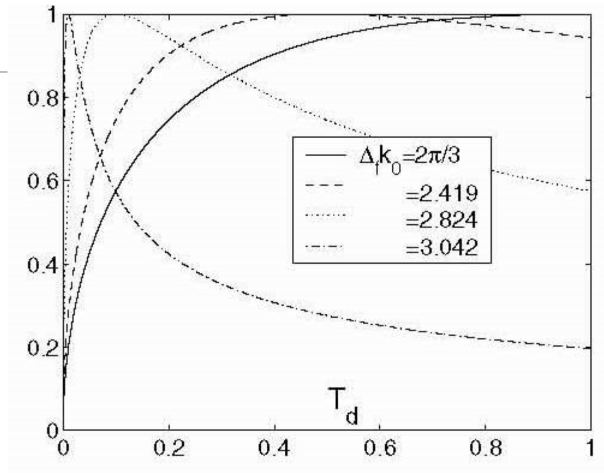


deterministic



$$V_{\det 2\chi_f} = \frac{4T_d \cos(\chi_f / 2)}{4\cos^2(\chi_f / 2) + T_d} \leq 1$$

stochastic



$$V_{sto 2\chi_f} = \frac{4\sqrt{T_d} \cos(\chi_f / 2)}{4\cos^2(\chi_f / 2) + T_d}$$

Robustness of topological phases

Berry-topological phase 1

$$|\psi(t)\rangle = e^{i\Phi(t)}|n(\vec{R}(t))\rangle$$

$|n(\vec{R}(t))\rangle$ …. eigenstates of the instantaneous Hamiltonian

$\Phi(t)$ ….. generalized phase

$$\hat{H}(\vec{R})|n(\vec{R}(t))\rangle = E_n(t)|n(\vec{R}(t))\rangle$$

for a closed path: $|\psi(\vec{R}(T))\rangle = |\psi(\vec{R}(0))\rangle$

$$\vec{H}(\vec{R})\,|\,\psi(t)> = i\hbar\frac{\partial\,|\,\psi(t)>}{\partial t}$$

$$\Phi(T) = \arg <\psi(T)|\psi(0)> = -\frac{1}{\hbar}\int_0^T <\psi(t)\,|\,\mathrm{H}\,|\,\psi(t)>dt + i\int<\phi(t)\,|\,\frac{d}{dt}\,|\,\phi(t)>dt$$

$$= \int_0^T E_n(\vec{R}(t))dt + i\oint d\vec{R} < n(\bar{R})\,|\,\nabla_{\vec{R}}\,|\,n(\vec{R})> = \delta + \gamma$$

dynamical phase   geometric phase

for a constant magnetic field:

$$|\psi> = \cos\frac{\theta}{2}|\uparrow> + \sin\frac{\theta}{2}|\downarrow> = \begin{pmatrix} \cos\theta/2 \\ e^{i\phi}\sin\theta/2 \end{pmatrix}$$

$$|\psi(t)> = e^{iHt/\hbar}\,|\,\psi(0)> = e^{-i\mu Bt/\hbar}\,|\,\psi(0)>$$

≐ 4π-symmetry of spinors

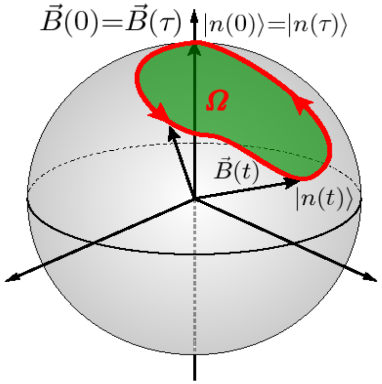
Geometric Phases

Berry Phase (adiabatic & cyclic evolution)

[Berry; Proc.R.S.Lond. A 392, 45 (1984)]

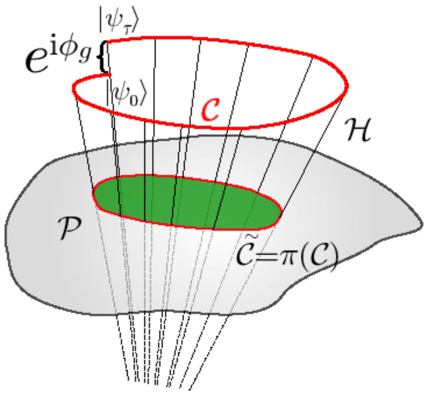
$$
\begin{aligned}
|\psi(t)\rangle &= e^{-i\phi_d} e^{i\phi_g} |n(R(t))\rangle \\
\phi_d(t) &= \frac{1}{\hbar}\int_0^t dt' E_n(t') \\
\phi_g &= -\frac{1}{2}\,\boldsymbol{\Omega}
\end{aligned}
$$

(for 2-level systems)



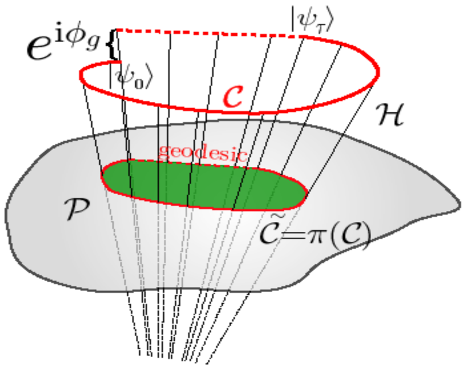
Non-adiabatic evolution

[Aharonov & Anandan, PRL 58, 1593 (1987)]

