Gravitational Cat State: Quantum Information in the face of Gravity

Bei - Lok Hu *(U. Maryland, USA)*

ongoing work with

Charis Anastopoulos (*U. Patras, Greece*)

*-- **EmQM15**, Vienna, Austria Oct. 23-25, 2015*

Based on C. Anastopoulos and B. L. Hu, *"Probing a Gravitational Cat State" Class. Quant. Grav. 32, 165022 (2015).* [*arXiv:1504.03103*]

***PITP** UBC - 2nd Galiano Island Meeting, Aug. 2015, **DICE2014** Castiglioncello, Italy Sept, 2014;*

***Peyresq 20**, France June 2015 (last few slides courtesy CA)*

- *RQI-N (Relativistic Quantum Information) 2014* *Seoul, Korea. June 30, 2014*

COST *meeting on Fundamental Issues, Weizmann Institute, Israel Mar 24-27, 2014*

Three elements: Q I G
Quantum, Information and Gravity

- Quantum ← Quantum Mechanics ← **Quantum Field Theory**
  Schroedinger Equation | | *micro*

- Gravity ← Newton Mechanics ← **General Relativity** | *Macro*

  - **GR+QFT= Semiclassical Gravity** (SCG)

- **Laboratory** conditions: | **Strong Field** Conditions:
  Weak field, nonrelativistic limit: | Early Universe, Black Holes

Newton Schrodinger Eq (NSE) | **Semiclassical Einstein Eq**

Two layers of theoretical construct:

(1 small surprise, 1 observation)

1) *Small Surprise?*:

NSE for single or multiple particles is not derivable from known physics

C. Anastopoulos and B. L. Hu, Problems with the Newton-Schrödinger Equations New J. Physics 16 (2014) 085007 [arXiv:1403.4921]

Newton-Schrodinger Eq <=/= **Semiclassical Einstein Eqn** of **Semiclassical Gravity**

(this nomenclature is preferred over **Mϕller-Rosenfeld Eq**)

Semiclassical Gravity

Semiclassical Einstein Equation (Moller-Rosenfeld):

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa\langle\hat{T}_{\mu\nu}\rangle_q + \kappa\,(T_{\mu\nu})_c$$

$\tilde{G}_{\mu\nu}$ is the Einstein tensor (plus covariant terms associated with the renormalization of the quantum field)

$\kappa = 8\pi G_N$ and G_N is Newton's constant

Free massive scalar field

$$(\Box - m^2 - \xi R)\hat{\phi} = 0.$$

$\hat{T}_{\mu\nu}$ is the stress-energy tensor operator

$\langle\,\rangle_q$ denotes the expectation value

Now bring in the most basic element in quantum information

Take the issue of **Quantum Entanglement**

Examine the **expectation value** not *wrt* a vacuum state (vev), but, say, a **cat state:**

$| +- \rangle = 1/\sqrt{2}\,(|\text{left}\rangle +- |\text{right}\rangle)$ |.....0.....|

-x +x

2) no Surprise:

One should know that **SCG is not sufficient for QI**, since it gives the **mean value** of the stress energy tensor T_{mn}, which predicts wrongly that the cat is at x=0. No Superposition, not quantum.

- Need to include contributions from the **fluctuations** in addition to the mean $\langle T_{mn} \rangle$ **(from SCG)**

- **Correlations of the stress energy tensor** $\langle T_{mn} T_{rs} \rangle$ is needed to address issues in **quantum information with gravity** (Relativistic QI, or **RQI**)

- There is such a theory, **Stochastic Semiclassical Gravity (SSG)**, **based solely on GR+QFT**.

  No new invention needed (or allowed).

- Just need to work things out carefully with experiments in mind. --- We are attempting this now:

Stochastic Gravity

Einstein- Langevin Equation (schematically):

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa \left(T^{\mathrm{c}}_{\mu\nu} + T^{\mathrm{qs}}_{\mu\nu}\right)$$

$T^{c}_{\mu\nu}$ is due to classical matter or fields

$$T^{\mathrm{qs}}_{\mu\nu} \equiv \langle \hat{T}_{\mu\nu} \rangle_{\mathrm{q}} + T^{\mathrm{s}}_{\mu\nu}$$

$T^{\mathrm{qs}}_{\mu\nu}$ is a new stochastic term
related to the quantum fluctuations of $T_{\mu\nu}$

