Atoms of Spacetime and the Nature of Gravity

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T.P., arXiv:1508.06286

‘Internal evidence’ suggests that classical gravity has the same conceptual status as elasticity/hydrodynamics

WHY ?

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How do we understand this macroscopically and microscopically?

The Reason

If gravity is immune to zero level of energy it must have a thermodynamic interpretation!

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Connects two features usually thought to be completely separate!

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Works for a wide class gravitational theories; entropy decides the theory.

Evolution arises from departure from holographic equipartition:

$$\text{Time evolution} \propto (N_{\text{sur}} - N_{\text{bulk}})$$

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All static geometries have

$$N_{\text{sur}} = N_{\text{bulk}}$$

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Cosmological constant arises as an integration constant.

Its value is determined by a new conserved quantity for the universe!

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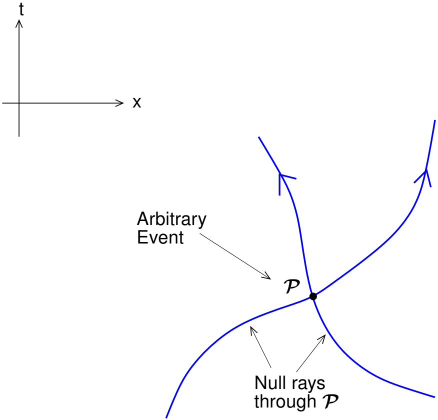
Points in a renormalized spacetime has zero volume but finite area!

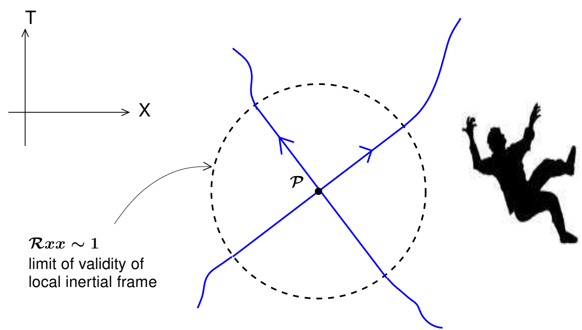
KINEMATICS: ***"How spacetime makes matter move"***

DYNAMICS: ***"How matter makes spacetime curve"***

Spacetime in Arbitrary Coordinates



Local Inertial Observers



Validity of laws of SR $\Rightarrow$ How gravity affects matter

Matter equations of motion $\Leftrightarrow$ $\nabla_a T^a_b = 0$

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Take non-inertial frames seriously: not "just coordinate relabeling".

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OBSERVERS WHO PERCEIVE A HORIZON ATTRIBUTE A TEMPERATURE TO SPACETIME

$$k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right)$$

[Davies (1975), Unruh (1976)]

Spacetimes, Like Matter, Can Be Hot

The most beautiful result in
the interface of quantum theory and gravity

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ATTRIBUTE A TEMPERATURE TO SPACETIME

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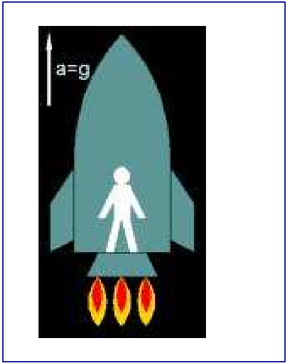
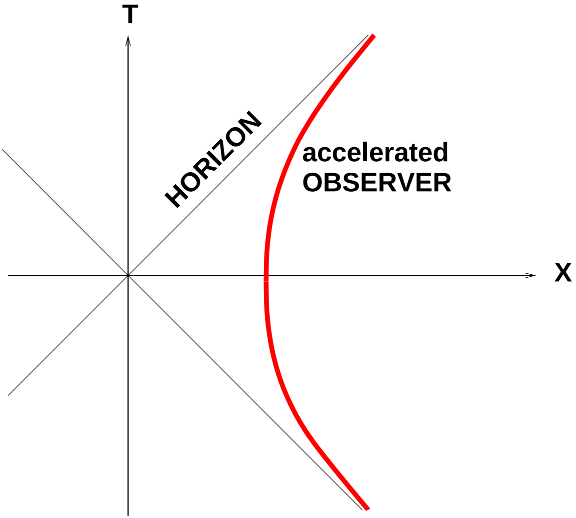
[Davies (1975), Unruh (1976)]

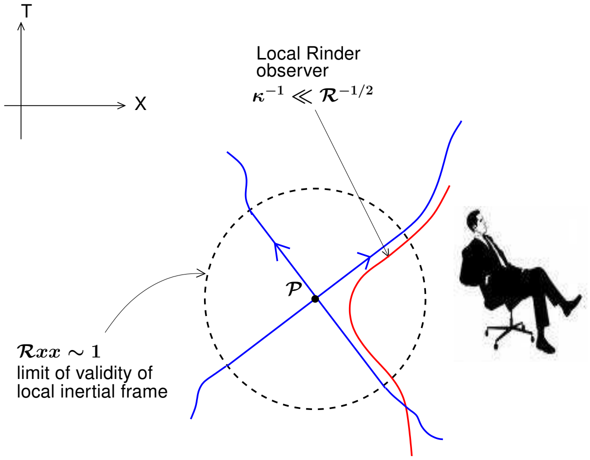
Temperature is independent of the
field equations of the theory!