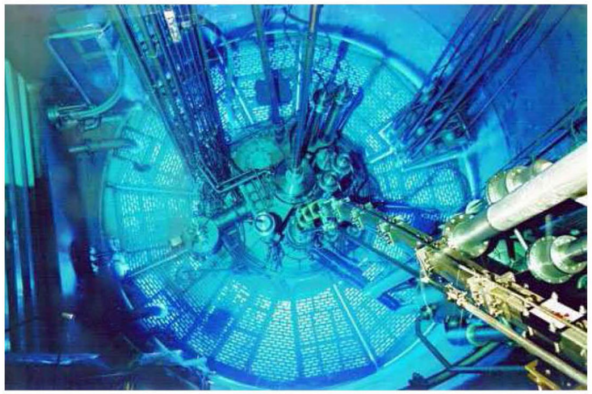


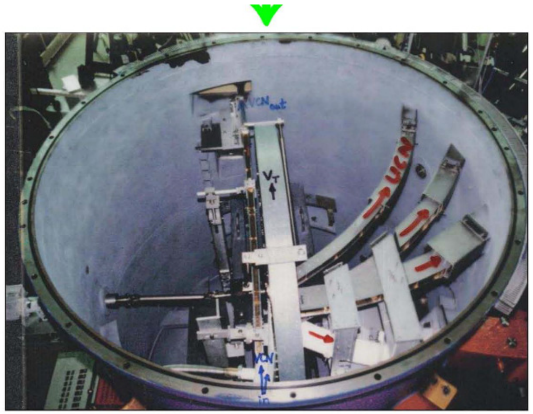
Non-adiabatic & non-cyclic evolution

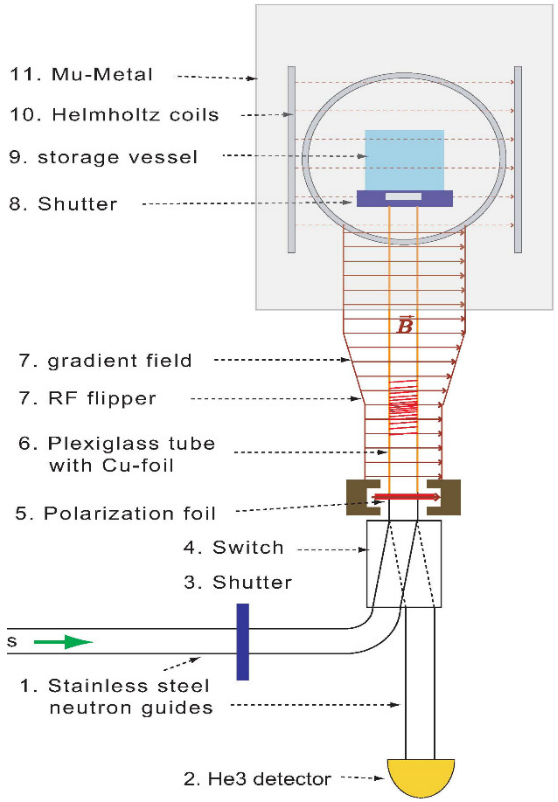
[Samuel & Bhandari, PRL 60, 2339 (1988)]



Ultra-cold neutrons at ILL



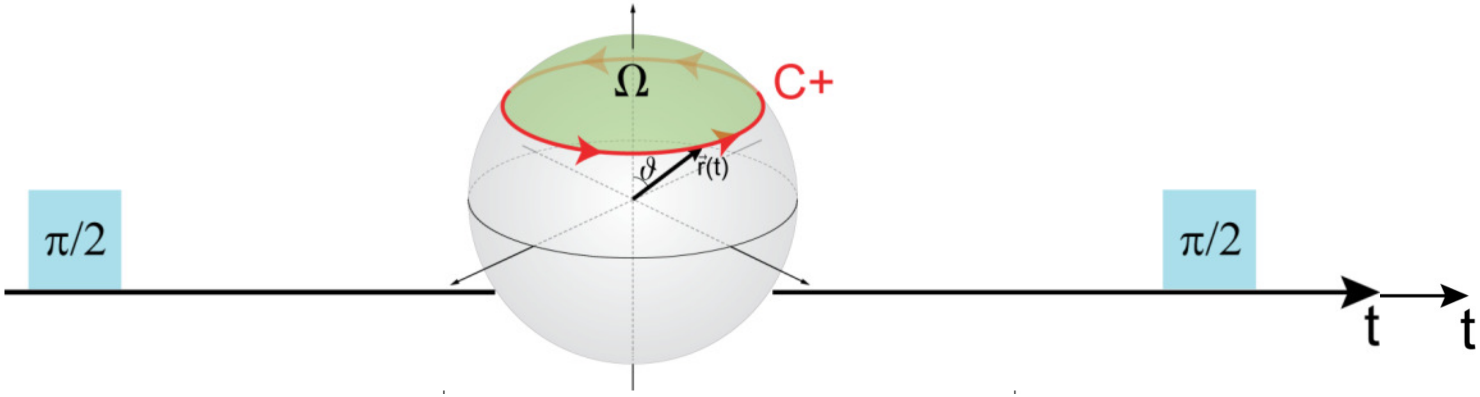




Spin echo to cancel dynamical phase

Ramsey experiment measures both geometric + dynamical phase

Constant B-field strength, but reverse path direction



+dynamic
+geometric

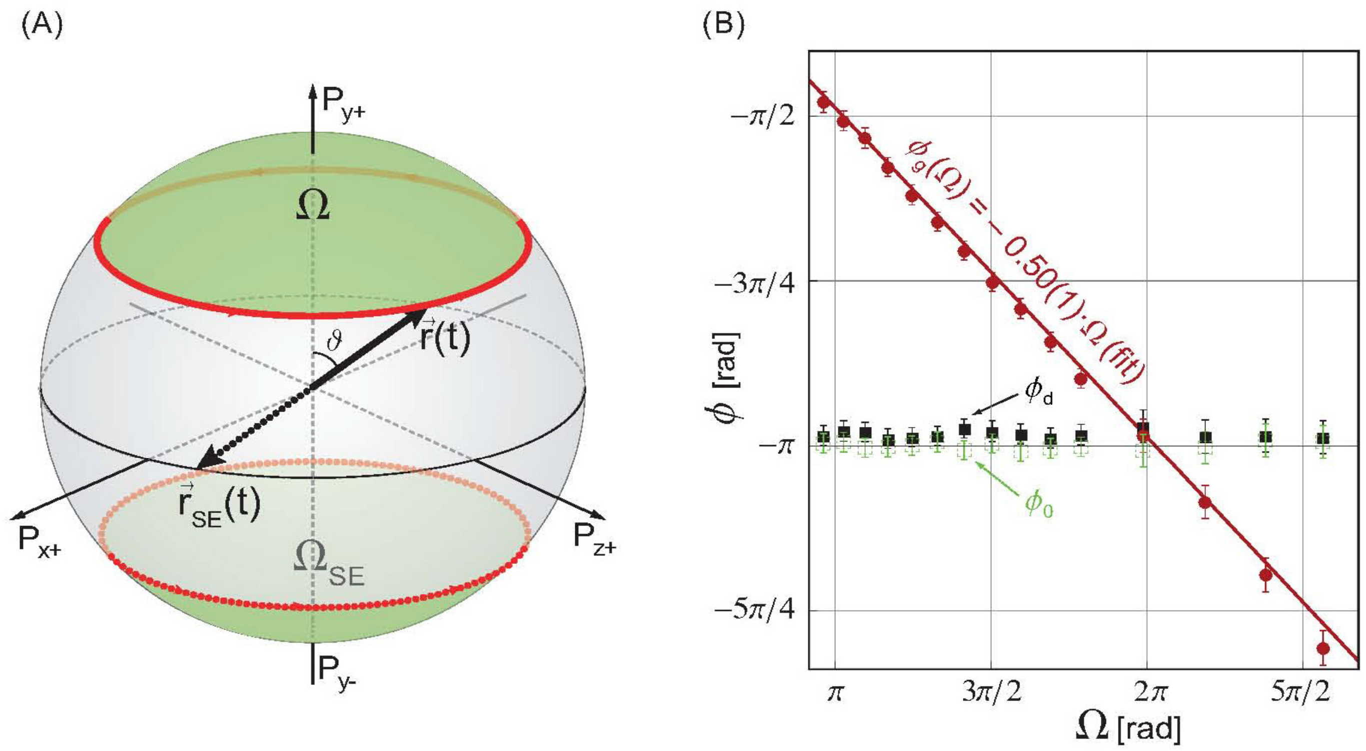
-dynamic
+geometric

= dynamic + geometric phase

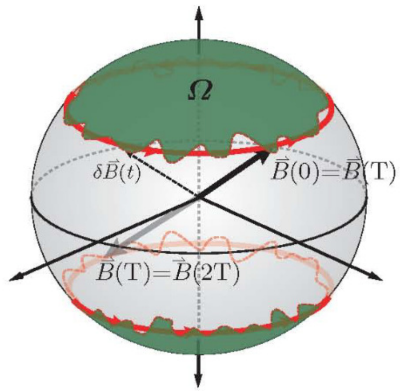
= 2x geometric phase

Compensation of the dynamical phase

ϕ_g for different ϑ, i. e. x-offset fields
$(\omega = 2\pi/T \approx 30 \text{ rads}^{-1} \,\widehat{=}\, \text{T} = 200 \text{ ms}, \; \omega_L = 1832 \text{ rads}^{-1} \,\widehat{=}\, \text{B} = 10\ \mu\text{T})$:



