Bohmian conditional wave functions

(and the status of the quantum state)

…some illustrative toy models in support of a half-baked idea…

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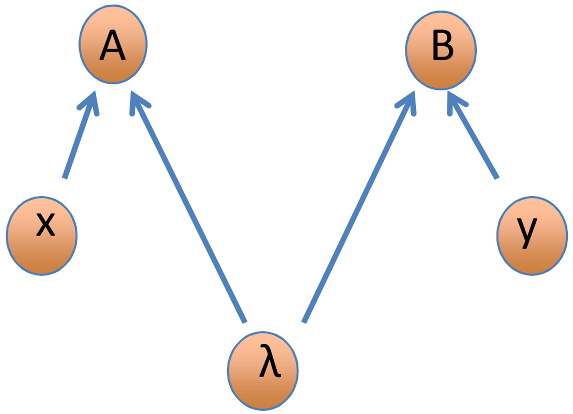
Context…

- Is the world local or non-local?
  - QFT?
  - Two Bell's theorems?
- …but which kind of nonlocality?
  - Nonlocal dynamics vs. nonlocal beables



Outline:

1. Bohmian Mechanics
2. Conditional Wave Functions (CWFs)
3. Semi-Classical Approximations
4. The Bohmian Double Semi-Quantum (BDSQ) Approximation
5. Why??

1. Bohmian Mechanics (aka dBB pilot-wave theory)

- Wave function $\Psi = \Psi(q,t)$ satisfying Schrödinger's equation:

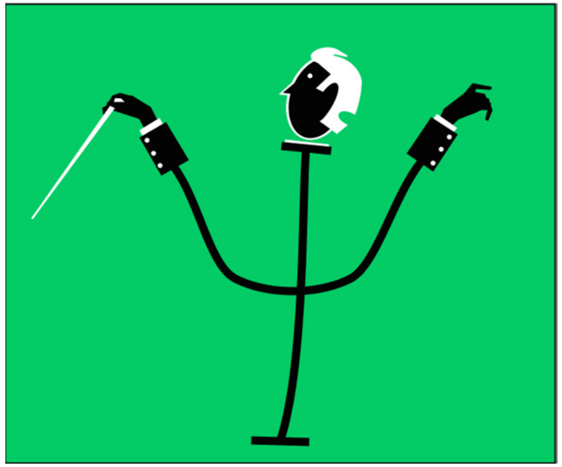
$$i\hbar\frac{\partial\Psi}{\partial t} = \sum_{i=1}^{N}\frac{-\hbar^2}{2m_i}\vec{\nabla}_i^2\Psi + V(x_1,...,x_N)\Psi$$

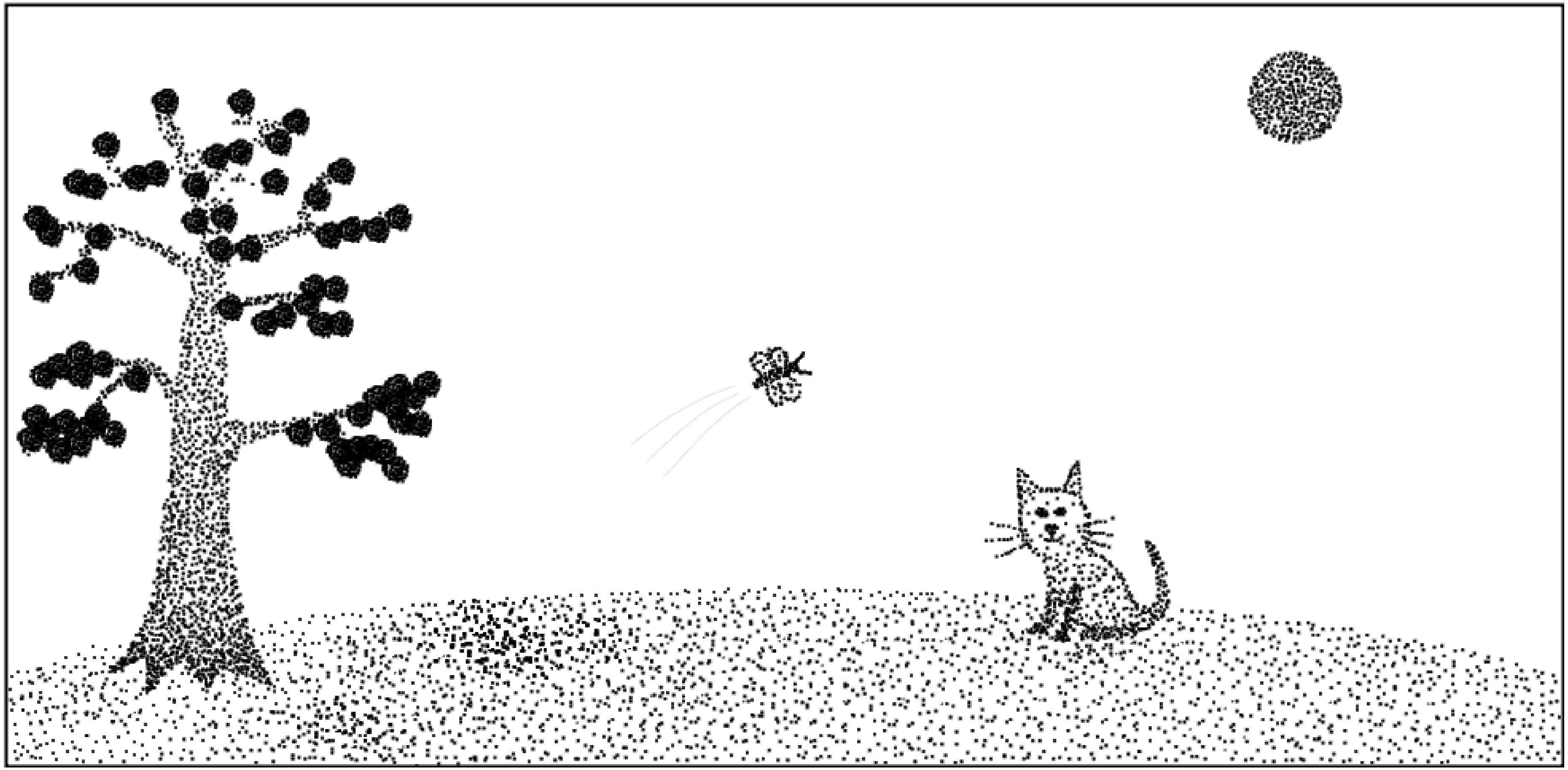
- N particles with configuration $Q = \left\{\vec{X}_1, \vec{X}_2, ..., \vec{X}_N\right\}$ evolving according to:

