TEXAS TECH UNIVERSITY

October 24, 2015

Emergent Quantum Mechanics 2015

Vienna, AUSTRIA

Quantum Mechanics Without Wavefunctions:
What happens when quantum worlds collide

Bill Poirier

Texas Tech University

Communication: Quantum mechanics without wavefunctions

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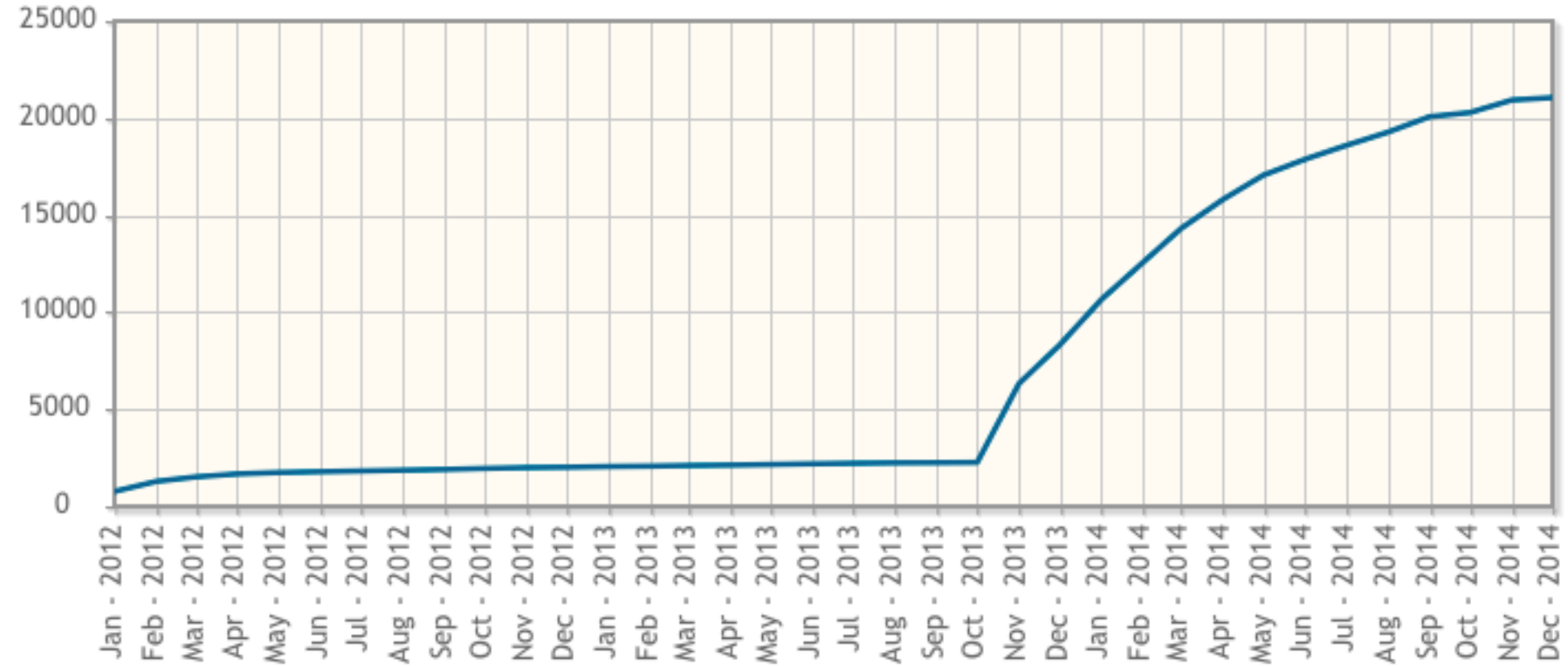

Editorial: Does Research on Foundations of Quantum Mechanics Fit into PRX's Scope?

And we have invited a Commentary by Bill Poirier from Texas Tech University that we hope will enhance your understanding of the paper and of our decision to publish it.

