EPR paradox, Quantum Nonlocality and Physical Reality

Marian Kupczynski
UQO

Vienna -EMQ15
Motto (Einstein)

Raffiniert ist der Herrgott
aber boshaft ist er nicht.

Subtle is the Lord
but devious he is not.

GOD ↔ NATURE
My philosophical stand point

• The moon does continue to exist even if I do not look at it.

• Probability is an objective contextual property of some random experiments and not a subjective belief of some intelligent agents.

• There is no need to reject causality and locality in order to explain EPR correlations.

http://w4.uqo.ca/kupcma01/homepage.htm
False Paradoxes

We roll a pair of dice. Each die on its own is random and fair, but its entangled partner somehow always gives the correct matching outcome. (IMPOSSIBLE!!!)

A particle passes through two neighboring slits at the same time. Therefore, an electron, is indeed both here and a meter to the right of here. (IMPOSSIBLE!!!)

Schrödinger Cat Paradox etc.
Quantum Nonlocality

Quantum nonlocality, whereby particles appear to influence one another instantaneously even though they are widely separated .....is a well established experimental fact. → MYSTERY

Correlations are coming out of space time!
Continuity Principle Violated

1. Explanation by causes belonging to the common past.

2. Explanation for correlations by a first event causing the next.

$\text{SPCE} \to \text{Explanations 1. and 2. failed} \to \text{Nature does not satisfy the continuity principle} \to \text{NATURE is nonlocal}$

WRONG CONCLUSION: Explanation 1 did not fail!
Law of Nature: nonlocal randomness?

We must accept... nonlocal randomness, an irreducible randomness that manifest itself in several widely separated places without propagating from one point of space to next. Producing at the same time strongly correlated outcomes.
Godwits migration from Alaska to New Zealand
Nonlocal randomness vs Physical Reality

• Bar-tailed Godwit *(Limosa lapponica baueri)* do the fall migration from Alaska to New Zealand in one hop.

• They *fly 11 000 km in about eight days* over the open Pacific Ocean, *without stopping to rest or refuel*.

• How could they do it if *nonlocal randomness* was a *law of Nature*?
Plan of this talk

• EPR Paradox and statistical Interpretation.
• SPCE and long range correlations.
• CFD and finite sample proofs of CHSH and BI.
• Impossible QRC of Vongher and Gill.
• Contextuality and local causality vs nonlocality.
• SPCE data and sample homogeneity loophole (SHL).
• Physical reality and its abstract description.
QT Orthodoxy 1935

INDIVIDUAL PHYSICAL SYSTEM: $t = T$

Pure State $\rightarrow$ Unique Wave Function

Instantaneous wave function reduction:

Any measurement causes a physical system to jump into one of the eigenstates of the dynamical variable that is being measured.
EPR PAPER (1935)

EPR: Two systems I + II in a pure quantum state which interacted in the past and which separated.

• A single measurement performed on one of the systems, for example on the system I, gives instantaneous knowledge of the wave function of the second system moving freely far away.

• By choosing two different incompatible observables to be measured on the system I it is possible to assign two different wave functions to the same physical reality (the second system after the interaction with the first)"
Schrödinger (1936)

No matter how far apart the particles are when we try to collect one of them they are not really “free” → Entanglement, Quantum Steering → Mystery, spooky action at the distance etc???
EPR- explained

• Einstein (1936): \( \Psi \) function does not, in any sense, describe the state of one single physical system. Reduced wave functions describe different sub-ensembles of the systems.

• Ballentine (1998) …the habit of considering an individual particle to have its own wave function is hard to break…. though it has been demonstrated strictly incorrect ..
A state vector or a density matrix describes an ensemble of the identical preparations of the physical systems.
EPR-B (1951) SINGLET STATE

\[ \Psi = \frac{1}{\sqrt{2}} (|+\rangle_\hat{p} \otimes |-\rangle_\hat{p} - |-\rangle_\hat{p} \otimes |+\rangle_\hat{p}) \]

where \(|+\rangle_\hat{p}\) and \(|-\rangle_\hat{p}\) are state vectors corresponding to the particle states in which the spin is "up" or "down" in the direction of \(\hat{p}\) respectively.
No EPR-B PARADOX IN SI

• The reduced quantum state $I^+ \rightarrow_\mathcal{P}$ describes the ensemble of partners of the particles which were analyzed to have “spin down” by the analyzer $\mathcal{P}$.

• For various directions $\mathbf{p}$ it is a different ensemble.

• Even if the anti-correlations were strict there is no paradox.