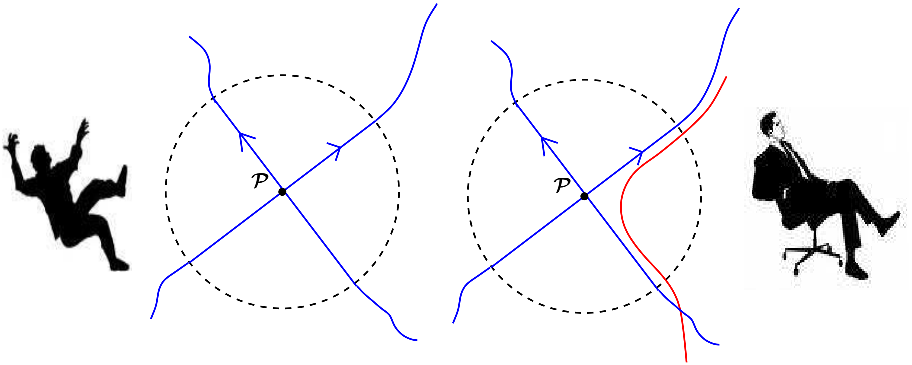
FLAT SPACETIME



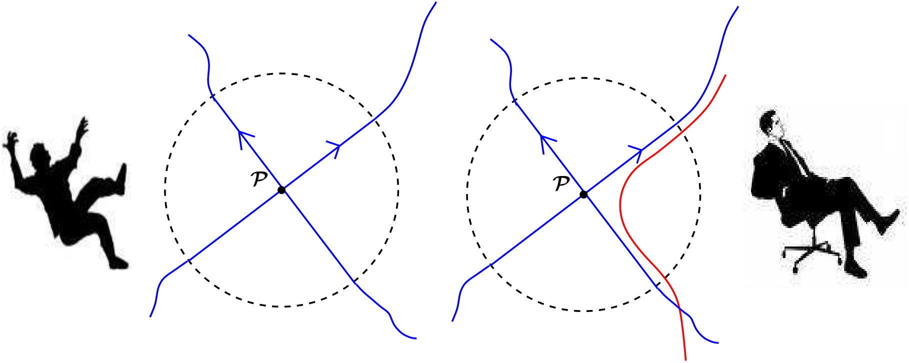


Local Rindler Observers





Vacuum fluctuations $\Longleftrightarrow$ Thermal fluctuations



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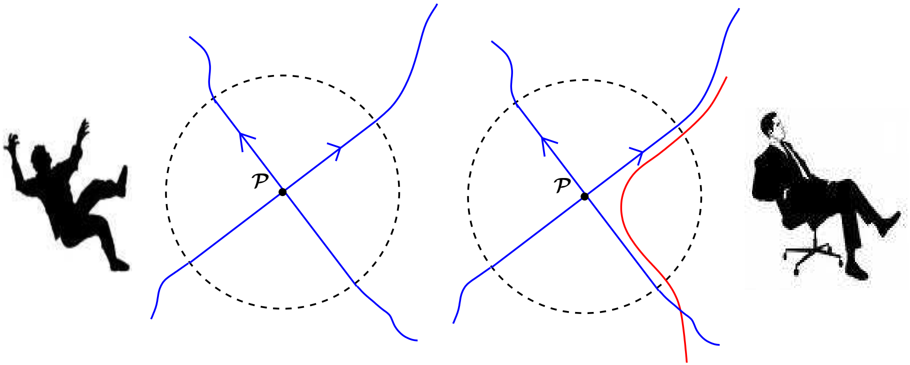
A VERY NON-TRIVIAL EQUIVALENCE!



Vacuum fluctuations $\Longleftrightarrow$ Thermal fluctuations

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QFT in FFF introduces $\hbar$; we now have $(\hbar/c)$ in the temperature

- **Heat transfered due to matter crossing a null surface:**

[T. Jacobson, gr-qc/9504004]

$$Q_m = \int \sqrt{\gamma}\, d^2x\, d\lambda\, (T_{ab}\ell^a \ell^b); \quad \mathcal{H}_m \equiv T_{ab}\ell^a \ell^b$$