The Editors

The Many Interacting Worlds Approach to Quantum Mechanics

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A Commentary on:
Quantum Phenomena Modeled by Interactions between Many Classical Worlds
Michael J. W. Hall, Dirk-André Deckert, and Howard M. Wiseman
Phys. Rev. X, 4, 041013 (2014)

About the Commentary author:

Bill Poirier is Chancellor's Council Distinguished Research Professor and also Barnie E. Rushing Jr. Distinguished Faculty Member at Texas Tech University, in the Department of Chemistry and Biochemistry and also the Department of Physics. He received his Ph.D. in theoretical physics from the University of California, Berkeley, followed by a chemistry research associateship at the University of Chicago. His research interest is in understanding and solving the Schrödinger equation, from both foundational and practical perspectives.

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Quantum Mechanics Without Wavefunctions: aka *"Many Interacting Worlds"* (MIW)

Bill Poirier

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Bohmian mechanics without pilot waves

Bill Poirier, *Chem. Phys.* **370**, 4-14 (2010).

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ABSTRACT

In David Bohm's causal/trajectory interpretation of quantum mechanics, a physical system is regarded as consisting of *both* a particle *and* a wavefunction, where the latter "pilots" the trajectory evolution of the former. In this paper, we show that it is possible to discard the pilot wave concept altogether, thus developing a complete mathematical formulation of time-dependent quantum mechanics directly in terms of real-valued trajectories alone. Moreover, by introducing a kinematic definition of the quantum potential,

"A host of important and beneficial ramifications—*theoretical*, *computational*, and *interpretational*—are discussed."

damental than Eq. (11). Basically, this implies that *no* quantum effects can be attributed to the behavior of a *single* trajectory alone. Rather, all quantum behavior in nature is due to an interaction amongst the different trajectories within a given ensemble, with

We conclude with a brief discussion of some of the potential *interpretive* ramifications of the new formulation. In Bohmian mechanics, there is only one system trajectory, whereas the present approach offers an entire ensemble of trajectories. If one presumes objective existence for a single trajectory only, then the remaining trajectories in the ensemble must be regarded as "virtual," in some sense. On the other hand, one might prefer to regard *all* trajectories in the quantum ensemble as equally valid and real. It is hard to imagine how this could be achieved, without positing that each trajectory inhabits a separate world. It must be emphasized, however, that this version of the many worlds interpretation would be *very* different from the standard form [19–21]. In a nutshell, the latter associates

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Quantum Mechanics Without Wavefunctions aka *"Many Interacting Worlds"*

Brief Outline:

1. First principles derivation / motivation for MIW.
2. Comparison between continuous and discrete MIW.
3. Various generalizations/extensions of the theory.
4. Entanglement and measurement.
5. Prospects for experimental validation.

Theoretical/Mathematical Justification
First Principles Derivation

- Start "from scratch"; assume almost no knowledge of:
  - classical mechanics (Newton's Laws)
  - quantum mechanics (TISE).
- "Trial" trajectory $x(t)$ completely unconstrained
  - i.e., $x(t)$ is a *path*, not yet a trajectory.
- Posit existence of two "functional forms" of $x(t)$.
  - $f[x]$ (depends on x; essentially potential energy)
  - $g[\dot{x}]$ (depends on $\dot{x}$; essentially kinetic energy)
  - *NO* assumptions are made about form of $f[x]$ and $g[\dot{x}]$!
  - Space x assumed to be 1D, homogeneous (for simplicity).

Metaphysical Assumption #1: Action Extremization

- For all possible smooth paths $x(t)$ that connect:
  - initial point (x_0,t_0) with final point (x_f,t_f)
  - dynamical solution trajectory = $x(t)$ path that extremizes action, S.
- Definition of action: $$S = \int_{t_0}^{t_f} L[x(t), \dot{x}(t)]dt = \int_{t_0}^{t_f} \left(g[\dot{x}] - f[x]\right)dt$$
- Solution $x(t)$ satisfies Euler-Lagrange equation:

$$\left[\frac{\partial f}{\partial x}\right] + \frac{d}{dt}\left[\frac{\partial g}{\partial \dot{x}}\right] = 0$$

- Note: $f[x]$ and $g[\dot{x}]$ are still completely unspecified!
  - e.g., $g[\dot{x}] = C\dot{x}^4$ would be permissible.
  - however, for any specific choice of f and g, the solution $x(t)$ is now completely determined.

Metaphysical Assumption #2:
Hamiltonian Energy Conservation

- For all possible smooth paths $x(t)$ with initial conditions:
  $x(t_0)=x_0$ and $\dot{x}(t_0) = \dot{x}_0$
  dynamical solution trajectory = $x(t)$ path that conserves Hamiltonian, H.