$$\frac{d\vec{X}_i}{dt} = \frac{\vec{j}_i(Q)}{\rho(Q)} = \frac{\hbar}{m_i}\,\mathrm{Im}\left(\frac{\vec{\nabla}_i\Psi}{\Psi}\right)\Bigg|_{\vec{x}_j=\vec{X}_j\forall j}$$

That's it!

The world according to dBB:





2. Bohmian CWF (Conditional Wave Function)

- The "Psi" appearing in the definition of the theory is not the wave function one normally deals with in QM. It is instead the wf of the universe. So it is not immediately obvious how the fundamental postulates of the theory relate to the textbook QM formalism!

- But it does… Bohmian Mechanics provides a very natural way of defining the wave function of a sub-system – the so-called "conditional wave function" [1] :

$$\phi(x,t) = \Psi(q,t)\big|_{y=Y(t)} = \Psi\left(x, Y(t), t\right)$$

- Note that the "guidance equation" (according to which the wave function "conducts" the motion of the particles) for each particle can be re-expressed in terms of that particle's CWF:

$$\frac{d\vec{X}_i}{dt} = \frac{\hbar}{m_i}\,\mathrm{Im}\left(\frac{\vec{\nabla}_i\phi(\vec{x}_i,t)}{\phi(\vec{x}_i,t)}\right)\Bigg|_{\vec{x}_i=\vec{X}_i(t)}$$

- Claim: the Bohmian CWF *is* the familiar wave function of textbook QM.

Case 1: Subsystem decoupled from environment:

Assume V(x,y) = V(x) + V(y) (…and no initial entanglement…)

Then the solution of the Schrödinger equation takes the form:

$$\Psi(q,t) = \alpha(\vec{x}_i,t)\beta(y,t)$$

so (up to an overall normalization factor) the CWF is just the obvious sub-system wave function:

$$\phi(\vec{x}_i,t) = \alpha(\vec{x}_i,t)\beta(Y(t),t) \sim \alpha(\vec{x}_i,t)$$

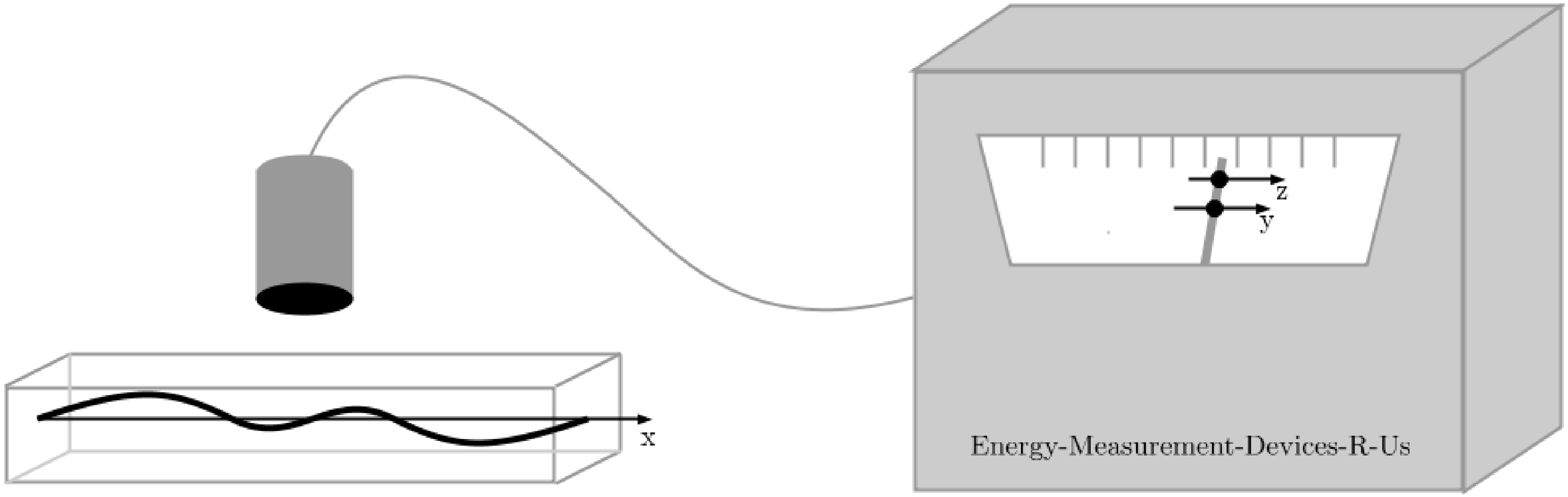
and the CWF obeys an obvious sub-system Schrödinger equation (with an extra time-dependent constant term in the potential since the CWF isn't normalized):