Compensation in the case of noise fields



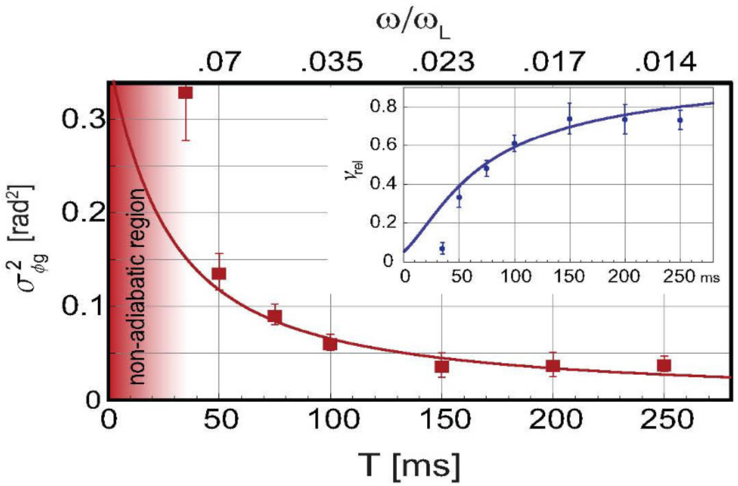
Spin-Echo Setup:

- One cycle: $\psi(\tau) = e^{i(\phi_d+\phi_g)}\psi(0)$
- Spin Echo: $\phi_d = 0$ (spin first in the positive and then in negative eigenstate of the magnetic field Hamiltonian).
- Geometric phase $\phi_g(2\tau) = 2\phi_g(\tau)$

Rubustness of the geometric phase

Predicted by:
G. De Chiara and G.M. Palma, PRL 91, 090404 (2003)
R.S. Whitney, Y. Gefer, Phys.Rev.Lett. 90(2003)190402

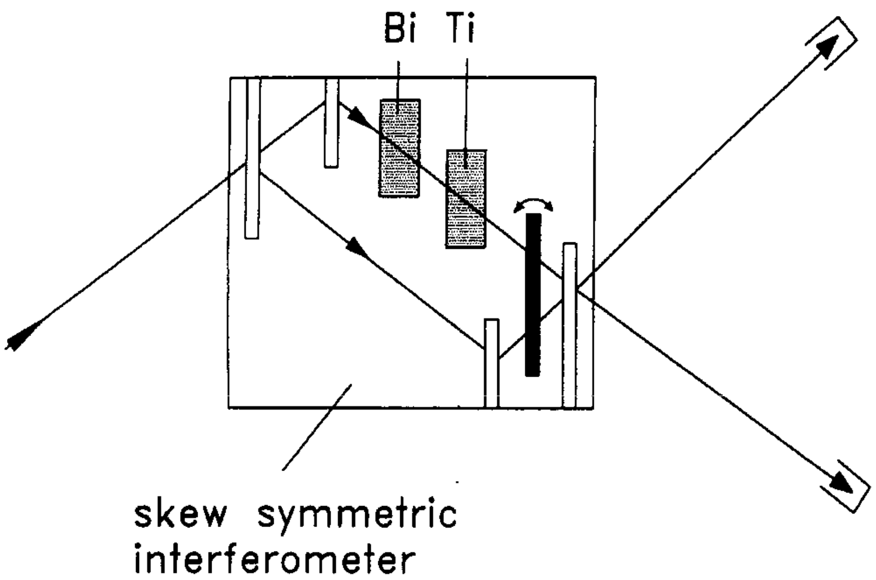
- $\phi_g^0 = 2.58 \text{ rads}^{-1}$, $B = 10\ \mu\text{T}(1832 \text{ rad/s})$
- noise rms $\sigma_P = 2\ \mu\text{T}(366 \text{ rad/s})$, bandwidth $\Gamma = 100$ rad/s
- $\overline{\phi}_g$: averaged over 300 cycles
- State tomography (6 cycles)
- measure degree of polarisation relative to noise-free evolution $\nu_{rel} = e^{-8\sigma^2_{\phi g}}/e^{-8\sigma^2_{g0}}$
- $\langle\overline{\phi}_g\rangle - \phi_g^0 = 0.0(1)$ rad

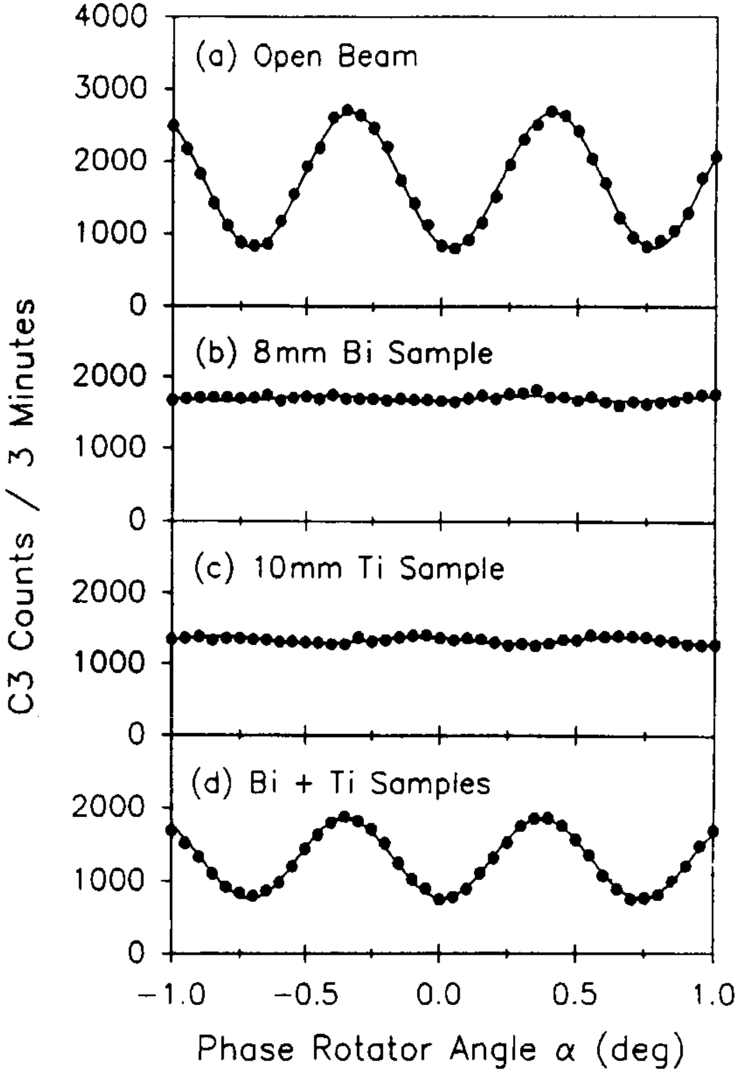


F. Filipp, J. Klepp, Y. Hasegawa, Ch. Plonka, P. Geltenbort, U. Schmidt, H. Rauch, Phys.Rev.Lett.102 (2009) 030404

Unavoidable Quantum Losses

Phase echo

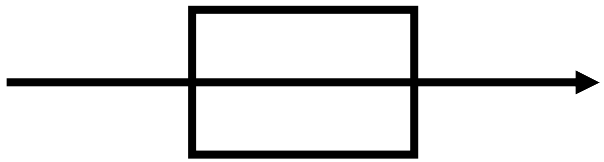




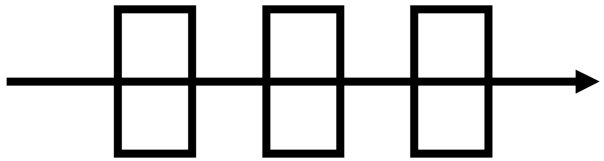
Clothier R., Kaiser H., Werner S.A., Rauch H., Woelwitsch H., Phys.Rev.A44 (1991)5357