EPR-B- Nonlocal Correlations Paradox

• A pulse of laser hitting the non linear crystal produces two correlated signals propagating in opposite directions.
• Clicks on the distant detectors are correlated.
• Bell and followers:

ALL LOCAL MODELS → BELL, CHSH, CH.. INEQ.

NOT ALL LOCAL MODELS!
SPCE IDEALIZED – STRONG CORRELATIONS

2 correlated signals $S_1$ and $S_2$ produced by a source $S$ are hitting the measuring devices $x$ and $y$ producing correlated outcomes $a=\pm 1$ and $b=\pm 1$.
Generalised joint probability distributions (GJPD)

2 random experiments x and y to measure A and B

two samples : \{a_1, a_2, ..., a_n \} and \{b_1, b_2, ..., b_n \}

math. stat.: observations of two time series of random variables \{A_1, A_2, ..., A_n \} and \{B_1, B_2, B_n \}

Problem how to pair the data in order to find a GJPD?

IN GENERAL: NON UNIQUE SOLUTION!

GJPD depends on a protocol how the pairing is done. In SPCE: \textit{width of the time windows, coincidence technique etc.}
Correlations depend on Pairing

• Pairing $S_{1k}$: \{(a_1, b_k), (a_2, b_{k+1}), (a_3, b_{k+2})\ldots \}

• Pairing $S_R$: \{ pairs $(a_s, b_t)$ chosen at random \}

Example $S_1 = 01010101\ldots$ $S_2 = 101010101\ldots$

Pairing $S_{1k}$ $k=$ odd \quad strong anti-correlation

Pairing $S_{1k}$ $k =$ even \quad strong correlation

Pairing $S_R$ no correlation!
Correlations

• Once the pairing is defined, GJPD and correlations may be estimated.

• In general, GJPD do not factorize:

\[ P(a, b \mid x, y) = P(A=a, B=b \mid x, y, S_1, S_2) \neq P(A=a \mid x, S_1) P(B=b \mid y, S_2) \]
Distant Correlations

• The correlations do not prove any causal relation between x and y.

• No communication or direct influence between x and y is needed for their existence.
QT→No strict anti-correlations!

• If $A$ and $B$ are the spin projections on two directions characterized by angles $\vartheta_A$ and $\vartheta_B$ respectively then for a perfect singlet state:

$$E(AB \mid \psi) = -\cos(\theta_A - \theta_B)$$

• No sharp directions exist in Nature. Thus QT prediction is:

$$E(AB \mid \psi) = -\int_{I_A} \int_{I_B} \cos(\theta_1 - \theta_2) \, d\rho_A(\theta_1) \, d\rho_B(\theta_2)$$

where $I_A$ and $I_B$ are small intervals around $\vartheta_A$ and $\vartheta_B$ (MK, Phys.Lett. 121, 51-53, 1987)
Quantum state produced in SPCE

- Werner State:
  \[ \rho = \left( V |\Psi\rangle\langle\Psi| + (1 - V) I \right)/4 \]

- Eberhard state:
  \[ |\Psi_r\rangle = (1 + r^2)^{-1/2} \left( |H\rangle\langle V| + r |V\rangle\langle H| \right) \]

- Köfler et al.:
  \[ \rho = \frac{1}{1 + r^2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & Vr & 0 \\ 0 & Vr & r^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

1 > r > 0 \quad 1 > V \quad \text{but in fact } V \text{ is complex and no zero elements in } \rho \quad \text{(r=0.297, V=0.965 good fit of the data)}
2 correlated signals $S_1$ and $S_2$ produced by a source $S$ are hitting the measuring devices $x$ and $y$ producing correlated outcomes $a=\pm 1$ and $b=\pm 1$.

A : 1 or 2

\[ -11-1... \leftarrow x \leftarrow 010... \leftarrow S \rightarrow 101... \rightarrow y \rightarrow 1-11 \]

Messages sent by S contain 50% of 0s and 1s.

- Random pairing \( S_R \) imposed by the randomizers A and B → CHSH
- Gill’s result: finite samples may violate CHSH: but the probability of large violation of CHSH is small.

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Counterfactual definiteness (CFD)

- A physical system is characterized by some well-defined attributive properties.
- A measuring apparatus is reading with possible mistakes the predetermined values of these properties.
- In SPCE the outcomes of all measurements on a ‘photon pair’ are predetermined by a source.

CFD contradicts QT.
CFD proof of CHSH by R. Gill (2014)

• The outcomes are predetermined attributes \((A, A', B, B')\) of the incoming “photon pairs”.