$$i\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{2m}\vec{\nabla}_i^2\phi + V(x)\phi + f(t)\phi$$

So the Bohmian CWF behaves just like an (unnormalized) textbook QM wf in the kind of situation where textbook QM says wfs obey Schrödinger's equation.

Case 2: What if the subsystem does interact strongly with its environment (as happens, for example, in a "measurement")?

Toy Model: Energy measurement on a 1D particle-in-a-box

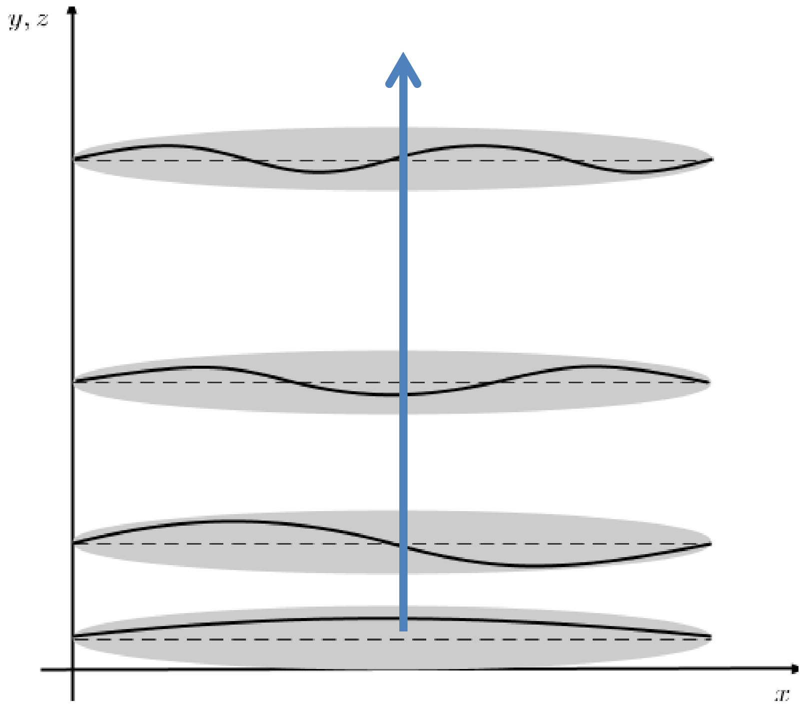


$$\hat{H} = \hat{H}_x + \hat{H}_y + \hat{H}_z + \lambda \hat{H}_x \left(\hat{p}_y + \hat{p}_z\right)$$

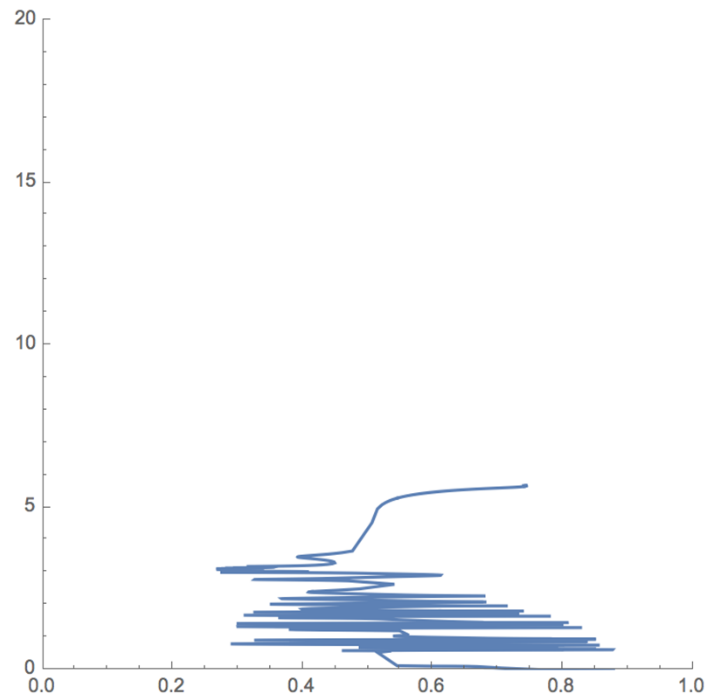
Initial condition: $$\Psi(x,y,z,0) = \left(\sum_n c_n \psi_n(x)\right) G_0(y) G_0(z)$$

Exact solution: $$\Psi(x,y,z,t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar} G_t(y - \lambda E_n t) G_t(z - \lambda E_n t)$$

Config. Space visualization:



Bohmian trajectory for some random initial condition X(0), Y(0), Z(0).