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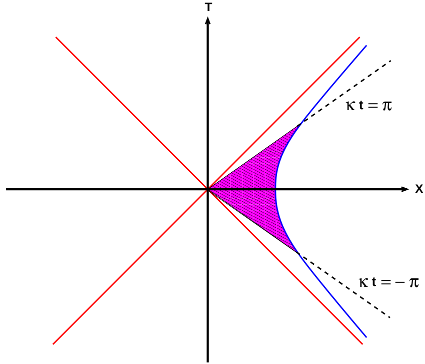
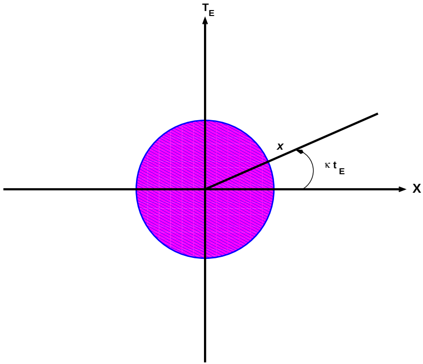
- **Note: Null horizon $\Leftrightarrow$ Euclidean origin**

$$X^2 - T^2 = 0 \Leftrightarrow X^2 + T_E^2 = 0$$





$$T = x \sinh \kappa t, \; X = x \cosh \kappa t \qquad T_E = x \sin \kappa t_E, \; X = x \cos \kappa t_E$$

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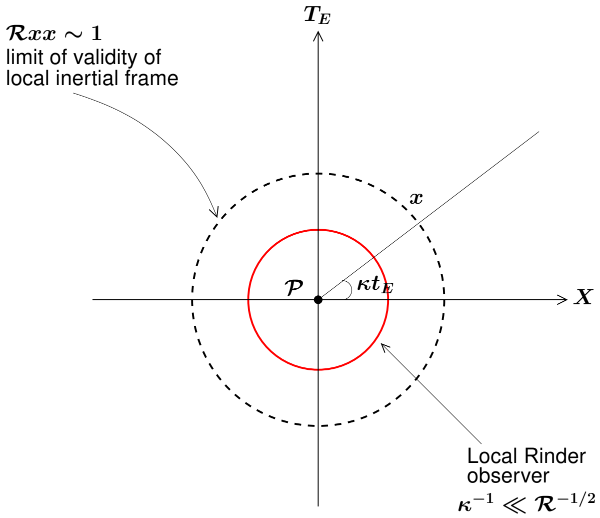
T_E
$\mathcal{R}xx \sim 1$
limit of validity of
local inertial frame
x
$\mathcal{P}$
κt_E
X
Local Rinder
observer
$\kappa^{-1} \ll \mathcal{R}^{-1/2}$

Guiding Principle For Dynamics

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Matter equations of motion remain invariant when a constant is added to the Lagrangian

Gravity must respect this symmetry

The variational principle for the dynamics of spacetime must be invariant under

$$T^a_b \to T^a_b + (\text{constant})\, \delta^a_b$$

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The variational principle cannot have metric as the dynamical variable!

Variational principle must have the form

$$Q = \int dV \; \mathcal{H}[T^a_b, g_{ab}, q_A]$$

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How can we vary q_A but get equations for g_{ab}?

How can $\mathcal{H}$ depend on T^a_b but yet be invariant under $T^a_b \to T^a_b$ + (constant) δ^a_b ?

- **Minimal possibility: We must have**

$$Q = \int dV \{\mathcal{H}_g[g_{ab}, n_a] + T^a_b n_a n^b\}$$

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- Demanding $(\delta Q / \delta n_a) = 0$ for **all** n_a at any given event should lead to:

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Can one find such a $\mathcal{H}_g[g_{ab}, n_a]$?

Dynamics Of Gravity

T.P., A. Paranjape [gr-qc/0701003]; T.P. [arXiv:0705.2533]

- Choose

$$\mathcal{H}_g = -\left(\frac{1}{16\pi L_P^2}\right)\left(4P^{ab}_{cd}\nabla_a n^c \nabla_b n^d\right)$$

with

$$P^{ab}_{cd} \propto \delta^{aba_2b_2\ldots a_m b_m}_{cdc_2d_2\ldots c_m d_m} R^{c_2d_2}_{a_2b_2} \ldots R^{c_m d_m}_{a_m b_m}$$

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- This gives

$$E^a_b \equiv P^{ai}_{jk}R^{jk}_{bi} - \frac{1}{2}\delta^a_b\mathcal{R} = (8\pi L_P^2)T^a_b + \Lambda\delta^a_b,$$

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- These are Lanczos-Lovelock models of gravity. In $d = 4$, it uniquely leads to GR

$$G^a_b = (8\pi L_P^2)T^a_b + \Lambda\delta^a_b$$

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Microscopically $\mathcal{H}_g(x^i, n_a)$ is the 'distribution function for atoms of space with momentum' n_a

The Thermodynamic Connection

- Macroscopically, identify $n_a \leftrightarrow \ell_a$ and

$$Q_{\rm tot} \equiv \int \sqrt{\gamma}\, d^2x d\lambda \left(\mathcal{H}_g[\ell] + \mathcal{H}_m[\ell]\right)$$

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Extremizing the heat densities of all null surfaces leads to gravitational dynamics!

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- ***The P^{ab}_{cd} is the entropy tensor of the spacetime which determines the theory***
- ***The entropy density of horizons:***

$$s = -\frac{1}{8}\sqrt{\gamma}P^{abcd}\epsilon_{ab}\epsilon_{cd}$$

[Iyer and Wald (1994)]

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- ***On-shell value of $Q_{\rm tot}$***

$$Q^{\rm on-shell}_{\rm tot} = \int d^2x (T_{\rm loc}\, s)\Big|_{\lambda_1}^{\lambda_2}$$

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- ***You can heat up spacetime!*** Do we have an equipartition law for the microscopic spacetime degrees of freedom?