• 4N subsequent pairs are described by 4N x 4 spreadsheet of numbers \(\pm 1\).

• The rows are labelled by an index \(j = 1, 2, \ldots, 4N\) and columns by names of 4 attributes: \(A, A', B, B'\).
# Table produced by R. Gill using CFD

<table>
<thead>
<tr>
<th>Pair</th>
<th>A</th>
<th>A’</th>
<th>B</th>
<th>B’</th>
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<td>4</td>
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Deduced sample for a setting \((x, y')\)

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</table>
CFD in conflict with QT

• The outcomes are not predetermined and 4N×4 cannot be constructed for SPCE.

• 4N×4 spreadsheet defines a random sample drawn from JPD for 4 random variables (A, A', B, B') like in LRHVM

• SPCE are the experiments for which GJPD do not exist. (Boole, Vorobev, Andrei, Hans, Itamar, Karl, Kristel, Luigi, Marian, Theo, Walter...)

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</table>
## Nx2 samples for subsequent settings

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<td>4</td>
<td>1</td>
<td>-1</td>
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Quantum Randi Challenges (QRC)


Bell’s Theorem Proof (VONGHER 2013)

• Photon pair → Pair of tennis balls

• Deterministic instructions on each ball tell whether output 0 or 1 according to the angle it encounters at the measuring station
Bell’s Theorem Proof (VONGHER 2013)

• For each pair of balls we may fill one row of the 800 x 4 counterfactual spreadsheet discussed by Gill but now instead of -1 we input 0.

• Strict anti-correlation for $a=b=0$ reduces the degrees of freedom to 3 and Bell’s theorem is proven etc. (CFD)
IMPOSSIBLE QRC (GILL 2014)

Conjecture:

\[
\Pr \left( \langle AB \rangle_{obs} + \langle AB' \rangle_{obs} + \langle A'B \rangle_{obs} - \langle A'B' \rangle_{obs} \geq 2 \right) \leq \frac{1}{2}
\]

• Create a 4Nx4 spreadsheet (e.g. N=800).
• Find \( \langle AB \rangle_{obs}, \langle AB' \rangle_{obs}, \langle A'B \rangle_{obs}, \langle A'B' \rangle_{obs} \) using Gill’s protocol.
• Violate systematically and significantly CHSH more than 50% of times.
• Get a Nobel prize.

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Finite sample proofs and QRC

Finite sample used are drawn from populations described by the probabilistic models used in:

- LRHVM or *Bertlmann’s Socks Model.*

\[ E(A_i B_j) = \sum_{\lambda \in \Lambda} P(\lambda_1, \lambda_2) A_i(\lambda_1) B_j(\lambda_2) \]

- SHVM or *Pairs of Dices Model*

\[ E(AB \mid x, y) = \sum_{\lambda \in \Lambda} P(\lambda_1, \lambda_2) E(A \mid x, \lambda_1) E(B \mid y, \lambda_2) \]

\[ P(a, b \mid x, y) = \sum_{\lambda \in \Lambda} P(\lambda_1, \lambda_2) P(a \mid x, \lambda_1) P(b \mid y, \lambda_2) \]
Nature’s local strategy (?)

• The outcomes are neither predetermined by a source or by a choice of the setting.
• The outcomes are not randomly and independently produced at the measuring stations
• The outcomes ±1 and 0 are produced in a deterministic way in the moment of a measurement in function of supplementary parameters describing (x, y) and the correlated signals.
a=0,±1 is determined in local deterministic way by the values of \( \lambda_1 \) and \( \lambda_x \) describing the signal \( S_1 \) and the measuring device \( x \) in a moment of measurement. In a similar way \( b=0, ± 1 \)
Meaning of hidden variables

Properties of correlated signals arriving to PBS-D module at time t as perceived by them are coded by \((\lambda_1, \lambda_2)\). Microstates of PBS-D modules at time t are coded by \((\lambda_x, \lambda_y)\). In function of these parameters we observe in a local and causal way a click or no click on corresponding detectors what is coded by:

\[ A(\lambda_1, \lambda_x) = 0, \pm 1 \] and \[ B(\lambda_2, \lambda_y) = 0, \pm 1 \]

If we change \((x, y)\) into \((x_1, y_1)\) the properties of the signals as perceived by them are different.
Contextuality-KOLMOGOROV