NOISE KERNEL

- Exp Value of 2-point correlations of stress tensor: bitensor

- Noise kernel measures quantum flucts of stress tensor

It can be represented by (shown via influence functional to be equivalent to) a classical stochastic tensor source $\xi_{ab}[g]$

$$\langle \xi_{ab} \rangle_s = 0 \qquad \langle \xi_{ab}(x)\xi_{cd}(y) \rangle_s = N_{abcd}(x,y)$$

- **Symmetric, traceless** (for conformal field), **divergenceless**

Einstein-Langevin Equation

- Consider a weak gravitational perturbation h off a background g $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu},$ The ELE is given by (The ELE is Gauge invariant)

$$G_{ab}[g+h] + \Lambda(g_{ab} + h_{ab}) - 2(\alpha A_{ab} + \beta B_{ab})[g+h]$$
$$= 8\pi G\left(\left\langle \hat{T}^{R}_{ab}[g+h]\right\rangle + \xi_{ab}[g]\right).$$

- Nonlocal dissipation and colored noise

Nonlocality manifests with **stochasticity**

because the gravitational sector is an open system

Stochastic (Semiclassical) Gravity for Strong Gravitational Field Conditions

- For problems in the early universe and black holes, one is interested in quantum processes related to the vacuum state, e.g., particle creation, vacuum fluctuations, vacuum polarization. (e.g., Hawking Effect).

- Vacuum Expectation Values of Tmn or Tmn Trs taken *wrt* a vacuum state.
- In analogous laboratory settings, with moving detectors mirrors (e.g. Unruh Effect, dynamical Casimir Effect)

Review: *B. L. Hu and E. Verdaguer, "Stochastic gravity: Theory and Applications", in* ***Living Reviews in Relativity*** *7 (2004) 3. Updated in LRR* ***11*** *(2008) 3 [arXiv:0802.0658]*

Weak field, non-relativistic Limit

- The Einstein-Langevin Equation should provide a legitimate stochastic source arising from the fluctuations of the matter quantum field because it is from GR+QFT

- This should be the cannon for all AQT (alternative quantum theories) invoking stochastic sources to be compared with

This work is beginning. See recent work of T P Singh & co.

Quantum Information Issues in gravitational quantum physics

- New emphasis: **not vacuum state**, but one particle and n particle states: That's OK
  [*Squeezed states* can be handled. In fact, cosmological expansion is squeezing.

  But for **quantum superposition** states
- Bell states, etc. SCG cannot handle

Look for the **Gravitational Quantum Cat** from the **fluctuations of energy density**, or the **correlator of the stress energy tensor**: the **Noise Kernel**

(Not the full Schrodinger cat – some quantum tributes of the cat)

Consider a scalar quantum field $\hat{\phi}(\mathbf{r})$ and its conjugate momentum $\hat{\pi}(\mathbf{r})$ of the creation and annihilation operators $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^{\dagger}$

$$\hat{\phi}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} \left[\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \hat{a}_{\mathbf{k}}^{\dagger} e^{-i\mathbf{k}\cdot\mathbf{r}}\right]$$

$$\hat{\pi}(\mathbf{r}) = i \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\omega_{\mathbf{k}}}{2}} \left[-\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \hat{a}_{\mathbf{k}}^{\dagger} e^{-i\mathbf{k}\cdot\mathbf{r}}\right].$$

For a free field, the Hamiltonian operator is

$$\hat{\mathrm{H}} = \int \frac{d^3k}{(2\pi)^3} \omega_{\mathbf{k}} \hat{a}_{\mathbf{k}}^{\dagger} \hat{a}_{\mathbf{k}},$$

where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$.

In the non-relativistic regime we define the fields

$$\hat{\psi}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}, \qquad \hat{\psi}^{\dagger}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \hat{a}_{\mathbf{k}}^{\dagger} e^{-i\mathbf{k}\cdot\mathbf{r}},$$

and to leading order in $|\mathbf{k}|/m$,

$$\hat{\phi}(\mathbf{r}) = \frac{1}{\sqrt{2m}} \left[\hat{\psi}(\mathbf{r}) + \hat{\psi}^{\dagger}(\mathbf{r})\right], \qquad \hat{\pi}(\mathbf{r}) = -i\sqrt{\frac{m}{2}} \left[\hat{\psi}(\mathbf{r}) - \hat{\psi}^{\dagger}(\mathbf{r})\right]$$

Mass density operator of a *non-relativistic N-Particle System*

The Hamiltonian then becomes

$$\hat{\mathrm{H}} = m \int d\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}) - \frac{1}{2m} \int d\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \nabla^2 \hat{\psi}(\mathbf{r}). \quad (11)$$

We will denote the second term in Eq.(11) as $\hat{H}_0$ because it corresponds to the Hamiltonian for N non-relativistic particles. The first term in Eq.(11) corresponds to Nm, for an N-particle state. Hence, the number operator $\hat{N}$ is

$$\hat{N} = \int d\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}) \quad (12)$$

This suggests that $m\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})$ can be identified as the mass-density operator $\hat{\mu}(\mathbf{r})$.