Equipartition with a surface-bulk correspondence

$$E_{\rm bulk} = \int_{\partial\mathcal{V}} \frac{dA}{L_P^2}\left(\frac{1}{2}k_B T_{loc}\right) \equiv \frac{1}{2}k_B \int_{\partial\mathcal{V}} dn\, T_{\rm loc}$$

Associates $dn = dA/L_P^2$ atoms (microscopic degrees of freedom) with an area dA

Holographic Equipartition

T.P. [gr-qc/0308070], [arXiv:0912.3165], [arXiv:1003.5665]

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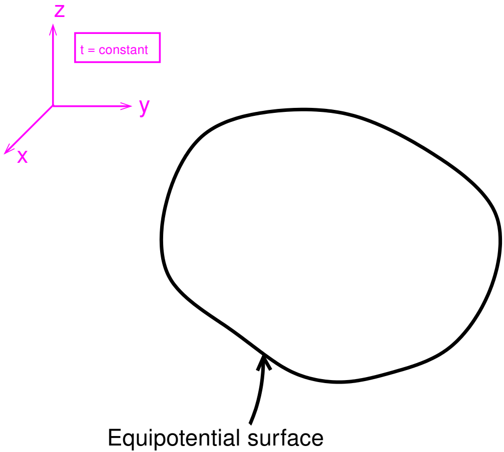

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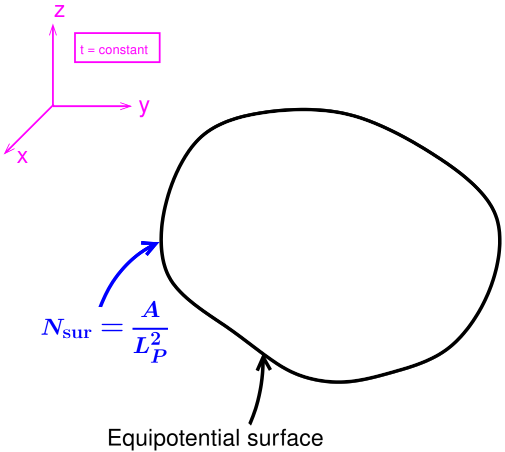

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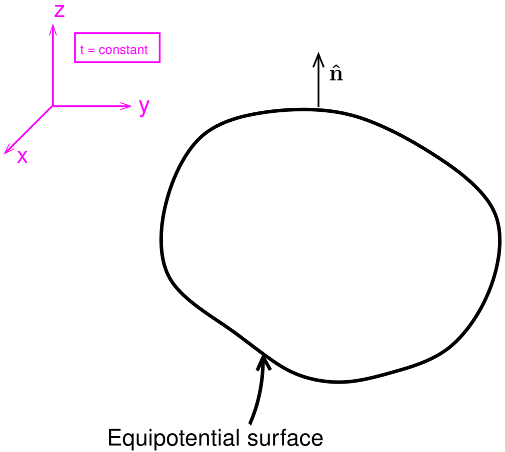

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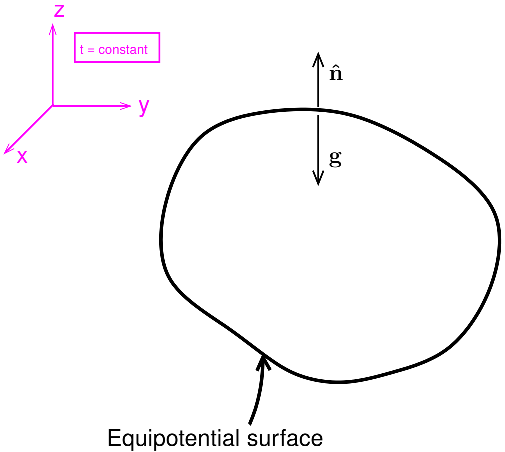

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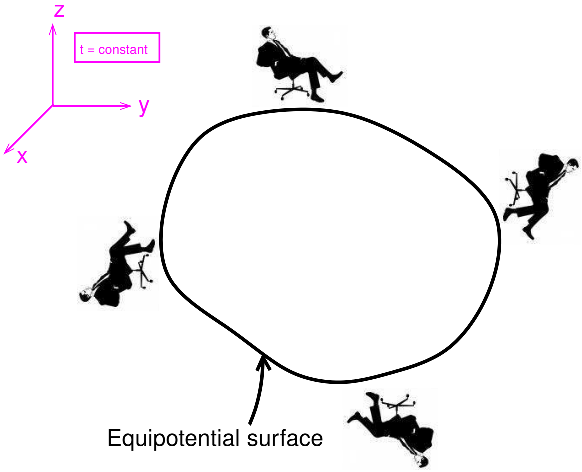