EACH RANDOM EXPERIMENT- HAS IT’S OWN

PROBABILITY SPACE Λ CONTAINING ALL

POSSIBLE OUTCOMES OF THE EXPERIMENT
Local Probabilistic Model for SPCE

$$E(AB \mid x, y) = \sum_{\lambda \in \Lambda_{xy}} P(\lambda) A(\lambda_1, \lambda_x) B(\lambda_2, \lambda_y)$$

where $A(\lambda_1, \lambda_x) = 0, \pm 1$ and $B(\lambda_2, \lambda_y) = 0, \pm 1$

$$P(\lambda) = P_{xy}(\lambda_1, \lambda_2) P_x(\lambda_x) P_y(\lambda_y)$$

Now there is no common hidden parameter space $\Lambda$ and $\Lambda_{xy}$ are different hidden parameter spaces for each pair $(x, y)$.

It is impossible to prove B-CHSH and CH inequalities!
CHSH-BELL PROOFS

THE EXISTENCE OF A COMMON PROBABILITY SPACE TAKEN FOR GRANTED

\[ \Lambda_{xy} \neq \Lambda_{x'y'} \neq \Lambda_{xy'} \neq \Lambda_{x'y} \neq \Lambda \]

FATAL CONTEXTUALITY LOOPHOLE (T.N.)
It was noticed by:
Accardi, Fine, Hess, Khrennikov, M.K, Michielsen, de Muynck, Niewenhuizen, Pitovsky, Philipp, De Raedt,...

Vorob’ev (1962): ‘Is it possible to construct always the joint probability distribution for any triple of only pairwise measurable observables?’

NON.
FREE WILL

• Choice of the setting $x$ does not depend on supplementary parameters $\Lambda(A) = (\Lambda_1, \Lambda_x)$

• However $\Lambda(A)$ depends strongly on $x$.

• Reasoning based on the symmetry is false!
Feasible quantum non-Randi challenge

Simulate the data preserving local causality and find agreement with QT or /and SPCE:

- Using

\[ E(AB \mid x, y) = \sum_{\lambda \in \Lambda_{xy}} P(\lambda)A(\lambda_1, \lambda_x)B(\lambda_2, \lambda_y) \]

where \( A(\lambda_1, \lambda_x) = 0, \pm 1 \), \( B(\lambda_2, \lambda_y) = 0, \pm 1 \) and

\[ P(\lambda) = P_{xy}(\lambda_1, \lambda_2)P_x(\lambda_x)P_y(\lambda_y) \]

- Using a different local model experimental outcomes of SPCE were successfully reproduced by K. De Raedt, H. De Raedt and, K. Michielsen, A computer program to simulate Einstein–Podolsky–Rosen–Bohm Computer Physics Communications 176 (2007) 642–651
Lessons from Vongher for SPCE

• Local realistic models exist in which CHSH may be violated 500 times when 1000 runs are simulated.

• In tests of Bell, CHSH and CH inequalities the results of only few long runs are presented.

• The ensembles of outcomes in SPCE are not homogenous.

• The sample inhomogeneity may invalidate statistical inference. MK and Hans de Raedt (2015)
CONCLUSIONS

VIOLATION OF B-CHSH

↓

NO IRRED. RANDOMNESS

↓

EMERGENT QT → DESCRIPTION OF SUBPHENOMENA?
Physical reality
Physical reality
Some visible phenomena

Visible picture of invisible elementary particles.


Sample Homogeneity Loophole

- Standard statistical inference and significance tests are reliable only if data sets are *simple random sample* what means:
  - all trials are independent
  - a sample is homogeneous.
- MK, Significance tests and sample homogeneity loophole, arxive:1505.063449[quant-ph]
Breakdown of Statistical Inference

• We generated 100 runs (each run containing $10^5$ data items)

• Tested a hypothesis $H_0: 1-B \geq 0$.

• For 3 runs the inequality was violated by more than 2000 SEM (standard error of the mean)

• The average over 100 runs ($10^7$ data items) gave $1-B = +0.95$ SEM. NO VIOLATION

Reason: Sample was not homogeneous.
Model 2 test of $H_0$: $1 - \langle A \rangle_s / \langle A \rangle \geq 0$

Protocol 3: $(N_1 = 4, N_2 = 25000)$ three runs containing $10^5$ data points compared with the average over a sample containing $10^7$ points