We include the effect of a confining potential $V(\mathbf{r})$, by modifying the field Hamiltonian

$$\hat{\mathrm{H}} = m \int d\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}) + \int d\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \left[-\frac{1}{2m} \nabla^2 + V(\mathbf{r}) \right] \hat{\psi}(\mathbf{r}). \quad (13)$$

Mass Density Correlations

2.2. Mass-density correlations, noise kernel

In the non relativistic limit, the dominant component of the stress tensor $T_{\mu\nu}$ is the energy density, which is dominated by the mass density, namely

$$T_{\mu\nu}(\mathbf{r},t) = \delta^0_\mu \delta^0_\nu \mu(\mathbf{r},t) \tag{14}$$

Thus, it suffices to calculate the correlation functions of the Heisenberg-picture operator

$$\hat{\mu}(\mathbf{r},t) = e^{i\hat{H}t}\hat{\mu}(\mathbf{r})e^{-i\hat{H}t}. \tag{15}$$

We assume an one-particle state

$$|\phi\rangle = \int d\mathbf{r}\phi(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})|0\rangle, \tag{16}$$

where $\phi(\mathbf{r})$ is the one-particle wave-function.

We find

$$\langle\mu(\mathbf{r},t)\rangle = \langle\phi|\hat{\mu}(\mathbf{r},t)|\phi\rangle = m\phi^*(\mathbf{r},t)\phi(\mathbf{r},t) \tag{17}$$

$$\langle\mu(\mathbf{r},t)\mu(\mathbf{r}',t')\rangle = \phi^*(\mathbf{r},t)\phi(\mathbf{r}',t')G(\mathbf{r},t;\mathbf{r}',t') \tag{18}$$

where $\phi(\mathbf{r},t)$ is the time-evolved single particle wave function and $G(\mathbf{r},t;\mathbf{r}',t')$ is the one-particle propagator,

$$G(\mathbf{r},t;\mathbf{r}',t') = \langle\mathbf{r}'|e^{-i\hat{H}(t'-t)}|\mathbf{r}\rangle. \tag{19}$$

Noise Kernel

For a free particle,

$$G(\mathbf{r}, t; \mathbf{r}', t') = \left(\frac{m}{2\pi i t}\right)^{3/2} \exp\left[\frac{im(\mathbf{r}-\mathbf{r}')^2}{2(t'-t)}\right] \tag{20}$$

We note that the two-point correlation function is complex valued. In general, it does not define a stochastic process. However, the real part,

$$\xi(\mathbf{r}, t; \mathbf{r}', t') = Re\langle\mu(\mathbf{r}, t)\mu(\mathbf{r}', t')\rangle, \tag{21}$$

known as the noise kernel, corresponds in some cases to the two-point correlation function of a stochastic process.

Of importance is also the connected two-point correlation function for the mass densities

$$\eta(\mathbf{r}, t; \mathbf{r}', t') = \langle\mu(\mathbf{r}, t)\mu(\mathbf{r}', t')\rangle - \langle\mu(\mathbf{r}, t)\rangle\langle\mu(\mathbf{r}', t')\rangle. \tag{22}$$

Smeared Mass-Density Function

- In realistic systems the mass density is not defined at a sharp spacetime point but smeared over a finite spacetime region.
- In actual experiments, the particles under consideration (atoms) have a finite size d and it is meaningless to talk about mass densities at scales smaller than d, unless one has a detailed knowledge of the particle's internal state.

- For this reason, rather than the exact mass density function, we consider a smeared mass density function:

$$\hat{\mu}_s(\mathbf{r}, t) = \int d\mathbf{r}' f(\mathbf{r} - \mathbf{r}') \hat{\mu}(\mathbf{r}', t), \tag{23}$$

for some smearing function $f(\mathbf{r})$ of dimension $[\text{length}]^{-3}$, centered around $\mathbf{r} = 0$. The smearing scale ℓ is defined by the condition $\ell^3 = 1/f(0)$.