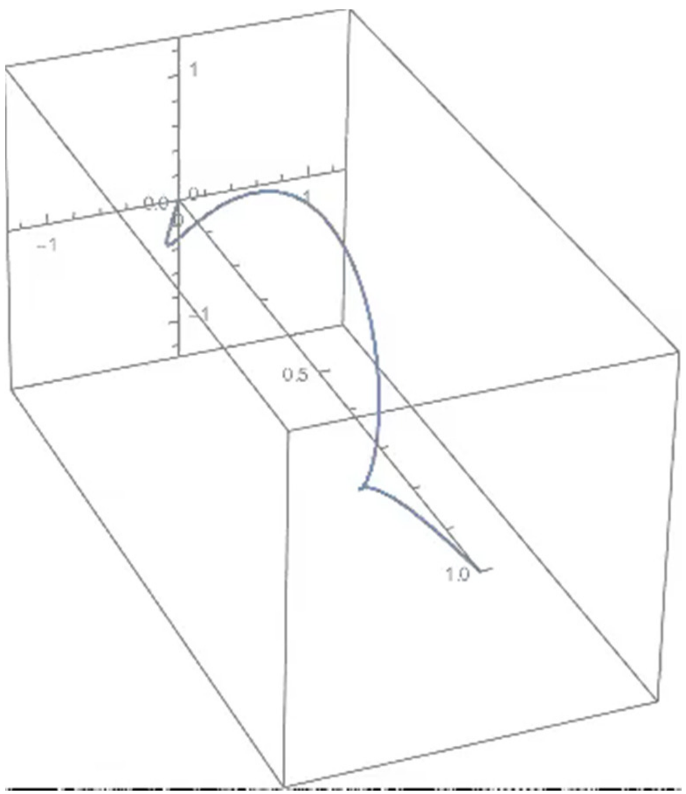


Note: the expression for the Bohmian velocity dQ/dt is a little unusual due to the unusual interaction term in the Hamiltonian.

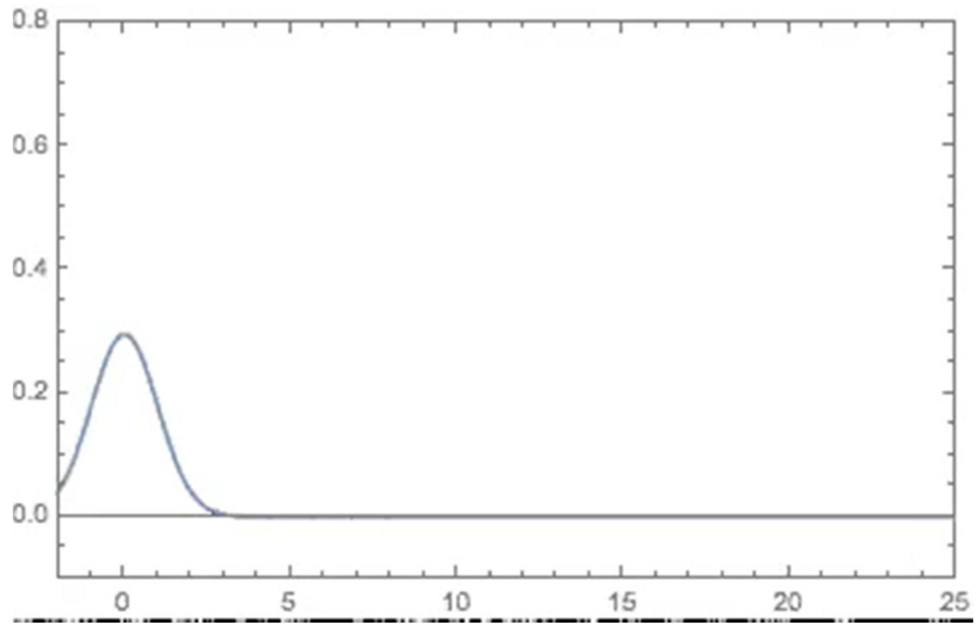
Movies of the CWFs for the PIB and one of the pointer particles…

(Note that for these initial conditions, the final positions of the pointer particles correspond to the “n=2” outcome for the energy measurement.)