REVERSIBILITY-IRREVERSIBILITY

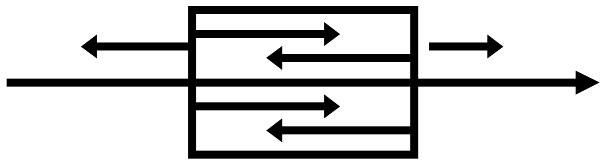
$$\psi_0$$



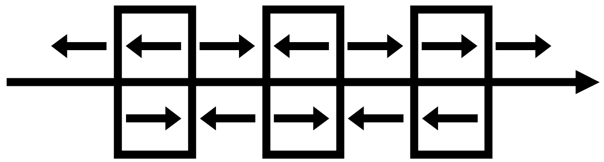
$$\psi_0 e^{i\chi}$$



$$\psi_0 e^{i\chi}$$

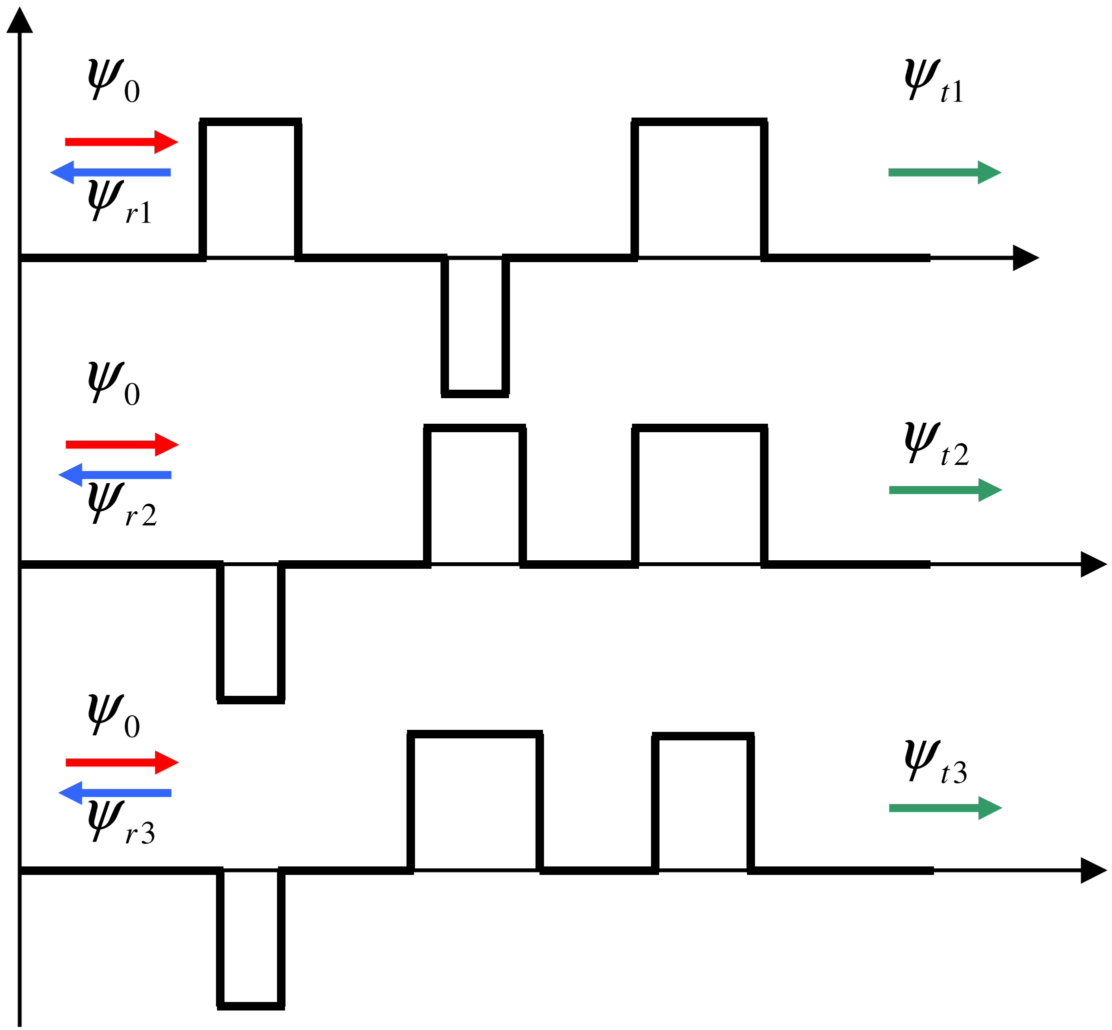


$$\psi_0 e^{i\chi} + \sum_{i,j}^{\infty} a_{ij} e^{i\left(\chi_j + \vec{k}(\vec{r}_i - \vec{r}_j)\right)}$$



$$\psi_0 e^{i\chi} + \sum_{i,j=1}^{\infty} b_{ij} e^{i\left(\chi_j + \vec{k}(\vec{r}_i - \vec{r}_j)\right)}$$