- Form of Hamiltonian:

$$H[x(t), \dot{x}(t)] = g[\dot{x}(t)] + f[x(t)]$$

- Solution $x(t)$ satisfies Hamiltonian energy conservation:

$$H[x(t), \dot{x}(t)] = H(t) = \text{constant}$$

- Note: $f[x]$ and $g[\dot{x}]$ are still completely unspecified!
  - however, for any specific choice of f and g, the solution $x(t)$ is now completely determined.

Combining Both Metaphysical Assumptions Together

- Either physical constraint by itself leads to a unique set of solution trajectories
  - In general, i.e. for arbitrary choice of $f[x]$ and $g[\dot{x}]$,

    Action extremizing trajectories are not the same as Hamiltonian conserving trajectories
- Satisfying *both* conditions simultaneously is very special:
  - Noether's theorem: explicit t invariance of L implies existence of a conserved energy quantity, denoted E.
  - Our condition: that Noether E be equal to the Hamiltonian H.
  - imposes *severe* restrictions on allowed forms for $f[x]$ and $g[\dot{x}]$.

Combining Both Constraints

- What are the most general possible forms consistent with both action extremization and Hamiltonian conservation?

  $f[x]$ = completely unconstrained = $V[x]$

  $g[\dot{x}] = A\dot{x}^2 = (m/2)\dot{x}^2 = T[\dot{x}]$

- These are *precisely* the most general possible forms that are considered in *classical mechanics*!
  - thus, classical mechanics satisfies both of the two physical constraints, that we have imposed (already known)
  - but *no* other choices for f[x] and $g[\dot{x}]$ (i.e., no other candidate dynamical laws) can do so.

Quantum Trajectories Derivation

1D Stationary Scattering States

- Requires modification of the $L[x, \dot{x}]$ and $H[x, \dot{x}]$ forms.
- Requires higher-order time derivatives
  - consider contact/point transformation from coordinate x to y.
  - transformed functionals now mix y and $\dot{y}$, however...
  - no new physics added, i.e. result still classical mechanics.
- Posit existence of higher-order functionals:

$$L = L[x, \dot{x}, \ddot{x}, \ldots]$$

$$H = H[x, \dot{x}, \ddot{x}, \ldots]$$

Quantum Contribution to *L* and *H* Functional Forms

- Space x assumed to be homogeneous (for simplicity).
- Posit existence of higher-order quantum correction, Q:

$$L[x, \dot{x}, \ddot{x}, \ldots] = T[\dot{x}] - V[x] - Q[\dot{x}, \ddot{x}, \ldots]$$

$$H[x, \dot{x}, \ddot{x}, \ldots] = T[\dot{x}] + V[x] + Q[\dot{x}, \ddot{x}, \ldots]$$

- Q resembles a *potential* energy:
  - connects to "quantum potential" of Bohm theory.
  - adds to H but subtracts from L, like a potential energy.
- Q resembles a *kinetic* energy:
  - kinematic quantity that cannot depend on x.
  - quantum "potential" actually comes from K.E. operator.

Functional Form of Q

- Technical Note:
  - action extremization via "generalized" Euler-Lagrange eqn:

$$\left[\frac{\partial L}{\partial x}\right] - \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}}\right] + \frac{d^2}{dt^2}\left[\frac{\partial L}{\partial \ddot{x}}\right] - \cdots = 0$$