$\phi_1(x, t)$



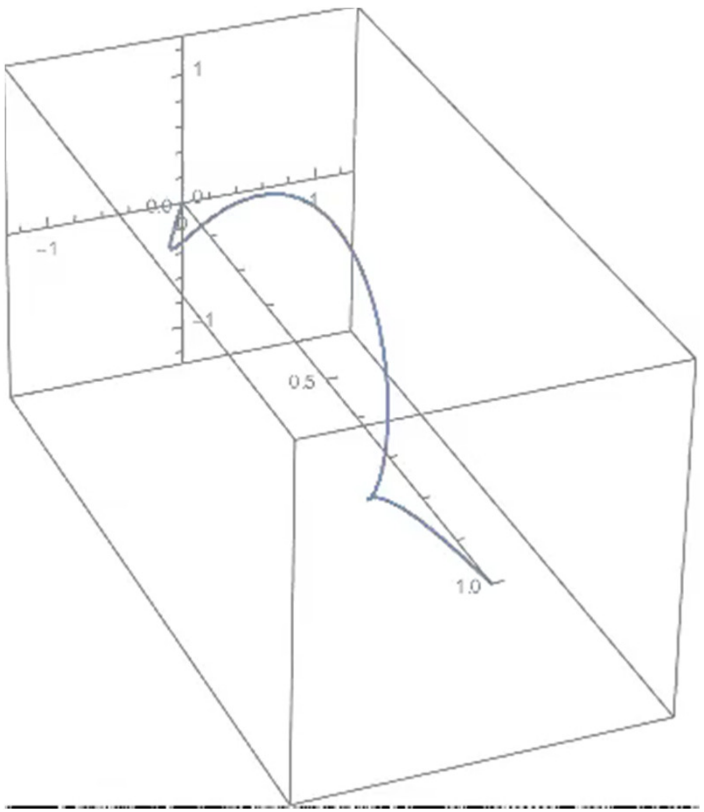
$|\phi_2(y, t)|^2$



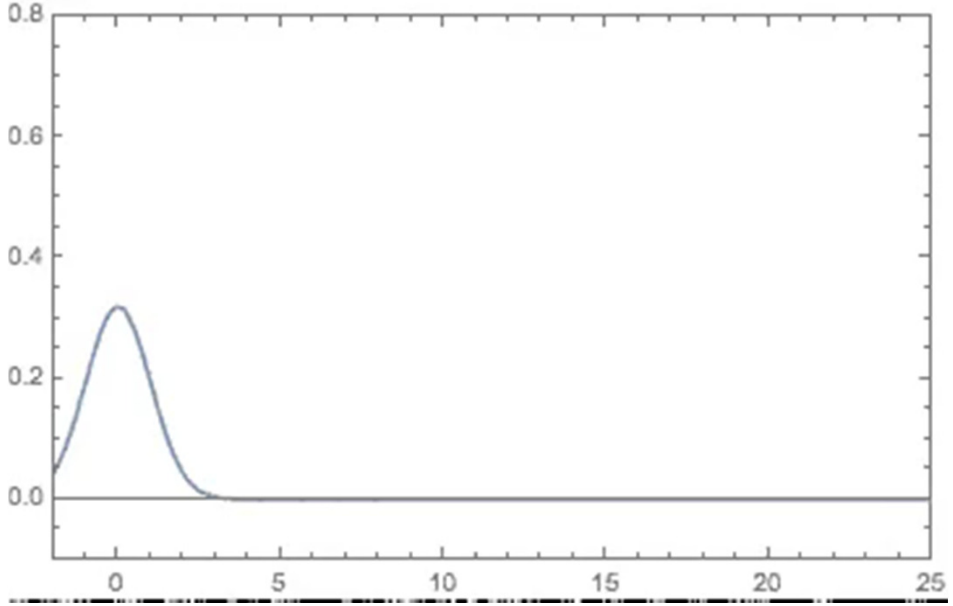
Same thing but for different X(0) which happens to give "n=4"…

(Note that both CWFs are the same, initially, as before!)

$\phi_1(x,t)$



$|\phi_2(y,t)|^2$



Summary so far:

- Bohmian CWFs not only evolve in accordance with Schrödinger's equation when the textbook wave function is supposed to, they also *collapse* when the textbook wave function is supposed to do that (and in exactly the right way, and with exactly the right statistics, etc…).

- But they do this "automatically" – without any fundamental distinction between 'process 1' and 'process 2' or any division of the world into 'speakable' and 'unspeakable' or anything like the ambiguities/inconsistencies that give rise to ordinary QM's infamous 'measurement problem'. They behave properly purely as a mathematical consequence of the two fundamental equations on the earlier slide.

- So… if one of the major questions of the conference is "Might QM emerge from some underlying theory, and if so, what might that look like?" Bohmian Mechanics should be considered very seriously as a viable answer. It provides a straightforward realist account of what the world might be like, and it genuinely *explains* the textbook quantum formalism (including wf collapse, but also including operators-as-observables [1] and other things I haven't stressed here).

- Now move from standard Bohmian Mechanics to extensions, approximations, and half-baked ideas…

3. Semi-Classical Approaches:

Consider a 2-particle system, say 2 interacting SHOs:

$$\hat{H} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2\right) + \left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial y^2} + \frac{1}{2}M\omega^2 y^2\right) + \lambda xy$$

If (say) M >> m, we might treat the system semi-classically:

$$\begin{cases} i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}m\omega^2 x^2\psi + \lambda Y\psi \\ \ddot{Y} + \omega^2 Y + \frac{\lambda}{M}Y = 0 \end{cases}$$

where … in the "orthodox" semi-classical treatment

$$\mathcal{X} = \langle\psi|\hat{x}|\psi\rangle$$

But … in the Bohmian semi-classical approach

$$\mathcal{X} = X \qquad \text{satisfying} \qquad \frac{dX}{dt} = \frac{\hbar}{m}\,\mathrm{Im}\left(\frac{\nabla\psi}{\psi}\right)\Big|_{x=X}$$

See Refs [2], [3], [4], [7] for discussion, numerical examples…

Derivation of BSC approximation:

Bohmian description of two-particle system:

$$i\hbar\frac{\partial\Psi(x,y,t)}{\partial t} = \left[\frac{-\hbar^2}{2m}\nabla_x^2 + \frac{-\hbar^2}{2M}\nabla_y^2 + V(x,y)\right]\Psi(x,y,t)$$

$$\frac{dY}{dt} = \frac{\hbar}{M}\,\mathrm{Im}\left(\frac{\nabla_y\Psi}{\Psi}\right)\Big|_{x=X(t),\,y=Y(t)} \quad \rightarrow \quad M\ddot{Y} = -\nabla_y\left(V+Q\right)|_{x=X(t),\,y=Y(t)}$$

$$\frac{dX}{dt} = \frac{\hbar}{m}\,\mathrm{Im}\left(\frac{\nabla_x\phi}{\phi}\right)\Big|_{x=X(t)}$$

CWF for particle 1 satisfies:

$$i\hbar\frac{\partial\phi(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\nabla_x^2\phi(x,t) + V\left[x,Y(t)\right]\phi(x,t) + i\hbar\frac{dY}{dt}\frac{\partial\Psi}{\partial y}\Big|_{y=Y(t)} + \frac{-\hbar^2}{2M}\frac{\partial^2\Psi}{\partial y^2}\Big|_{y=Y(t)}$$

(See [7] for applications to field theory, gravity, etc.)

4. Bohmian Double Semi-Quantum (BDSQ) Approximation

There is no particular reason that we have to treat one of the two particles *classically*…

We can instead split the two-particle quantum system into two (interacting) one-particle systems, each of which we treat just as we treated the quantum half in the semi-classical treatment:

$$i\hbar\frac{\partial\Psi(x,y,t)}{\partial t} = \cdots$$

$$i\hbar\frac{\partial\phi_1(x,t)}{\partial t} = -\frac{\hbar^2}{2m_1}\frac{\partial^2\phi_1}{\partial x^2} + V[x,Y(t)]\phi_1 + i\hbar\frac{dY}{dt}\frac{\partial\Psi}{\partial y}\bigg|_{y=Y(t)} - \frac{\hbar^2}{2m_2}\frac{\partial^2\Psi}{\partial y^2}\bigg|_{y=Y(t)}$$

$$i\hbar\frac{\partial\phi_2(y,t)}{\partial t} = -\frac{\hbar^2}{2m_2}\frac{\partial^2\phi_2}{\partial y^2} + V[X(t),y]\phi_2 + i\hbar\frac{dX}{dt}\frac{\partial\Psi}{\partial x}\bigg|_{x=X(t)} - \frac{\hbar^2}{2m_1}\frac{\partial^2\Psi}{\partial x^2}\bigg|_{x=X(t)}$$

…and of course Bohmian particles, each guided by its own one-particle wave-function.

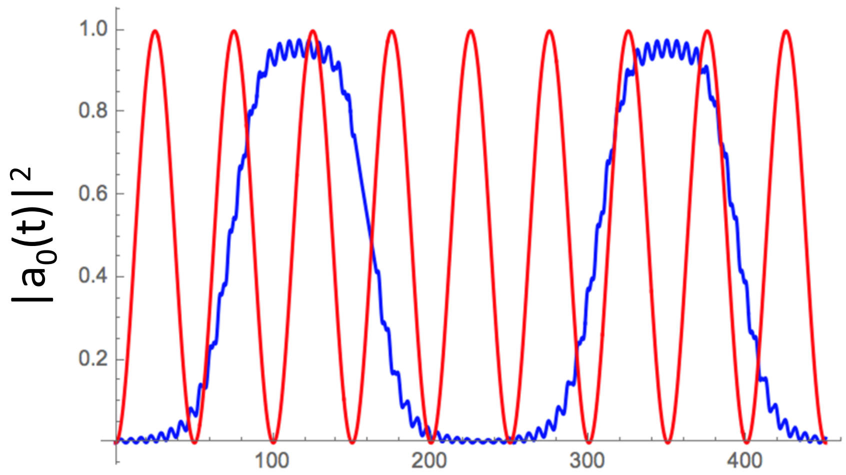
Note that this scheme will exactly reproduce the ordinary Bohmian particle trajectories (and hence the empirical predictions of ordinary QM) if the two particles are (and remain!) unentangled. So it is in some sense a "small entanglement approximation" (SEA). [6]

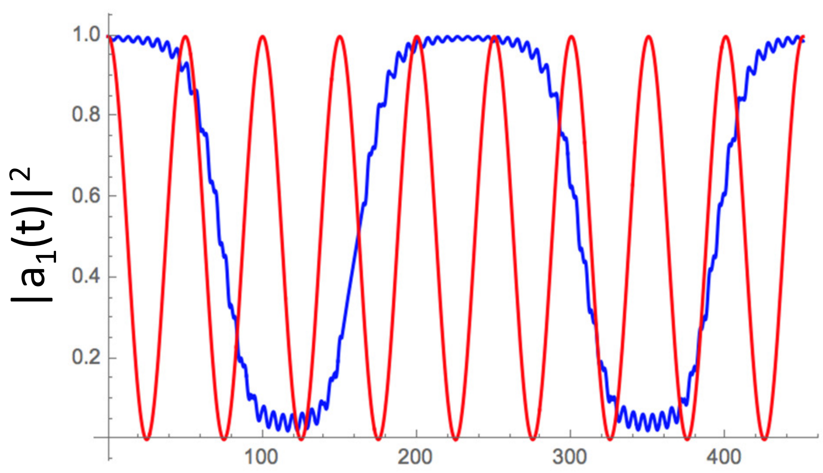
Simple example: two interacting (equal mass) SHOs:

$$\hat{H} = \hat{H}^1_{SHO} + \hat{H}^2_{SHO} + \lambda \hat{x}\hat{y}$$

Exact Solution: $|\Psi(t)\rangle = \cos\left(\frac{\lambda t}{\hbar\omega}\right)|1,0\rangle - i\sin\left(\frac{\lambda t}{\hbar\omega}\right)|0,1\rangle$

BDSQ results: $\phi_1(x,t) = a_0(t)\psi_0^{SHO}(x,t) + a_1(t)\psi_1^{SHO}(x,t)$

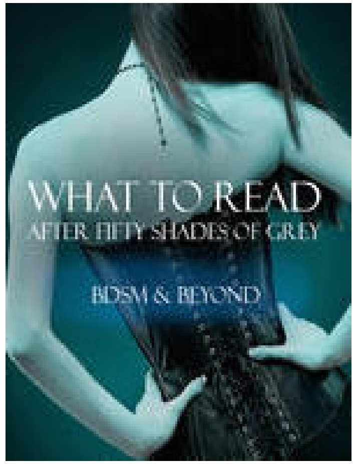




Qualitative behavior is right, but quantitative details aren't quite right…
… not shocking since this was a (rather uncontrolled!) approximation scheme.

Beyond BDSQ

not to be confused with →



Include the neglected terms by introducing new "fields":

$$\phi_1'(x,t) = \left.\frac{\partial \Psi(x,y,t)}{\partial y}\right|_{y=Y(t)} \qquad \phi_1''(x,t) = \left.\frac{\partial^2 \Psi(x,y,t)}{\partial y^2}\right|_{y=Y(t)}$$

(etc.) obeying their own dynamical Schroedinger-type equations (e.g.)

$$i\hbar\frac{\partial \phi_1'(x,t)}{\partial t} = -\frac{\hbar^2}{2m_1}\frac{\partial^2 \phi_1'}{\partial x^2} + V[x,Y(t)]\phi_1' + \frac{\partial V}{\partial y}[x,Y(t)]\phi_1 + i\hbar\frac{dY}{dt}\phi_1'' - \frac{\hbar^2}{2m_2}\phi_1'''$$

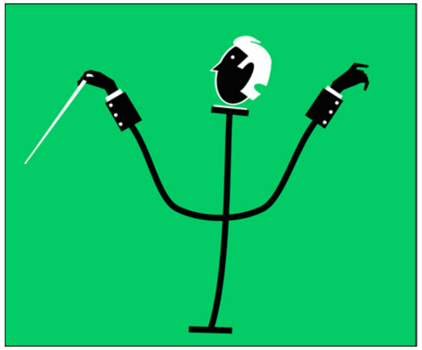
…or maybe some other way of packaging/sorting all the structure that's left out of BDSQ…

This remains work in progress… but there must exist some sensible, converging expansion here which moves the BDSQ approximation toward the exact results by introducing additional beables which capture (what is described in ordinary QM as) entanglement.

5. Who cares? Why bother??

- May prove useful as novel (uniquely Bohmian) approximate treatment of many-body systems
- But my interest is more metaphysical:

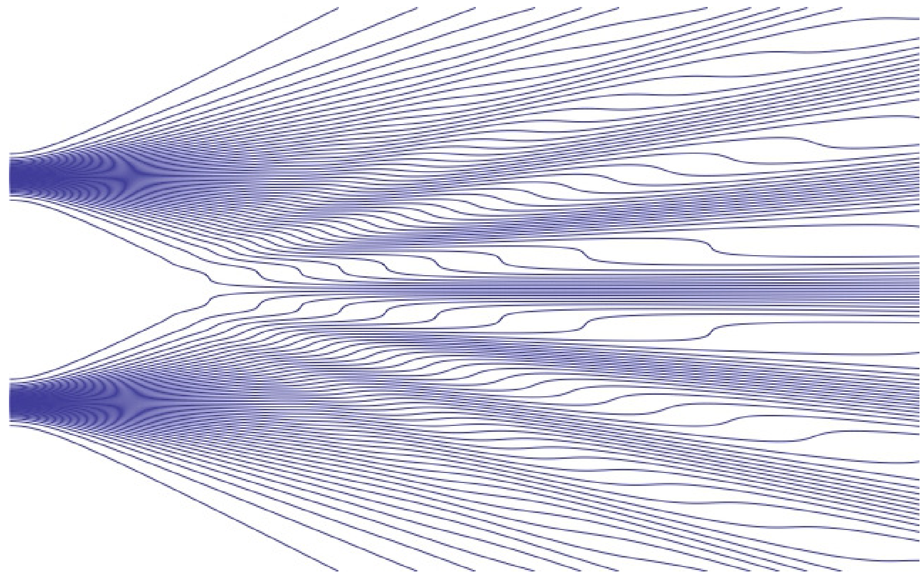
This guy is weird →



To me it is certain that QM emerges from something deeper. (Measurement problem!) Bohmian mechanics seems to me to provide the best available candidate for an underlying theory, largely because it gives a perfectly clear and comprehensible account of local beables (the particles) in terms of which we can recognize the familiar 3D world. But the universal wave function is, to me, a very strange kind of thing (living as it does in configuration space) and I never got the hang of interpreting it as a "law". Reformulating BM in terms of single-particle pilot-wave (3-space) fields(and more…) seems like a promising avenue toward a Theory of Exclusively Local Beables (TELB).