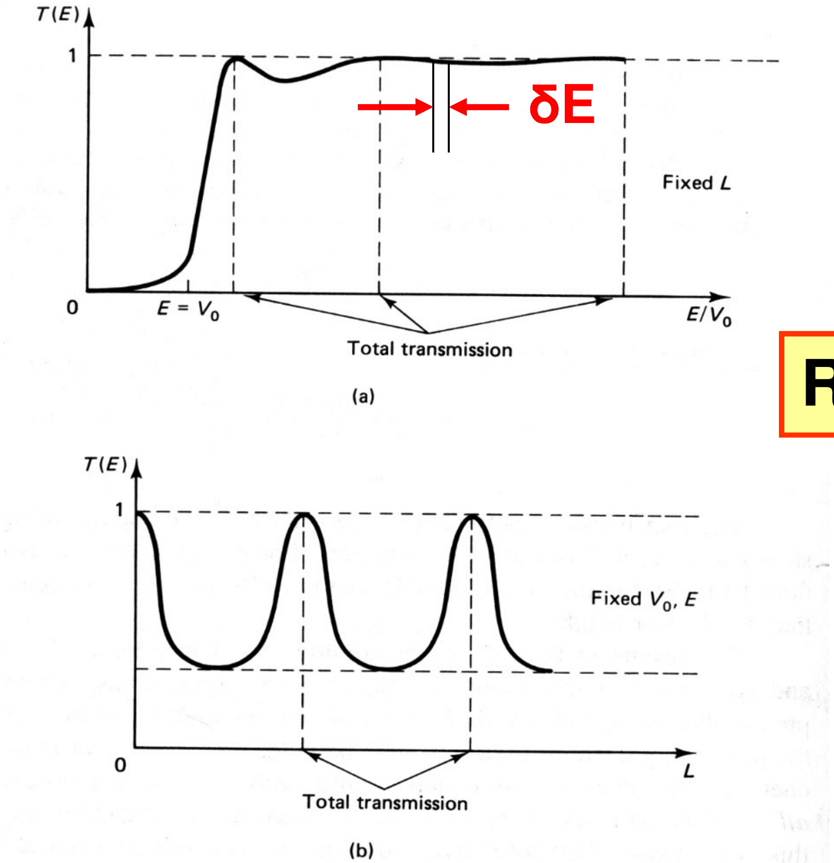
Source) $\longrightarrow$ **(Detector**



and many other combinations…

Barrier Reflectivity

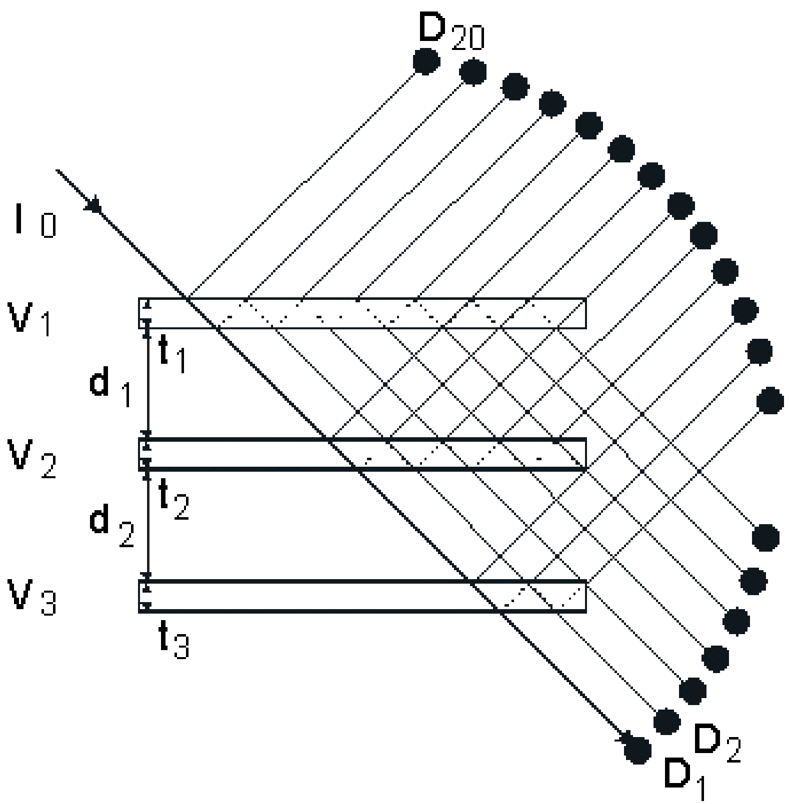
T + R = 1



T < 1

$$R_{Min} = (V/2E)^2 \delta k^2 L^2 > 0$$

Parasitic (unavoidable) reflections



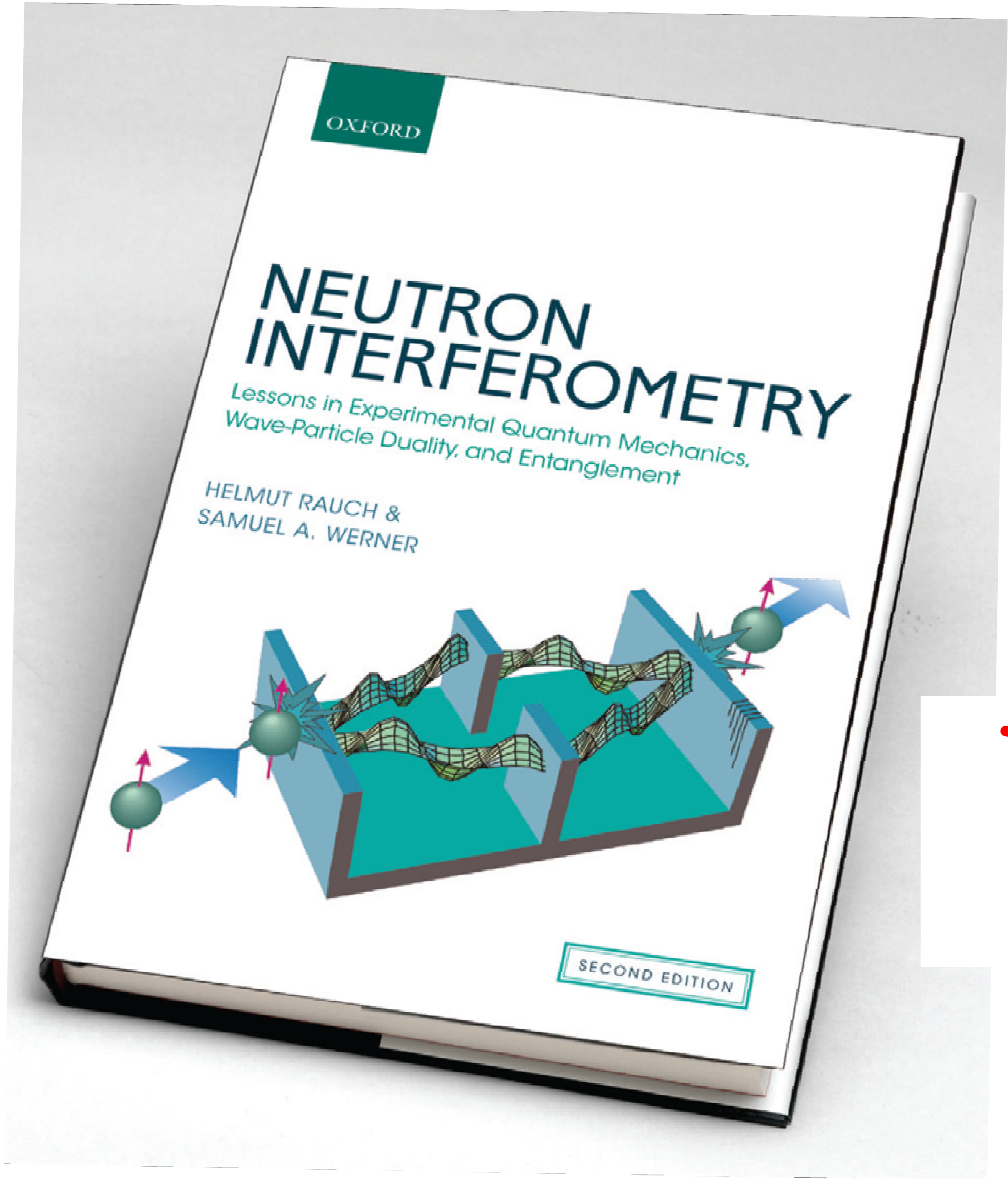
D_2 –D_{20} weak measurement channels

D_1 quantum input state

Overall phase shift
(overall thickness, potential height)

What means ignorance?

- There are no quantum complete experiments.
- Plane wave components of wave packets are arbitrary non-local.
- Loss of interference must not be a loss of coherence.
- Topological phases are less sensitive to disturbances than dynamic ones.
- Quantum losses in any interaction are unavoidable.


OXFORD
NEUTRON INTERFEROMETRY
Lessons in Experimental Quantum Mechanics, Wave-Particle Duality, and Entanglement
HELMUT RAUCH &
SAMUEL A. WERNER
SECOND EDITION

Thank you!