We define the positive operator

$$\hat{P}_{\mathbf{r}} = \int d\mathbf{r}' g(\mathbf{r} - \mathbf{r}') |\mathbf{r}'\rangle\langle\mathbf{r}'|, \tag{24}$$

where $g(\mathbf{r}) := f(\mathbf{r})/f(0)$.

Wigner function representation

- For a free particle, we can express the correlation functions in terms of the Wigner function $W_0(\mathbf{r}, \mathbf{p})$ of the initial state.

- For scales of observation much larger than *l*, we have

$$\langle \mu_s(\mathbf{r},t)\rangle = \frac{m}{(2\pi)^3}\int d\mathrm{p} W_0(\mathbf{r}-\frac{\mathbf{p}t}{m},\mathrm{p})$$

$$\langle \mu_s(\mathbf{r},t)\mu_s(\mathbf{r}',t')\rangle = \frac{m^5}{(2\pi)^3(t-t')^3}W_0(\frac{\mathbf{r}+\mathbf{r}'}{2}-\frac{(\mathbf{r}-\mathbf{r}')(t+t')}{2(t-t')}, m\frac{\mathbf{r}-\mathbf{r}'}{t-t'})$$

- For an initial state with vanishing mean momentum, we obtain a stationary process.

$$\langle \mu_s(\mathbf{r},t)\rangle = m|\psi_0(\mathbf{r})|^2.$$

$$\langle \mu_s(\mathbf{r},t)\mu_s(\mathbf{r}',t')\rangle = m^2|\psi_0(\mathbf{r})|^2\delta^3(\mathbf{r}-\mathbf{r}').$$

Quantum feature: A classical charge distribution would involve $|\psi_0(r)|^4$.

Key features of correlations of quantum systems

Mass density fluctuations are

- Of the **same order of magnitude as the mean mass density**
  This property seems to be generic in stress-energy fluctuations
  (Kuo+Ford 93, Phillips+Hu 97,00).

- Highly **non-Markovian.** They are unlike any classical stochastic process.

Fluctuations of the mass density generate fluctuations of the Newtonian force through Poisson's equation. ➔ Exploration of Newton-Schrodinger Eq and AQT

Beware of these features

Measured values of correlations

Temporal correlation functions of quantum systems are **highly contextual** (Anastopoulos 04,05).

A characteristic feature of quantum correlations exemplified by the Leggett-Gard inequality (or temporal Bell inequalities).

- By **contextual**, we mean that the measured values depend strongly on the context of measurement, i.e., on the specific set-up through which the correlations are measured.

- To compare, all samplings of position correspond to probabilities that closely approximate the **ideal distribution** $|\psi(\mathbf{r})|^2$.

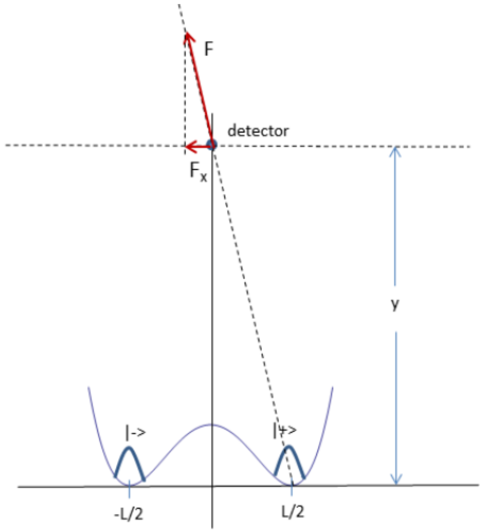
- **There are no ideal distributions** for generic multi-time measurements. Probabilities are highly sensitive to the details of the sampling.

- Hence, there is no **intrinsic** stochastic process that describes the mass density fluctuations of a particle, but

- Any stochastic process that describes the experimental data depends on the specific procedure through which the measurement is carried out.

Gravitational Cat State:

Direct bearing of Quantum Optomechanics



A particle of **mass m** in a confining potential with two minima. Assume that the **distance L** between the minima is much larger than the width of the localization region.

The system can be approximated by a qubit with defining states |+> and |->.

Hamiltonian $\hat{H} = \nu\,\hat{\sigma}_1$, where ν is the tunneling rate between the two minima.