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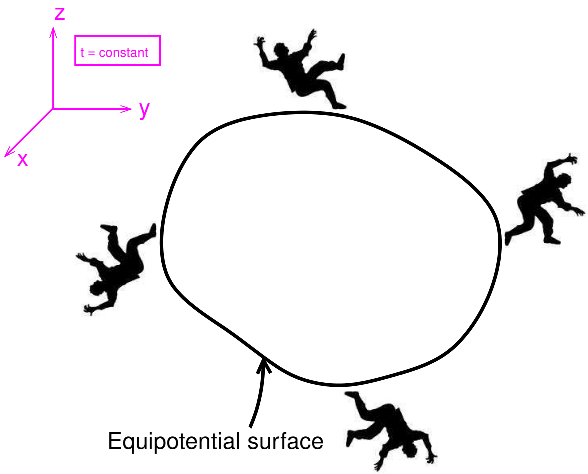

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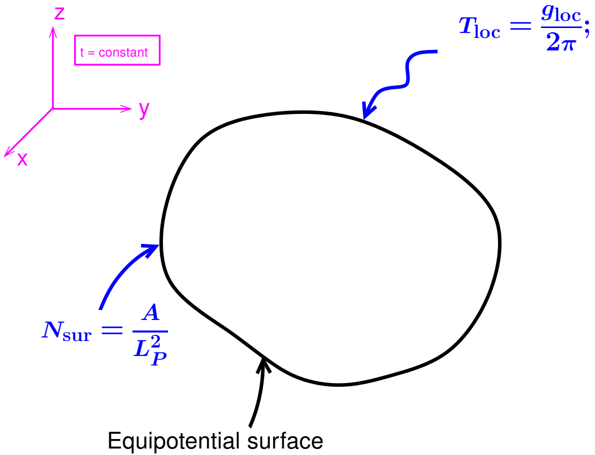

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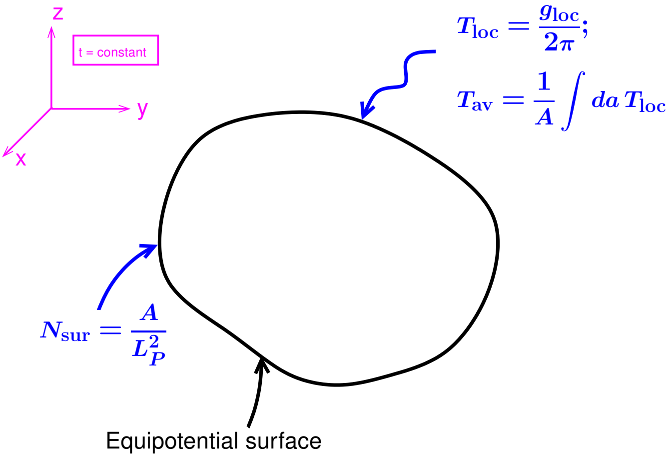

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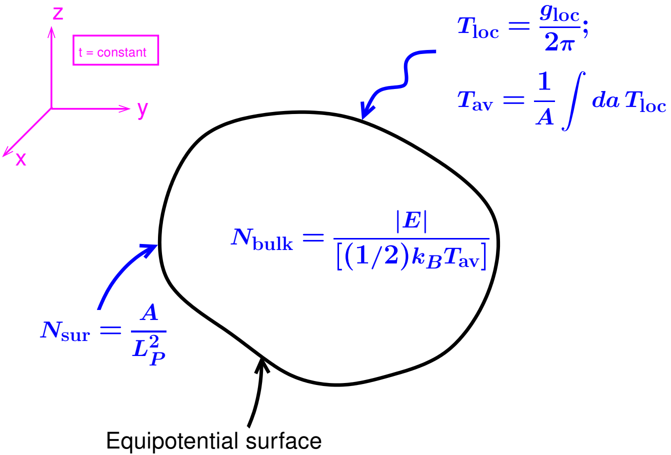

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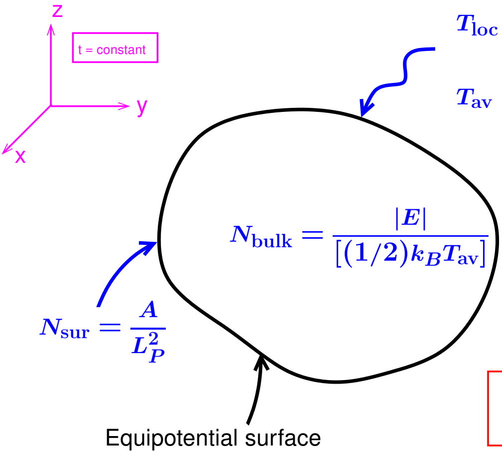

$$T_{\rm loc} = \frac{g_{\rm loc}}{2\pi}; \qquad T_{\rm av} = \frac{1}{A}\int da\, T_{\rm loc}$$

$$N_{\rm bulk} = \frac{|E|}{[(1/2)k_B T_{\rm av}]}$$

$$N_{\rm sur} = \frac{A}{L_P^2}$$

$$N_{\rm sur} = N_{\rm bulk}\,!$$

We must be able to express — and interpret — the field equation in a purely thermodynamic language !

$$\boldsymbol{q^{ab} \equiv \sqrt{-g}\, g^{ab}}$$

Geometry ⇔ Thermodynamics

K. Parattu, B.R. Majhi, T.P. [arXiv:1303.1535]

$$q^{ab} \equiv \sqrt{-g}\, g^{ab}$$

$$p^a_{bc} \equiv -\Gamma^a_{bc} + \frac{1}{2}(\Gamma^d_{bd}\delta^a_c + \Gamma^d_{cd}\delta^a_b)$$

These variables have a thermodynamic interpretation

$$(q\delta p, p\delta q) \Leftrightarrow (s\delta T, T\delta s)$$

What Makes Spacetime Evolve ?