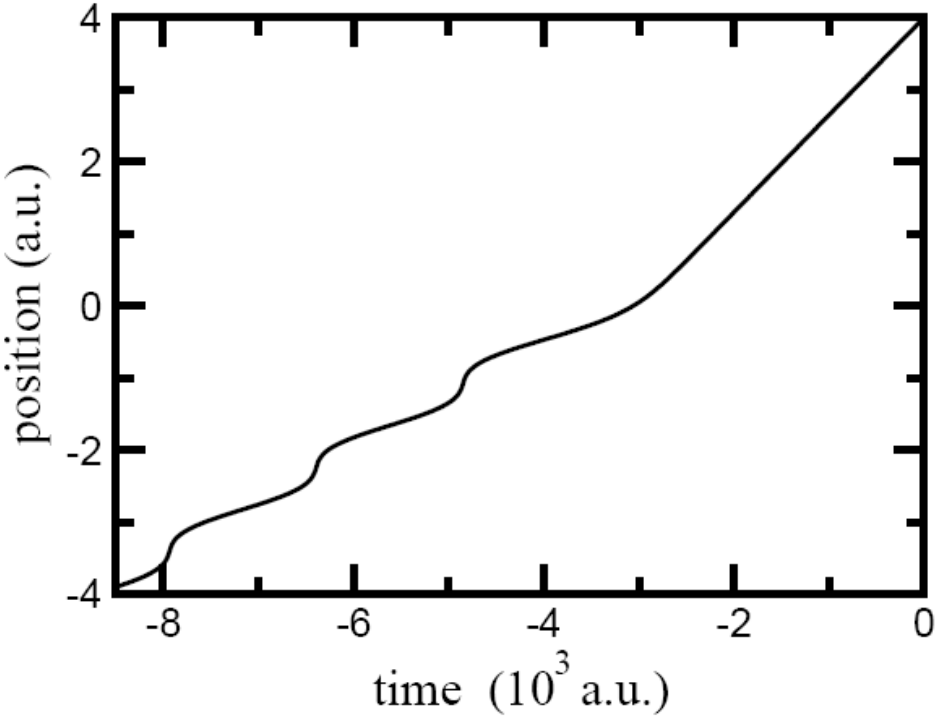
- Allowed meromorphic solutions (dynamical laws):

$V[x]$ = completely unconstrained.

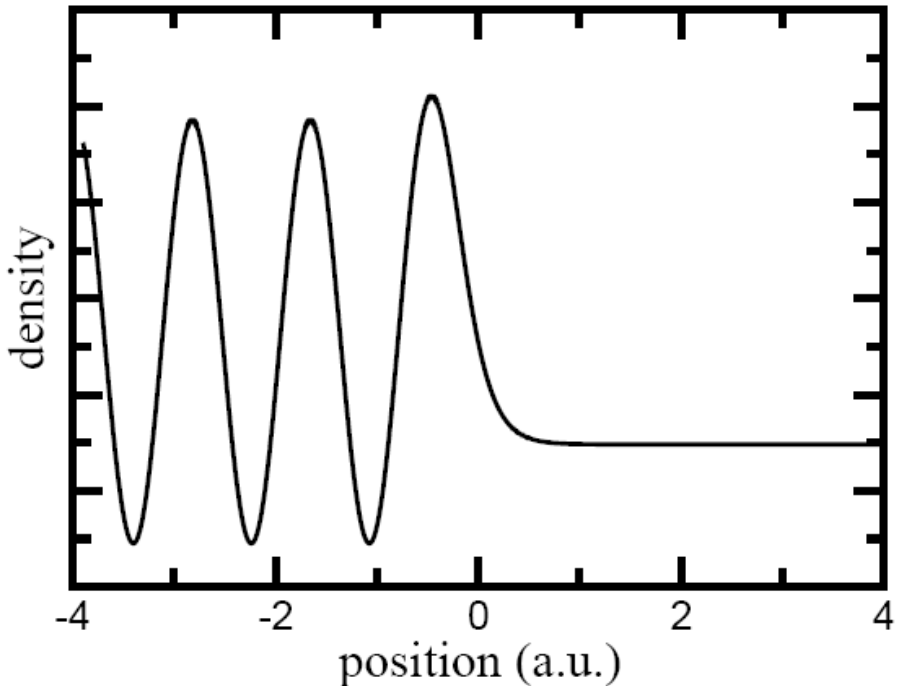
$T[\dot{x}] = (m/2)\dot{x}^2$

$$Q[\dot{x},\ddot{x},\ldots] = \begin{cases} \Delta E = \text{constant} & \text{order 0 (classical mechanics)} \\ \text{no solutions} & \text{order 1} \\ \text{no solutions} & \text{order 2} \\ -\dfrac{B}{2m}\left(\dfrac{5}{4}\dfrac{\ddot{x}^2}{\dot{x}^4} - \dfrac{1}{2}\dfrac{\dddot{x}}{\dot{x}^3}\right) & \text{order 3 (quantum mechanics, } B = \hbar^2) \end{cases}$$