Matt Liefer's "Explanatory conservatism" (applied in a way he would probably disagree with!):

"If there is a natural explanation for a quantum phenomenon then we should adopt an interpretation that incorporates it."



→ Particles guided by fields in 3D.

The Bohmian "TELB" vision in a picture:

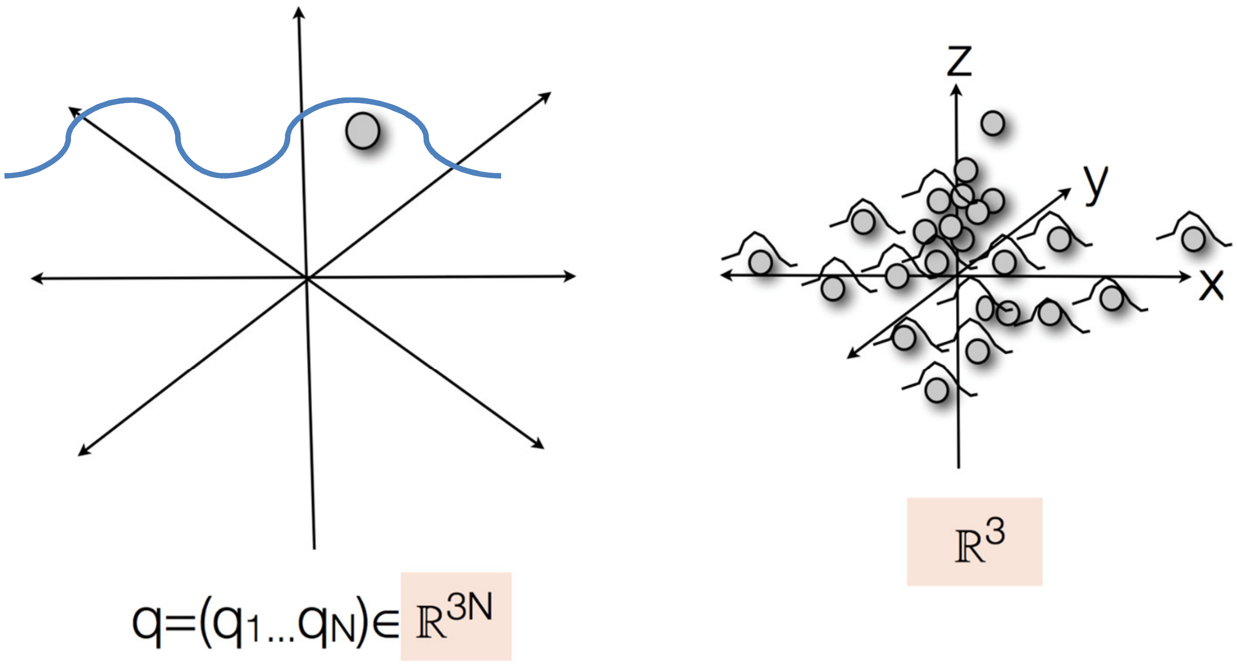


Figure 2.2: Albert vs Norsen

(From C. Callender, "One World, One Beable", Synthese 2014)

References:

[1] Detlef Dürr, Sheldon Goldstein, and Nino Zanghi, "Quantum Equilibrium and the Origin of Absolute Uncertainty", J. Stat. Phys. Vol. 67 (1992) pp. 843-907 as well as "Quantum Equilibrium and the Role of Operators as Observables in Quantum Theory", J. Stat. Phys. Vol. 116 (2004) pp. 959-1055.

[2] E. Gindensperger, C. Meier and J.A. Beswich, "Mixing quantum and classical dynamics using Bohmian trajectories", J. Chem. Phys. Vol. 113 (2000), 9369-9372

[3] Oleg Prezhdo and Craig Brooksby, "Quantum Backreaction Through the Bohmian Particle" PRL Vol. 86:15 (April 9, 2001) pp. 3215-3219

[4] Xavier Oriols, "Quantum Trajectory Approach to Time-Dependent Transport in Mesoscopic Systems with Electron-Electron Interactions", PRL Vol. 98 (2007) 066803

[5] Travis Norsen, "The theory of (exclusively) local beables", Foundations of Physics Vol. 40 (2010), pp. 1858-1884 and arXiv:0909:4553v2 [quant-ph]

[6] Travis Norsen, Damiano Marian and Xavier Oriols, "Can the wave function in configuration space be replaced by single-particle wave functions in physical space?", Synthese (2015) and arXiv:1410.3676 [quant-ph]

[7] Ward Struyve, "Semi-Classical Approximations based on Bohmian Mechanics" arxiv:1507.04771