T.P., Gen.Rel.Grav (2014) [arXiv:1312.3253]

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$$\int \frac{d\Sigma_a}{8\pi L_P^2}\,[q^{\ell m}\partial\, p^a_{\ell m}] = -\frac{1}{2}k_B T_{\rm av}\ \left(N_{\rm sur} - N_{\rm bulk}\right)$$

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time evolution of spacetime

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time evolution of spacetime

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deviation from

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This replaces the field equation for gravity

Newton's law of gravitation

T.P. [hep-th/0205278]

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Temperature $\Rightarrow (\hbar/\boldsymbol{c})$; Entropy $\Rightarrow \boldsymbol{L_P^2}$

$$\boldsymbol{F} = \left(\frac{\boldsymbol{c}^3 \boldsymbol{L_P^2}}{\hbar}\right)\left(\frac{\boldsymbol{m_1 m_2}}{\boldsymbol{r^2}}\right)$$

Newton's law of gravitation

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Temperature $\Rightarrow (\hbar/c)$; Entropy $\Rightarrow L_P^2$

$$F = \left(\frac{c^3 L_P^2}{\hbar}\right)\left(\frac{m_1 m_2}{r^2}\right)$$

Gravity, like matter, is intrinsically quantum and cannot exist in the limit of $\hbar \to 0$!

Momentum of Gravity

T.P. [arXiv:1506.03814]

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$\nabla_a(P^a + M^a) = 0$ for all observers imply field equations

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Variational principle has a physical meaning:

$$Q_{\rm tot} = -\int d\mathcal{V}\,\ell_a\,[P^a(\xi) + M^a(\xi)]$$

Three projections $P^a\ell_a,\ P^a k_a,\ P^a q^b_a$ on a null surface give

- Navier-Stokes equation [T.P., arXiv:1012.0119]
- $TdS = dE + PdV$ [T.P., gr-qc/0204019; D. Kothawala, T.P., arXiv:0904.0215]
- Evolution equation for the null surface

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- Determine $\mathcal{H}_g$; use alternative, dimensionless, form:

$$\mathcal{H}_g \equiv -\frac{1}{8\pi}(L_P^2 R_{ab} n^a n^b)$$

Algebraic Aside

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- Alternative, dimensionless, form:

$$\mathcal{K}_g \equiv -\frac{1}{8\pi} (L_P^2 R_{ab} n^a n^b)$$

The Challenge

How can we get $\mathcal{H}_g$ from a microscopic theory without knowing the full QG?

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We need to recognize discreteness and yet use continuum mathematics!

Atoms Of A Fluid

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The $\mathcal{H}_g$ is proportional to the $f(x^i, n_j)$ for the number of atoms of space "at" x^i with "momentum" n_i. In dimensionless form:

$$\frac{d(Q/E_P)}{d(V/L_P^3)} \equiv \mathcal{H}_g(x^i, n_j) \propto f(x^i, n_j)$$

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Use equi-geodesic surfaces to make this idea precise

Geodesic Interval

D. Kothawala, T.P. [arXiv:1405.4967]; [arXiv:1408.3963]

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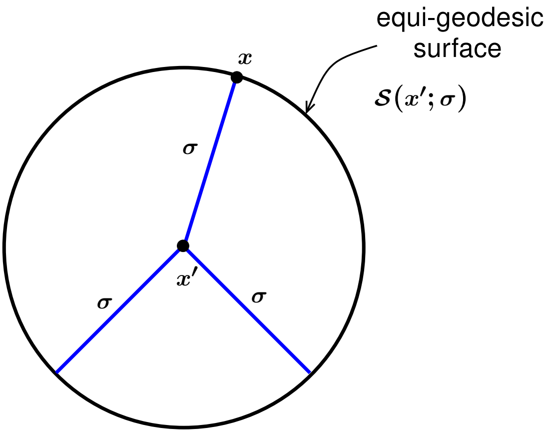
$$\sigma(x, x') = \int_x^{x'} \sqrt{g_{ab} n^a n^b} d\lambda$$