Numerical Solution of the 1D TISE:
Eckart Barrier



Trajectory, $x(t)$



Wavefunction density, $\rho(x)$

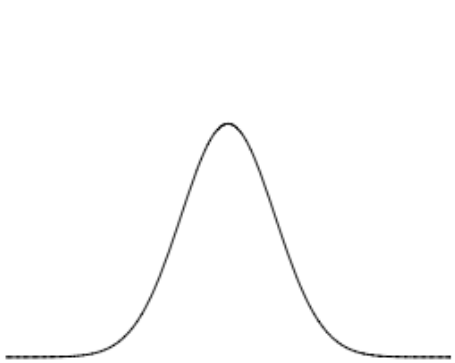
Solve 4th order real-valued ODE in t, to obtain $x(t)$.

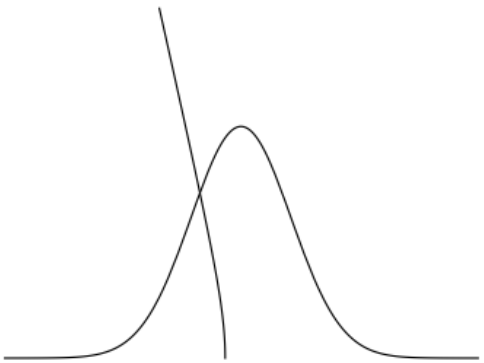
- similar to Newton's second law, w/ extra terms.
- two initial conditions specify E and x_0.
- remaining two specify boundary conditions of solution ψ

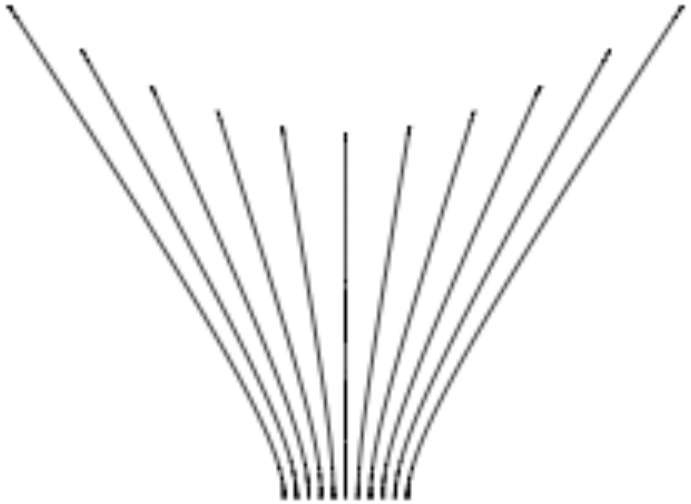
1D Time-Dependent Generalization

- The wavefunction $\Psi(x,t)$ is replaced with an *ensemble* (family) of trajectories, $x(C,t)$.
  - parameter C labels individual trajectories within the ensemble.
  - resembles classical statistical mechanics/trajectory simulations.
- Trajectories governed by their own self-contained PDE.
  - we now have "spatial" derivatives in terms of C, (i.e., across trajectories), in addition to time derivatives.
  - allowed forms of $T[]$, $V[]$, and $Q[]$ are *identical* to time-independent case, except with C rather than t derivatives for $Q[]$.
  - all quantum effects/quantum forces arise from C derivatives, i.e. stem from interaction across nearby worlds.

Copenhagen quantum mechanics	Bohmian mechanics	Quantum trajectory-based formulation (wavefunction-free)
Ψ represents the state of the system. TDSE drives evolution of $\Psi(x,t)$.	Ψ and $x(t)$ together represent the state of the system. Ψ leads to quantum potential Q, driving trajectory via: $m\ddot{x} + \frac{\partial V(x)}{\partial x} + \frac{\partial Q(x,t)}{\partial x} = 0$	There is no Ψ. $x(t, C)$ (trajectory ensemble) alone represents the state of the system, and leads to Q. $x(t, C)$ satisfies its own PDE that replaces the TDSE (with $'$ denoting partial derivative w/ respect to C.)







$$Q[x',x'',x'''] = -\frac{\hbar^2}{2m}\left(\frac{5}{4}\frac{x''^2}{x'^4} - \frac{1}{2}\frac{x'''}{x'^3}\right)$$

$$m\ddot{x} + \frac{\partial V(x)}{\partial x} + \frac{\hbar^2}{4m}\left(\frac{x''''}{x'^4} - 8\frac{x'''x''}{x'^5} + 10\frac{x''^3}{x'^6}\right) = 0$$