Compton frequency

$$\nu_c = \frac{c}{\lambda_c} = \frac{mc^2}{h}$$

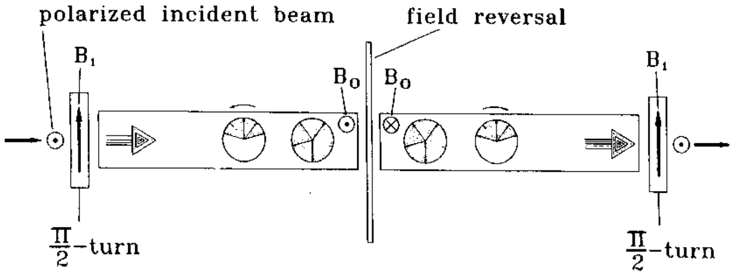
as an internal clock ?

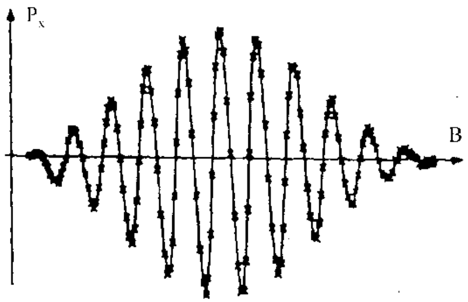
initiated by: H. Müller, A. Peters, S. Chu, Nature 463 (2010) 926

Larmor interferometry

static:

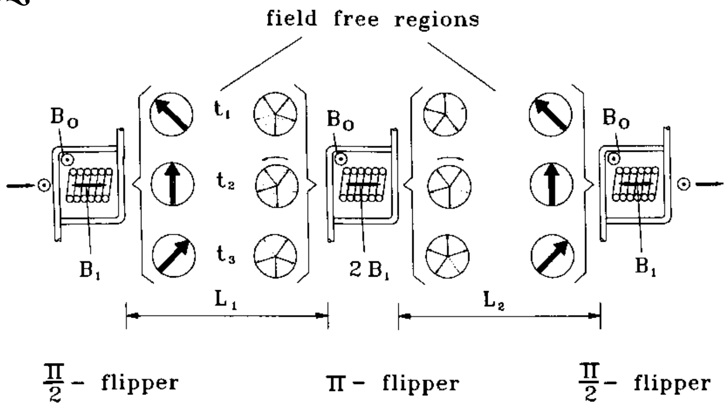
$$\omega_L = \frac{2\mu B_0}{\eta} \approx 1MHz$$

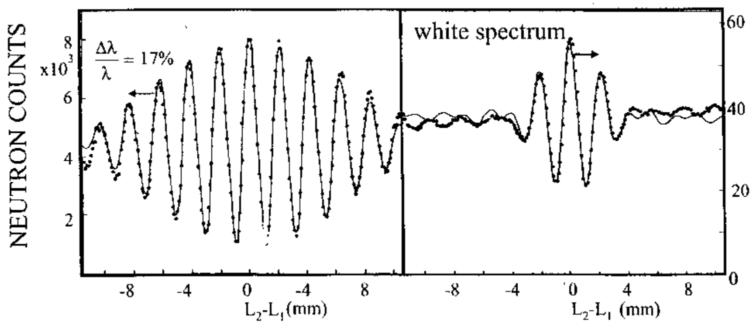




F. Mezei, Z. Physik 255 (1972) 146

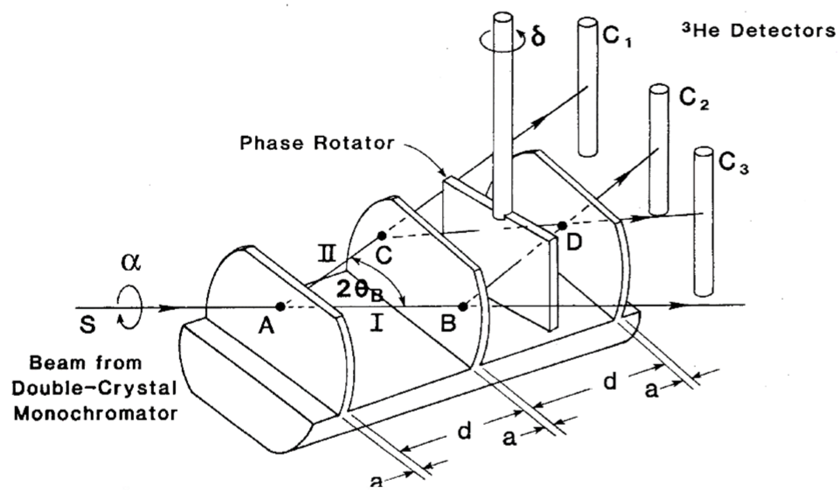
time dependent:

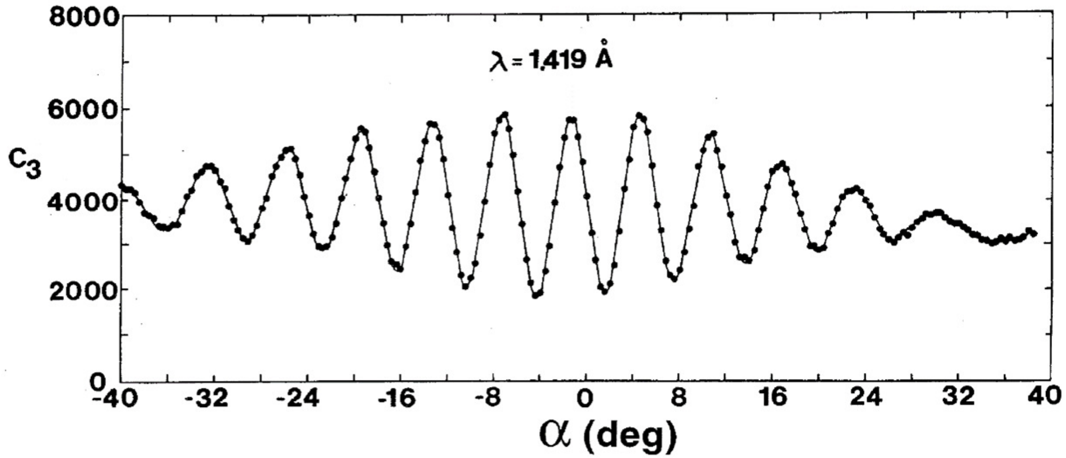




R. Gähler, Golub, J.Phys. France 49 (1988) 1195

COW-Experiment
(Colella, Overhauser, Werner)





$$E_o = \frac{\eta^2 k_o^2}{2m} = \frac{\eta^2 k^2}{2m} + mgH(\alpha)$$

E_0= 20 meV; mgH ~ 1.003 neV

$$\Delta k = (k - k_o) \cong -\frac{m^2 gH}{\eta^2 k_o} \sin\alpha$$

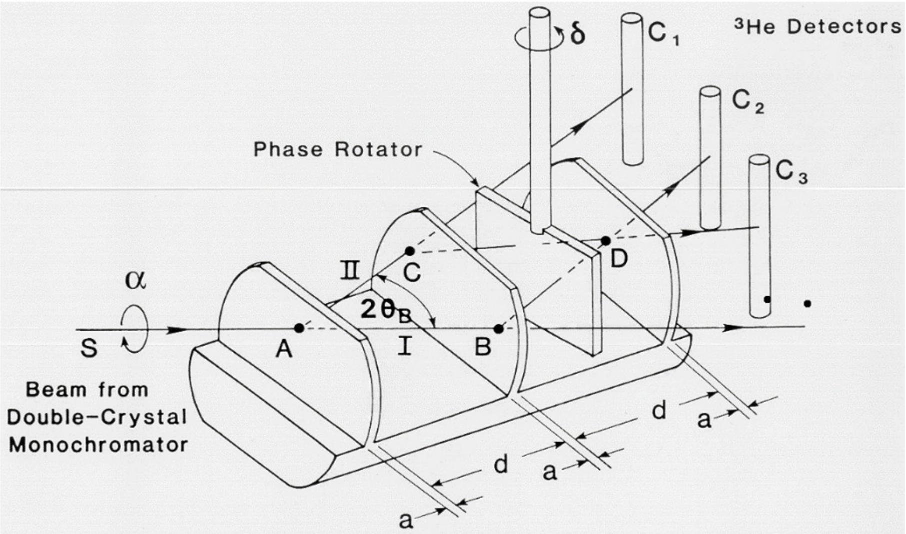
$$\Delta\Phi_{cow} = \Phi_{II} - \Phi_I = \Delta k \cdot S$$