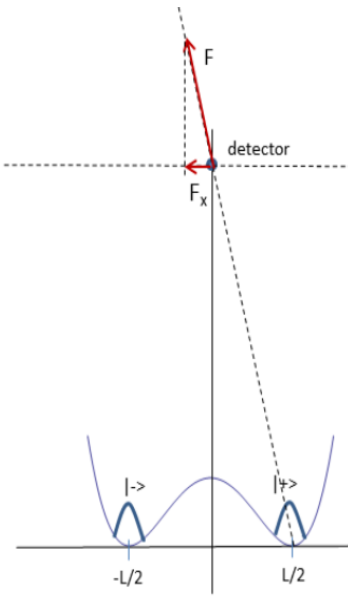
- The famous ***atomic*** cat of Wineland et al (1996) had L = 80nm and m = 8 amu.
  The cattiness record seems to come from the **Ardnt** 2012 diffraction experiment with L = 100nm and m =1300 amu. Bassi's review has more recent data
  Record for weakest force measured from **CalTech** ? (2014), ~ 4 x 10^{-23} N.

- **Recent experiments** on **entanglement between massive objects**: **Aspelmeyer**'s work- Indirect (entanglement with third party measured); Direct (Calvendish expt)

* **Romero-Isart**'s superconducting microsphere gives the most stringent limit on NSEq ?

Measurement by a classical probe

Consider a particle of mass m_0 near the particle of mass m that was prepared in a cat state.



We evaluate the x component of the Newtonian force exerted by the two-level system on the test particle. Again denote $a = \pm$. If the system lies on the minimum of the potential at $x = aL/2$, the force $F_x(a)$ exerted on the test particle in the x direction is

$$F_x(a) = -\frac{Gmm_0 L}{2(y^2 + L^2/4)^{3/2}} a = -f_0 a \tag{62}$$

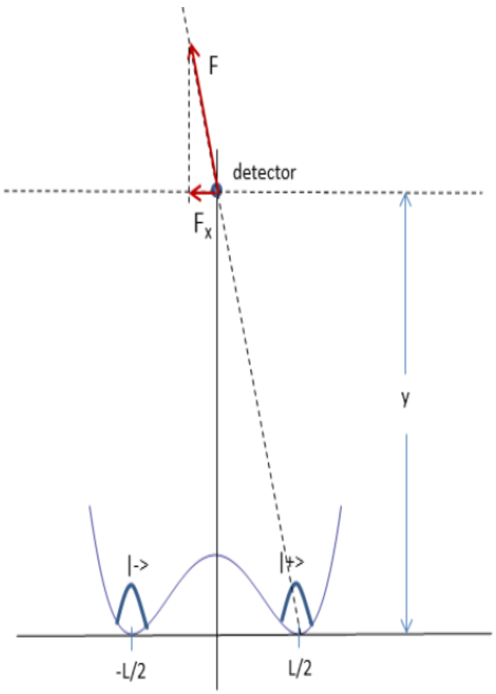
Assuming that the test mass is not allowed to move, the force F_x takes only two values f_0 and $-f_0$. These values are correlated with the projectors $\hat{P}_a$, Eq. (48). Thus F_x corresponds to a self-adjoint operator

$$\hat{P}_+ = \int_0^\infty dx \int_\infty^\infty dy \int_\infty^\infty dz |x,y,z\rangle\langle x,y,z|$$

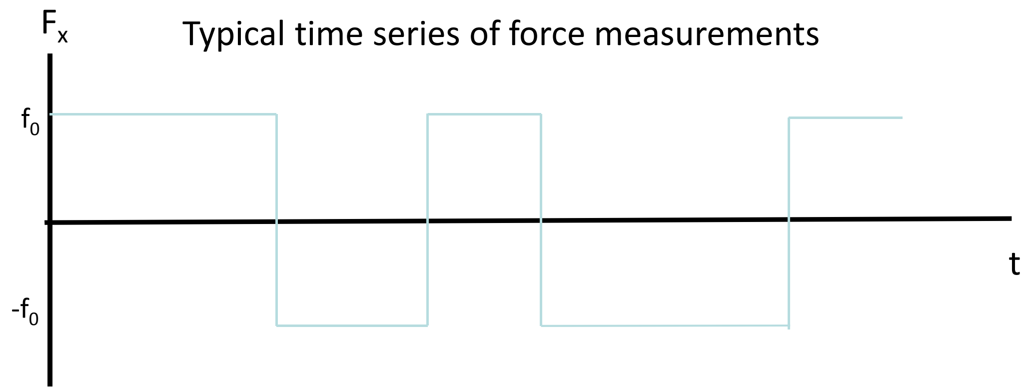
$$\hat{F} = -f_0 \hat{P}_+ + f_0 \hat{P}_- = -f_0 \hat{\sigma}_3, \qquad \hat{P}_+ = \int_{-\infty}^0 dx \int_\infty^\infty dy \int_\infty^\infty dz |x,y,z\rangle\langle x,y,z|. \tag{63}$$