Continuous vs. Discrete MIW

Continuous MIW	Discrete MIW
continuous ensemble, $x(C,t)$	discrete ensemble, $x_i(t)$
exact solution of PDE	approximate discretization
unique dynamical law	dynamical law unspecified
action extremization principle	unclear at present
invariance/symmetry principle	unclear at present
relativistic generalization	unclear at present
Heisenberg/many-D/spin	under development
probability measure required	"emergent" probability
natural classical limit	natural classical limit
no trajectory crossing	no trajectory crossing

Trajectories for Wavepacket Dynamics
Arbitrary Dimensionality

- Generalization for arbitrary dimensionalities, and non-Euclidean metrics, has been thoroughly worked out.
  - trajectory ensemble now vector field, $\mathbf{x}(\mathbf{C},t)$
  - symmetries, conservation laws, stress-energy tensors, etc.
- Simplest Euclidean forms presented below:

$$\mathbf{K}^{-1} = \mathbf{J} = \text{ Jacobian matrix } (J^i_j = \partial x^i / \partial C^j)$$

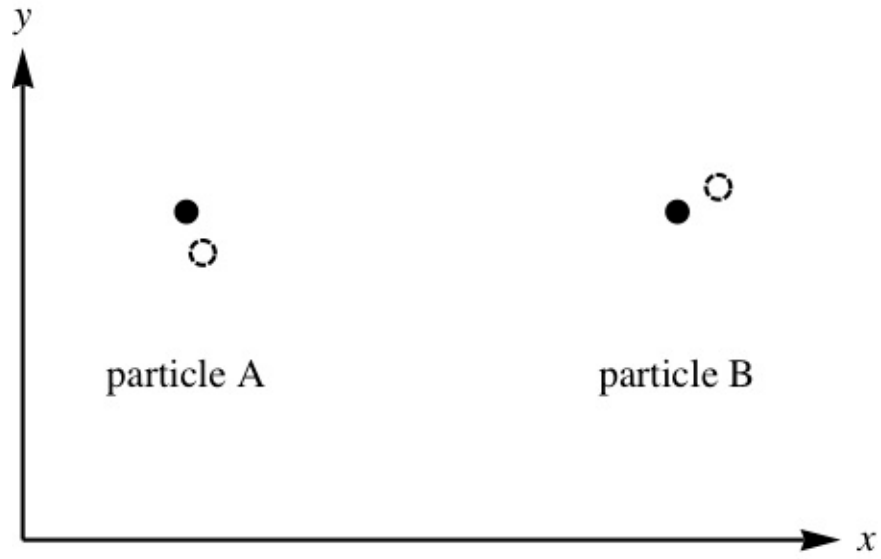
$$L = \frac{m}{2}\dot{\mathbf{x}}\cdot\,\dot{\mathbf{x}} - V(\mathbf{x}) - Q$$

$$Q = -\frac{\hbar^2}{4m}\left(K^k_j \frac{\partial^2 K^l_j}{\partial C^k \partial C^l} + \frac{1}{2}\frac{\partial K^k_j}{\partial C^k}\frac{\partial K^l_j}{\partial C^l}\right)$$

$$0 = m\ddot{x}^i + \frac{\partial V(\mathbf{x})}{\partial x^i} - \frac{\hbar^2}{4m}\frac{\partial}{\partial C^m}\left(K^k_i K^m_j \frac{\partial^2 K^l_j}{\partial C^k \partial C^l}\right)$$