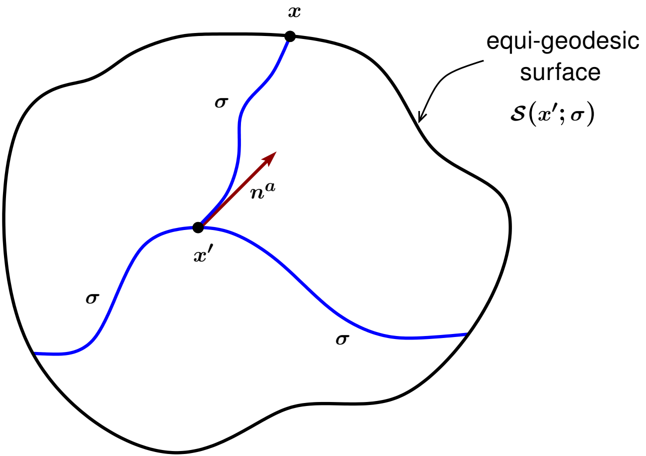
$$\frac{1}{2}\nabla_a \nabla_b \sigma^2 = g_{ab} - \frac{\lambda^2}{3}\mathcal{E}_{ab} + \frac{\lambda^2}{12} n^i \nabla_i \mathcal{E}_{ab} + \mathcal{O}(\lambda^4)$$

$$n_j = \nabla_j \sigma, \qquad \mathcal{E}_{ab} \equiv R_{akbj} n^k n^j$$



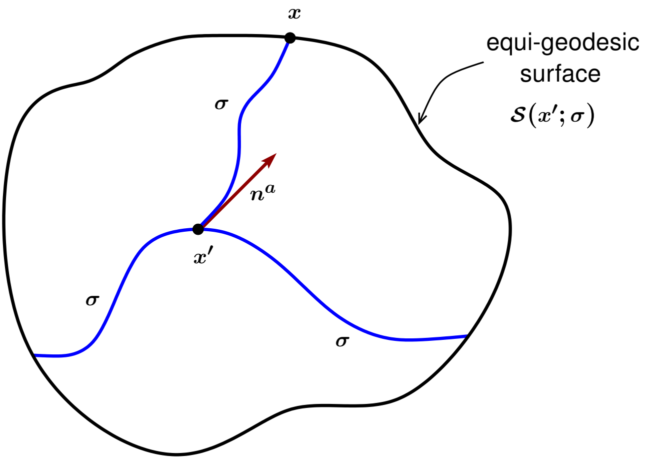
$$ds^2 = d\sigma^2 + \sigma^2 d\Omega^2_{(S3)}$$

$$\sqrt{g} \propto \sigma^3 \qquad \sqrt{h} \propto \sigma^3$$



$$ds^2 = d\sigma^2 + h_{\alpha\beta}dx^\alpha dx^\beta$$

The $\sqrt{g} = \sqrt{h}$ will pick up curvature corrections



$$ds^2 = d\sigma^2 + h_{\alpha\beta} dx^\alpha dx^\beta$$

$$\sqrt{h}(x,x') = \sqrt{g}(x,x') = \sigma^3 \left(1 - \frac{\sigma^2}{6}\mathcal{E}\right) \sqrt{h_\Omega}; \quad \mathcal{E} \equiv R_{ab} n^a n^b$$

Zero-Point Length

T.P. Ann.Phy. (1985), 165, 38; PRL (1997), 78, 1854

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Quantum spacetime has a zero-point length:

$$\sigma^2(x,x') \;\to\; S(\sigma^2) = \sigma^2(x,x') + L_0^2$$
$$g_{ab}(x) \;\to\; q_{ab}(x,x';L_0^2)$$

D. Kothawala, T.P. [arXiv:1405.4967]; [arXiv:1408.3963]

The number of atoms of space at x^i with attribute ("momentum") n_i scales as volume or area measure of the equigeodesic surface in the quantum Euclidean space when $x' \to x$

$$f(x^i, n_j) \propto \sqrt{g}(x^i, n_j) \text{ OR } \sqrt{h}(x^i, n_j)$$

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The $\sigma^2 \to 0$ limit picks null vectors! Euclidean origin maps to local Rindler horizons.

Area Of A Point

T.P. [arXiv:1508.06286]

$$\sqrt{q} = \sigma\left(\sigma^2 + L_0^2\right)\left[1 - \frac{1}{6}\mathcal{E}\left(\sigma^2 + L_0^2\right)\right]\sqrt{h_\Omega}$$

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Aside: Spacetime becomes two-dimensional at Planck scales

Distribution Function For Atoms Of Space

The area measure gives exactly what we need, along with a zero-point-contribution

$$f(x^i, n_a) = 1 - \frac{1}{8\pi}\mathcal{E}L_P^2 = 1 - \frac{1}{8\pi}L_P^2 R_{ab}n^a n^b$$