on the 2-state system's Hilbert space. Thus, the gravitational force behaves as a quantum variable, its probabilities and correlations determined by quantum mechanics.

Measurement by a classical probe



Since Newton's law is instantaneous, a force will be recorded by the macroscopic probe at all times. Thus we have a **continuous-time measurement** for a qubit.



Essentially similar to the **quantum jump** expts of Dehmelt et al (86).

Calculate the correlation functions of the force from the quantum probabilities for a continuous-time measurement

$$\langle F(t) \rangle = -f_0 e^{-\Gamma t}$$
$$\langle F(t') F(t) \rangle = f_0^2 e^{-\Gamma |t'-t|}.$$

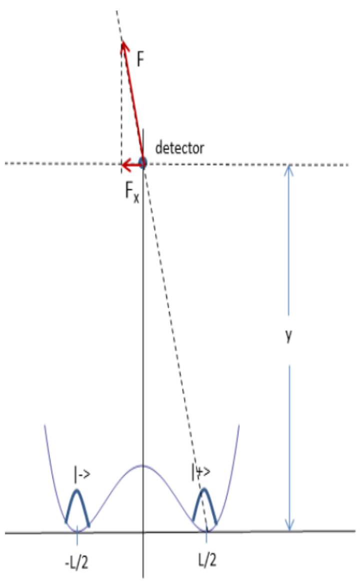
$$\Gamma = \frac{\nu^2 \tau}{2},$$

τ is the temporal resolution of the probe.

Non-Markovian, obtained for $\nu\tau << 1$.

Measurement by a quantum probe 1

- Coupling through the Newtonian force to a quantum harmonic oscillator constrained to move along the x-axis.



Now consider a quantum probe made of a harmonic oscillator of frequency ω that is constrained to move along the horizontal axis as in Fig. 1. The Hamiltonian of the harmonic oscillator probe is

$$H_P = \omega \hat{a}^\dagger \hat{a} \tag{68}$$

If the amplitude of the oscillations is much smaller than L, the length scale of the cat state, the force acted upon the oscillator along the x direction is approximately constant and equal to Eq. (62). This corresponds to an interaction Hamiltonian

$$\hat{H}_I = -f_0 \hat{\sigma}_3 \hat{x} = -\frac{f_0}{\sqrt{2m_0\omega}} \hat{\sigma}_3 (\hat{a} + \hat{a}^\dagger), \tag{69}$$

Thus, the total Hamiltonian of the two state system interacting with the oscillator probe is

$$\hat{H} = H_S + H_P + H_I = \nu \hat{\sigma}_1 + \omega \hat{a}^\dagger \hat{a} + g \hat{\sigma}_3 (\hat{a} + \hat{a}^\dagger), \tag{70}$$

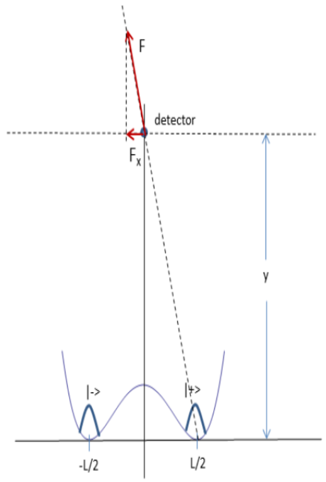
where the system Hamiltonian H_S is given by Eq. (56) specialized to $\chi = 0$, i.e., it is equivalent to the Hamiltonian of a single-mode Jaynes-Cummings model with a coupling constant

Eq. (56) H_S $\hat{H} := \nu(\cos\chi \hat{\sigma}_1 + \sin\chi \hat{\sigma}_2)$

$$g = -\frac{f_0}{\sqrt{2m_0\omega}}. \tag{71}$$

Equivalent to the Jaynes-Cummings (JC) model of quantum optics.

quantum probe 2



- If the oscillator is to act as a measurement, the coupling term should be strong, it cannot be treated as a small perturbation.
- Thus we cannot use the commonly employed Rotating Wave Approximation.
- JC model was recently shown to be integrable (Braak 11), but the solution is not helpful in finding time evolution.