Measurement/Interpretational Ramifications

Nonlocality and Quantum Entanglement



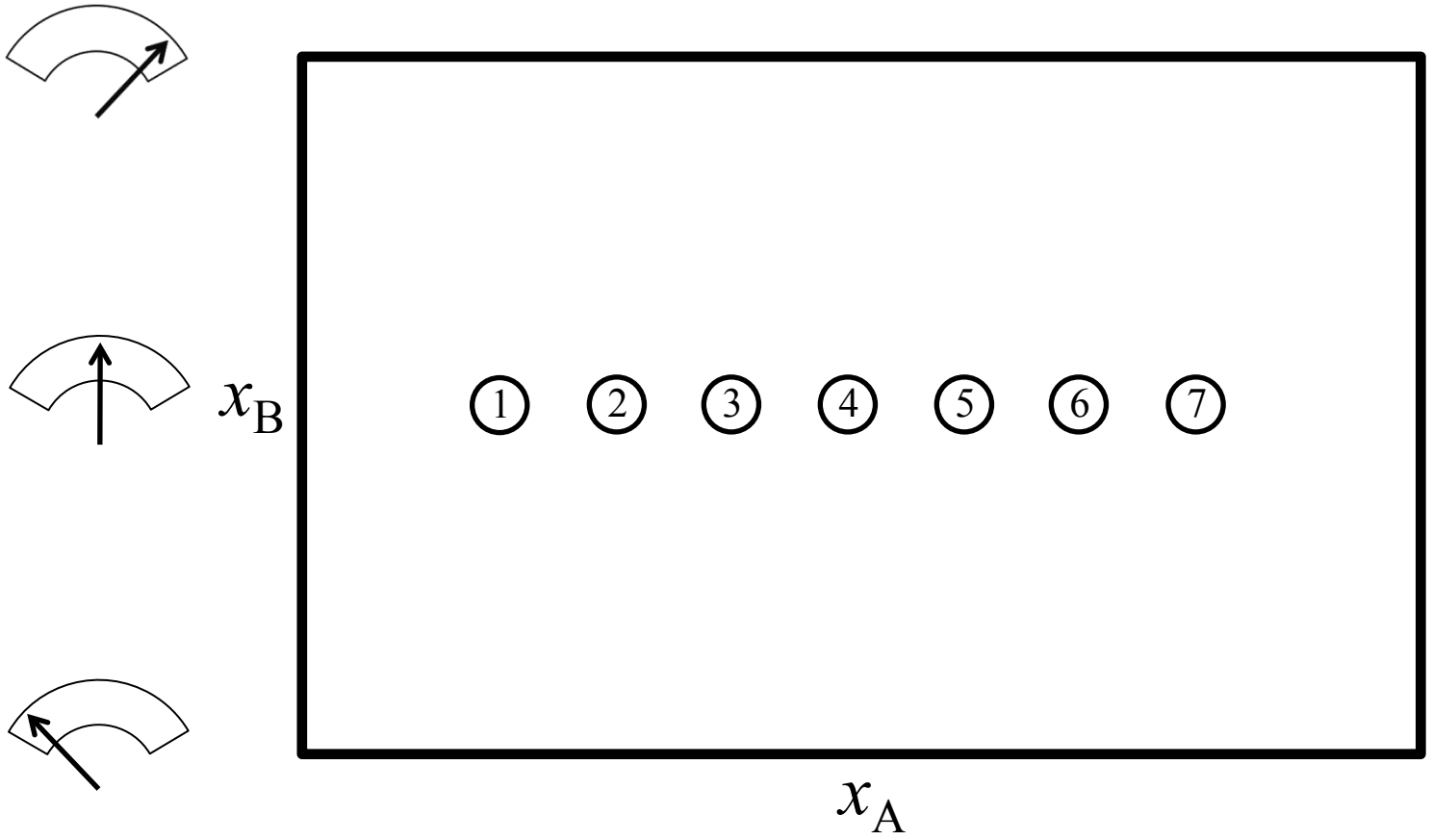
position space: (x,y)

configuration space: (x_A,y_A,x_B,y_B)

Nonlocality in the MIW picture: Consider two "entangled" quantum particles, A and B. A change in particle A is associated with an instant change in faraway particle B, which seems to violate relativity. The MIW picture describes this nonlocality paradox as follows. Let the two black discs represent the positions of particles A and B in our world. There is also a neighboring world in which particles A and B also exist, but at slightly displaced positions (represented by the open, dashed circles). The two *worlds* are close to each other, even though the two *particles*, A and B, are not.