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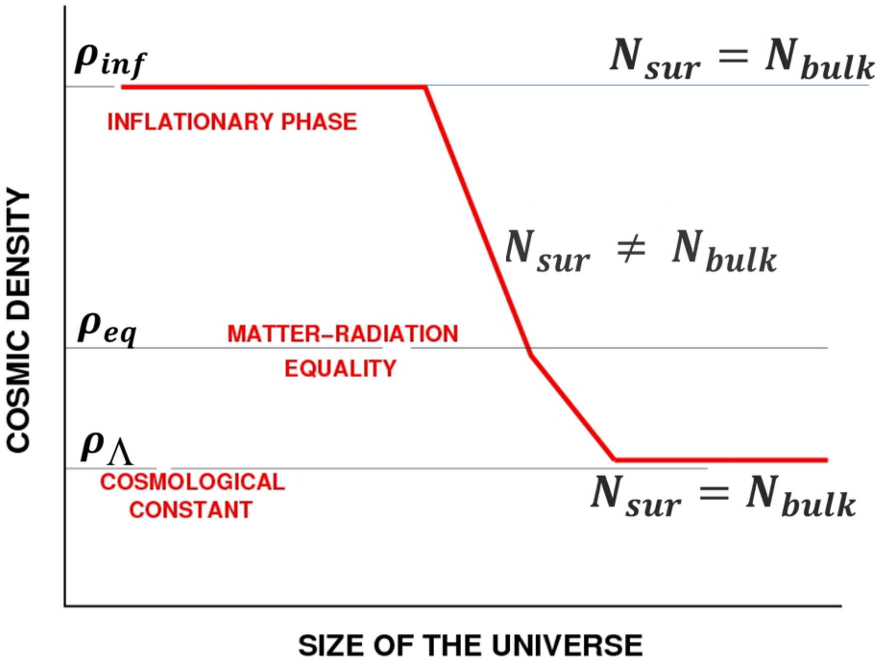
Zero-point contribution is important; degrees of freedom of Planck 2-sphere: $4\pi L_P^2/L_P^2 = 4\pi$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$N_{sur} = N_{bulk}$
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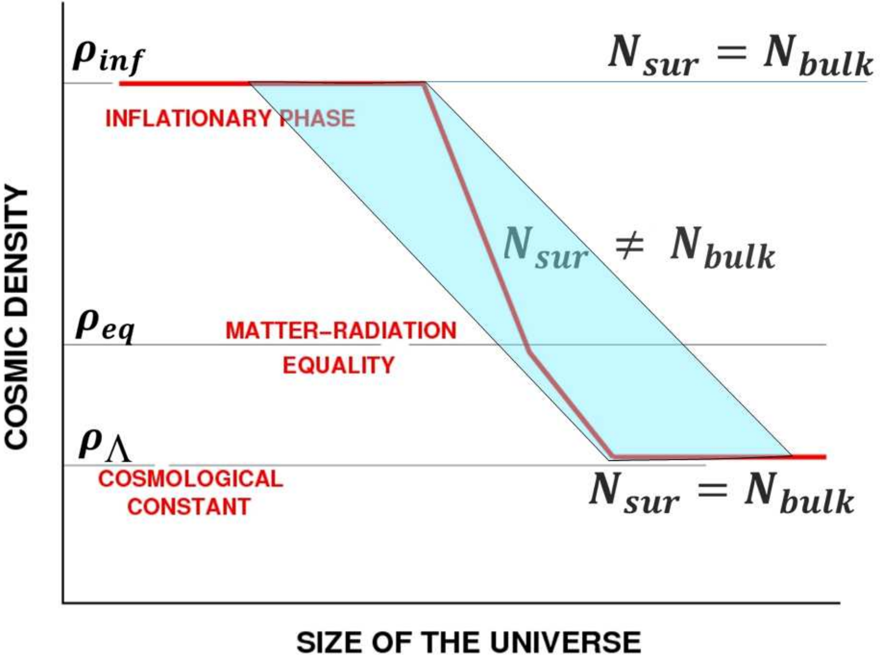

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Value of the Cosmological Constant

Hamsa Padmanabhan, T.P. [arXiv:1302.3226]

$$\rho_\Lambda = \frac{4}{27}\frac{\rho_{inf}^{3/2}}{\rho_{eq}^{1/2}}\exp(-36\pi^2)$$

$$e^{S(x^i)} \propto \int \mathcal{D}n_i P(x^i, n_a) \exp[\mu L_P^4 T_{ab} n^a n^b]$$

Speculation

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$$P(x^i, n_a) \propto \exp[\mu f(x^i, n_a)] \propto \exp\left(-\frac{\mu L_P^2}{8\pi} R_{ab} n^a n^b\right)$$

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MAY BE ONE SHOULD NOT THINK OF COSMOLOGY AS PART OF GENERAL RELATIVITY!

Open Questions

- Matter and Geometry need to emerge together for proper interpretation of $\boldsymbol{T^{ab}n_a n_b}$ at the microscopic scale. How do we do this?

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- Generalisation to Lanczos-Lovelock models with $R_{ab} \to \mathcal{R}_{ab}$: What happens at microscopic scales?

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- If we take number of atoms of spacetime at an event to be proportional to the area measure in a spacetime with zero-point length, we get the correct variational principle
- A Planck scale 2-sphere has 4π degrees of freedom which allows the determination of the cosmological constant

References

T.P, *General Relativity from a Thermodynamic Perspective*, *Gen. Rel. Grav.*, **46**, 1673 (2014) [arXiv:1312.3253].

T.P, *Distribution function of the Atoms of Spacetime and the Nature of Gravity*, (2015) [arXiv:1508.06286].

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