Consider **adiabatic regime** { =0 (vanishing tunneling). Then for the oscillator probe initially in the vacuum and the cat particle in c_+|+> +c_-|->,

$$|\Psi(t)\rangle = e^{i\frac{g^2}{\omega^2}[\omega t - \sin(\omega t)]} \begin{pmatrix} c_+|\zeta(t)\rangle \\ c_-|-\zeta(t)\rangle \end{pmatrix}, \tag{74}$$

where the path

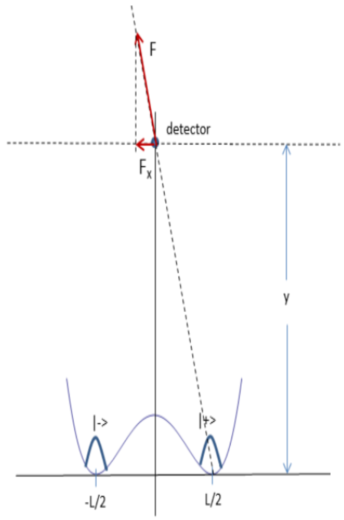
$$\zeta(t) = -\frac{g}{\omega}(1 - e^{-i\omega t}), \qquad \text{Coherence state representation} \tag{75}$$

describes an oscillation centered around $\zeta_0 = -\frac{g}{\omega} = \frac{f_0}{\sqrt{2m_0\omega^3}}$. The center of the oscillation corresponds to position $x_0 = \frac{f_0}{m_0\omega^2}$ and momentum $p_0 = 0$.

We obtain a superposition of two oscillations around different centers. The centers are distinguished only if $|\langle\zeta_0|-\zeta_0\rangle| << 1$, or

$$\omega^3 << \left(\frac{f_0}{m_0}\right)^2 m_0.$$

quantum probe 3



Treat small values of ν as perturbations of the adiabatic solution.

4.2.2. Rabi oscillations A finite value of ν allows for transitions between the two gravitational quantum states, which induce transitions among the phase space paths of the oscillator. While the model is not exactly solvable, we can estimate the rate of such transitions using perturbation theory with respect to the tunneling rate ν. In Appendix B, we show that to leading order in ν, $e^{-i\hat{H}t} = e^{-i\hat{H}_0 t}\hat{O}_t$, where

$$\hat{O}_t = \begin{pmatrix} \cos\nu t & -i\sin\nu t \hat{D}(2\zeta_0) \\ -i\sin\nu t\hat{D}(-2\zeta_0) & \cos\nu t \end{pmatrix}. \tag{79}$$

As an estimate of the transition between the two gravitational quantum states, we compute the amplitude $\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle$, between the stationary states $|\zeta_0, +\rangle$ and $|-\zeta_0, -\rangle$. We find

$$\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle = -i\sin\nu t, \tag{80}$$

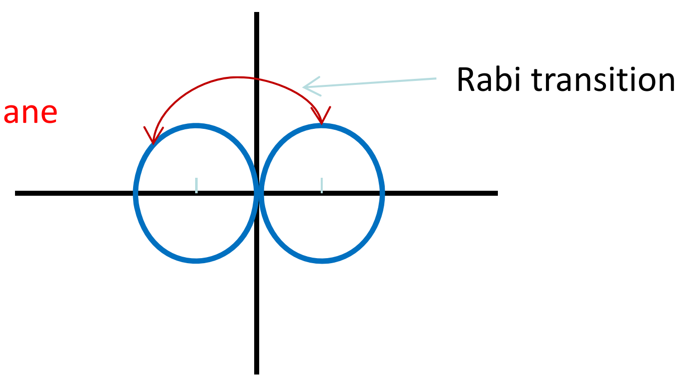
and thus the associated probability

$$p(t) = |\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle|^2 = \sin^2\nu t, \tag{81}$$

exhibits Rabi-type oscillations, with frequency ν.

Then we obtain Rabi oscillations of frequency ν between the two centers ζ_0 and $-\zeta_0$

Coherent state plane



Implications

Since the gravitational field is slaved to matter, the gravitational force is represented by an operator on the Hilbert space of the matter field.