$$\Delta\Phi_{cow} = -2\pi\lambda \frac{m^2}{h^2} g A_o \sin\alpha$$

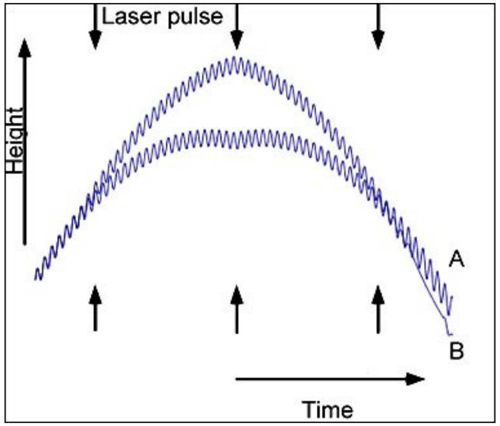
R. Colella, A.W. Overhauser, S.A. Werner, Phys.Rev.Lett. 34 (1974) 1472

Use of Compton Frequency

$$\lambda_C = h/mc \longrightarrow \omega_C = \frac{mc^2}{\eta} \approx 10^{25}\,Hz$$



Collela R., Overhauser A.W., Werner S.A. Phys.Rev.Lett. 34 (1975) 1053



Peters A., Chung K.Y., Chu S. Nature 400 (1999) 849

Müller H., Peters A., Chu S. Nature 463 (2010) 926

Gravity phase shift

classical motion

$$L_{cl} = \frac{GMm}{r_\oplus} - mgz + \frac{1}{2} m\dot{z}^2$$

$$g = GM / r_\oplus^2 \quad and \quad r = r_\oplus + z$$

$$\varphi = \oint k.ds$$

$$\frac{\eta^2 k^2}{2m} + mgz = \frac{\eta^2 k_0^2}{2m}$$

Schwarzschild metric for motion

$$L_{gr} = -mc^2 + \frac{GMm}{r_\oplus} - mgz + \frac{1}{2} m\dot{z}^2 + O\left(\frac{1}{c^2}\right)$$

$$g = GM / r_\oplus^2 \quad and \quad r = r_\oplus + z$$

$$\tau = \tau_0 (1 + \Delta U / mc^2)$$

$$(\Delta U = -mgH)$$

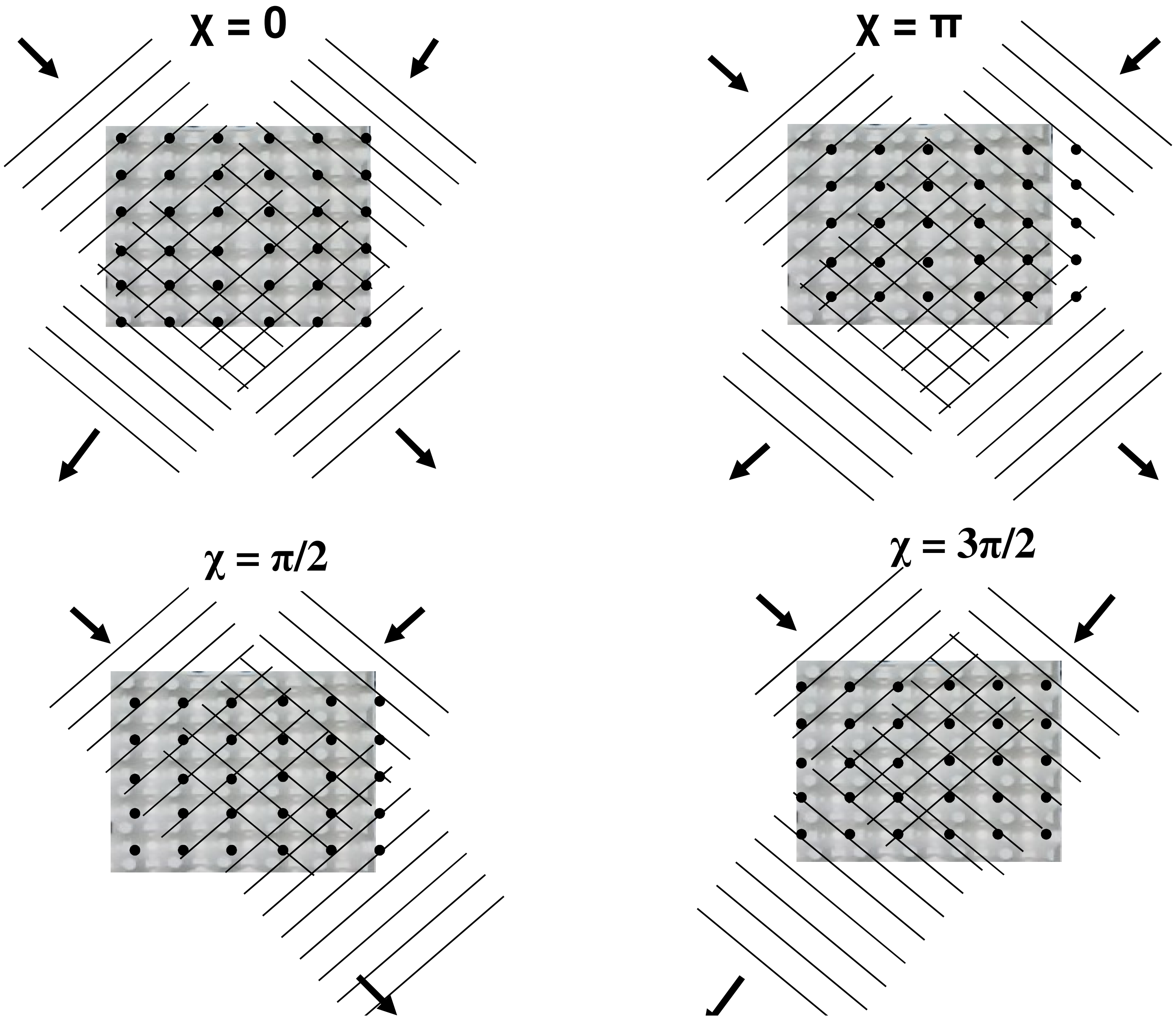
$$\varphi = \frac{1}{\eta} \oint mc^2 d\tau = \omega_c \oint d\tau$$

$$\Delta\varphi = \frac{mc^2}{\eta} \frac{\Delta U}{mc^2} \tau_0 = -\frac{mgH}{\eta} \frac{L}{v} = -2\pi\lambda \frac{m^2}{h^2} gA_0 \sin\alpha$$