Measurement/Interpretational Ramifications

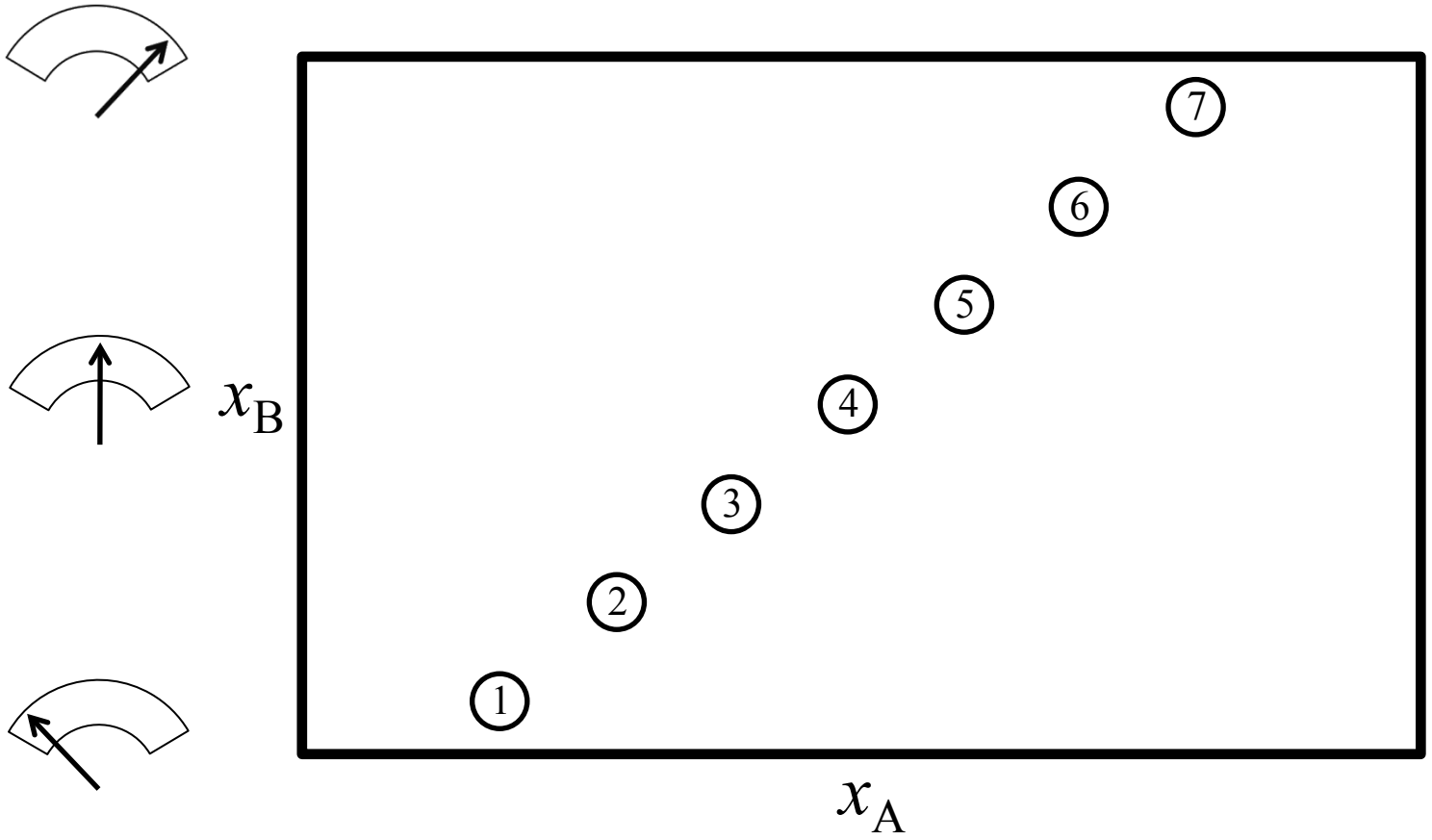
Measurement and "Collapse"



Before measurement: no correlation, worlds close

Measurement/Interpretational Ramifications

Measurement and "Collapse"



After measurement: perfect correlation, worlds far apart

Experimental Validation of *Many Interacting Worlds* ?

Three potential avenues by which MIW might lead to experimental outcomes that differ from those predicted by standard quantum theory:

1. Higher order contributions (i.e., beyond 4^{th}) to the trajectory-based dynamical law (continuous MIW).
2. Single particle relativistic quantum trajectory predictions (continuous MIW).
3. "Aliasing" effects due to discretization (discrete MIW).

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