<table>
<thead>
<tr>
<th></th>
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<th>3</th>
<th>4</th>
<th>100 runs</th>
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</thead>
<tbody>
<tr>
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<td>0.1304×10^1</td>
<td>0.1303×10^1</td>
<td>0.9727</td>
</tr>
<tr>
<td></td>
<td>0.1396×10^{-3}</td>
<td>0.1395×10^{-3}</td>
<td>0.1397×10^{-3}</td>
<td>0.2851×10^{-1}</td>
</tr>
<tr>
<td></td>
<td>-2177 SEM</td>
<td>-2236 SEM</td>
<td>-2168 SEM</td>
<td>+0.95 SEM</td>
</tr>
</tbody>
</table>
CONCLUSIONS

Deviations from homogeneity can invalidate a statistical inference. Homogeneity tests should become a standard part of statistical analysis of any large sample of experimental data in any domain of science.
Giustina et al. experiment


Eberhard inequality tested $H_0: J \geq 0$

- 1 long run divided into **30 bins** giving a sample size $N=30$
- **69 $\sigma$** violation found
- Result inconclusive: run has to be divided in many bins and its homogeneity tested.
Lesson

Subquantal detailed descriptions of quantum phenomena and of physical reality cannot be naïve and if they exist they have to be complicated.

In order to check predictive completeness of QT one has to search for fine structures of experimental time series of data which were not predicted by QT.

Marian Kupczynski
EMQ15
CFD and Bell’s game
Settings $x$ and $y$ chosen randomly
Outcomes $a$ and $b$ determined locally
in function of $x$ and $y$
Bell’s Game: Hardware

\begin{align*}
\text{Alice} & \\
\text{Time } x & \quad \text{a} \\
9\text{h00 left} & \quad 0 \\
9\text{h01 left} & \quad 1 \\
9\text{h02 right} & \quad 1 \\
9\text{h03 left} & \quad 1 \\
9\text{h04 right} & \quad 1 \\
9\text{h05 right} & \quad 0 \\
\ldots
\end{align*}

\begin{align*}
\text{Bob} & \\
\text{Time } y & \quad b \\
9\text{h00 left} & \quad 0 \\
9\text{h01 left} & \quad 1 \\
9\text{h02 left} & \quad 1 \\
9\text{h03 right} & \quad 1 \\
9\text{h04 right} & \quad 0 \\
\ldots
\end{align*}
Local Protocols

Alice’s box:
• i=1: a=0 for all x
• i=3: a=x
• i=2: a=1 for all x
• i=4: a =1-x

Bob’s box:
• j=1: a=0 for all y
• j=3: a=y
• j=2: a=1 for all y
• j=4: a =1-j

We have 16 combinations of programs. Programs can change at each minute.

Programs (i,j) determine the outcomes (a,b)

N. Gisin: Quantum Chance. Springer 2014
Bell’s Game: Rules

\[ (a + b)^2 = xy \]  \hspace{1cm} (1)

Rules:

• If \((x, y) = (1, 1)\) and \(a \neq b\) \(\rightarrow\) 1 point
• If \((x, y) \neq (1, 1)\) and \(a = b\) \(\rightarrow\) 1 point
• Otherwise no point is gained

CLAIM: average score is always smaller than 3

QT: average score is 3.41 (nonlocal randomness)
<table>
<thead>
<tr>
<th>Alice’s program</th>
<th>Bob’s program</th>
<th>Results for choice ((x, y) = (0, 0))</th>
<th>Results for choice ((x, y) = (0, 1))</th>
<th>Results for choice ((x, y) = (1, 0))</th>
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<tbody>
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Winning strategy=Score 4

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Counterfactual reasoning

• One assumes that the same pair of protocols is used for all the possible settings.
• For each trial outcomes are predetermined for any choice of the settings.
• For each photon pair spin projections in all possible directions are predetermined we have a sample drawn from some joint multivariate probability distribution.
• IN SPCE : three outcomes a=±1 or 0 etc.
Vongher simulations

• 1000 runs and each run contains 800 pairs.

• For each run correlations are found and Bell and CHSH inequalities tested.

• QT - Bell and CHSH violated 99%

• LHVM 1- no violation of BI and CHSH

• LHVM 2- Not all instructions used BI and CHSH violated 50%

• LHVM 3- 13% anti-correlations missed BI and CHSH

1000 runs - Bell and CHSH violated 87% and 50% of times respectively.
IMPOSSIBLE VONGHER QRC

• 1000 runs using QT singlet model with strict anti-correlations: Bell and CHSH violated 99% and 100% of times respectively.
• QRC: construct 800x4 spreadsheet and a computer model violating Bell 99% of times.

Photon pairs are not tennis balls, there is no strict anti-correlation and the 4Nx4 spreadsheets considered by Gill and Vongher cannot be used to describe the outcomes of SPCE