Müller H., Peters A., Chu S. Nature 463 (2010) 926

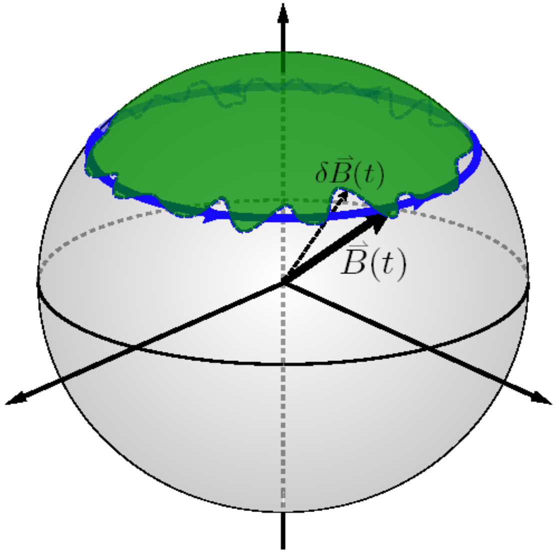
debate with: Wolf P., Blanchhet L., Borde C.J., Raynaud S., Salomon C., Cohen-Tannoudji, Class.Quantum Grav. 28 (2011) 145017

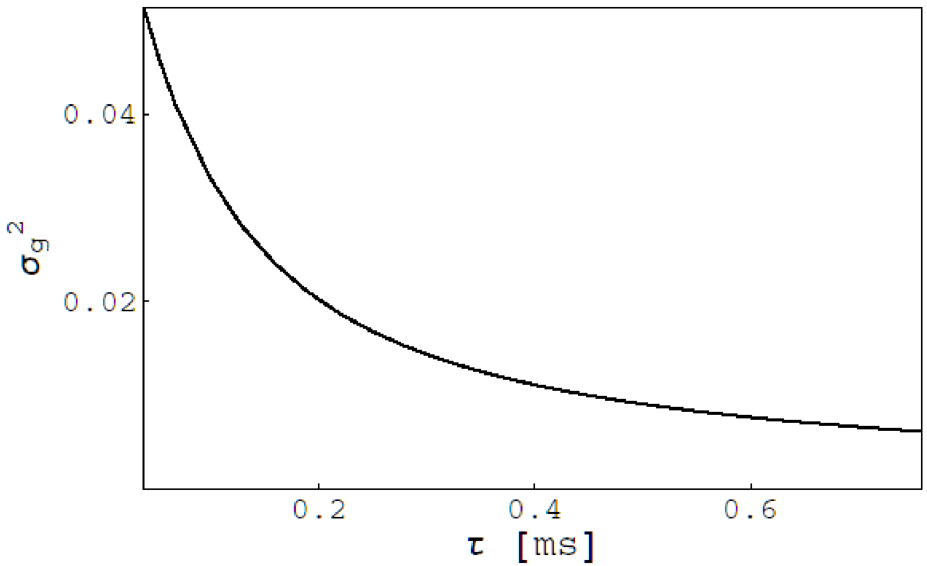
Wave – Lattice Interaction



Dephasing - Decoherence

G. De Chiara and G.M. Palma, PRL 91, 090404 (2003)
R.S. Whitney, Y. Gefer, Phys.Rev.Lett. 90(2003)190402

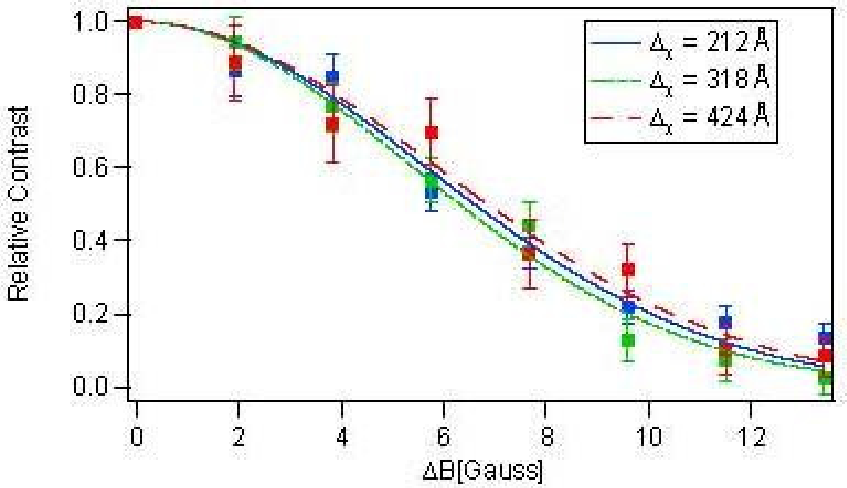




Variance of geometric phase (σ_g^2) tends to 0 for increasing time of evolution in a magnetic field.

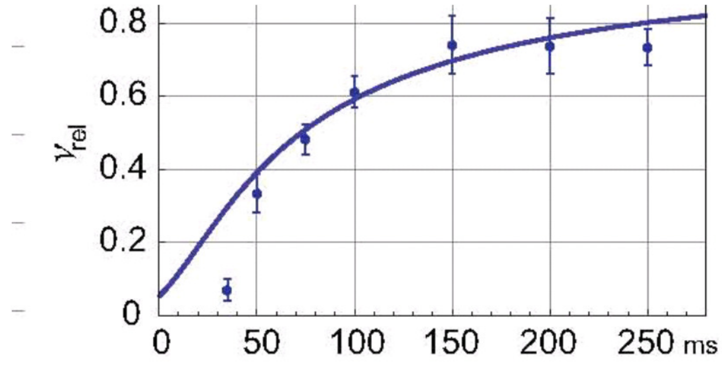
Rubustness of the geometric phase

dynamic

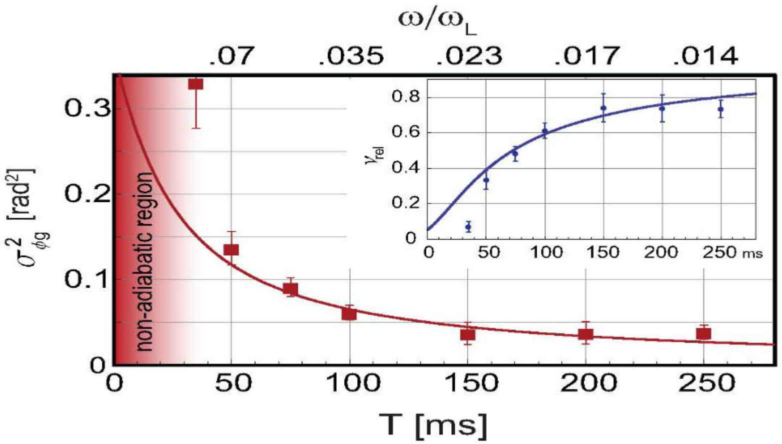


→ strength ΔVΔτ

geometric



→ strength ΔVΔτ



F. Filipp, J. Klepp, Y. Hasegawa, Ch. Plonka, P. Geltenbort, U. Schmidt, H. Rauch, Phys.Rev.Lett.102 (2009) 030404

Visualisation of the robustness of geometric phases

- Stereographic plots of sample noise realizations (simulation):

- ✘ effective frequency changes
- ✘ less fluctuations in enclosed area for longer T
- ✘ less dispersion of ϕ_g