Thus, the standard operational procedures in QM can be invoked for measuring a gravitational force. But what does this mean?

Standard interpretation: weak field

- Once we measure a force **F** on a test particle of mass m, we can calculate the field strength **g** = **F**/m.
- The field strength corresponds to a gravitational potential φ.
- In the weak field limit of GR, the potential appears in the g_{00} component of the metric tensor.

From the vantage point of GR:

Spacetime & quantum matter intimately linked

Do quantum fluctuations of the force define quantum fluctuations of the spacetime geometry? [stochastic gravity addresses this issue]

Operational definitions of spacetime geometry seem to agree on that.

If this is true, the state we considered here is a **genuine gravcat,** a quantum superposition of two spacetime geometries.

Discussions

- *Does the gravitational force remain slaved to the mass density as classical GR dictates* , even if the latter behaves quantum mechanically? (it has fluctuations, it is subject to quantum measurements, etc.)

We can only answer this questions by attempting to construct gravcats, or other non-classical states for Macroscopic systems. Optomechanical systems seems to be the most promising route.

- In principle, we can construct probes that record quantum jumps of the gravitational force. Can we talk about q jumps on the gravitational potential? And then about **jumps of** (not just **in**) **the induced quantum spacetime**?

Does this idea even make sense?

The conceptual tension between GR and QM such as spelled out by Penrose, already manifest in the Newtonian regime.

- Invoking gravitational decoherence (grav field as environment to quantum systems) to kill gravcats may solve the problem above, but the intrinsic tension between GR +QM remains.

The interface between macroscopic quantum phenomena and gravitational quantum physics is of fundamental significance from this perspective.

Perspective

- In view of advances in AMO,CMP and Optomechanics **precision experiments in weak gravitational fields**

  -- *Gravitational Quantum physics (Focus Issue in NJP 2014)*

- it pays to reexamine the WF-NR limit of

  1) semiclassical Einstein Equation, (in relation to NSEq etc)

  2) Noise kernel, or stress energy density correlators (new)

Bringing gravity into consideration of issues in

- **quantum foundations** such as the Born Rule; and
- **quantum information** such as the Cat State with gravity

Conclusion: *Investigation of Q Information issues of gravitational systems using quantum probes*

- *Quantum Gravity (theories for the microscopic structures of spacetime) is not needed.*
- *Focus on systems under laboratory conditions: nonrelativistic systems, weak gravitational field.*

- *Semiclassical Gravity is inadequate.*
- *Focus on fluctuations and correlations of mass density -- incorporated in Stochastic Gravity Theory*

Big Quest: *Can we infer attributes of spacetime fluctuations from quantum experiments even at the level of Newtonian gravity without appealing to new theories of QM or GR?*

Thank you for your attention!

& the Organizers for their nice work

Theoretical Motivations:
Frontier & Foundational Issues

0. **Innate conflict** between the fundamental principles of **general relativity** and of **quantum mechanics**

1. Macroscopic Quantum Phenomena [Dice 10,12]

Different MQ behavior in e.g., BEC, Superconductivity, crystals: How the micro-constituents are organized.
Issues: ***Quantum Coherence, Q. Correlation, Q. Entanglement***

2. Emergent Quantum Mechanics

- Are there fundamental changes as one traverses between the micro and the macro domains? **New laws for meso-physics**? (e.g., Leggett)
- **Sublevel structure** which in some limit shows up as quantum mechanics
- **Emergent gravity**: Both emergent from the same sub-structure.

3. Quantum Information: *How it enters in* ***Physical Reality*** *and the formulation or even the advent of* ***Physical Laws*** [It-Bit]

Alternative Q Theories

- L **Diosi** (84, 87, 89) R. **Penrose**, *Phil. Trans. R. Soc. Lond.* A (1998) **356**, 1927-1939 / GRG (96)

-Advocate gravity as the source of decoherence of quantum particles.

*-Proposed different forms of **Newton-Schrodinger Equation** NSE*

*- But we find that **NSE cannot be derived from QFT + GR***

- **GRWP**: G.C. Ghirardi, R. Grassi, A. Rimini, Weber and Pearle Phys. Rev. A42, 1057 (1990).; Pearle. *Changing QM,* We view this class of theories as expressing a wish: That at a certain scale between the micro and macro, the wave function collapses: " localization". Less concerned with Why

*Both classes of theories are **Phenomenological, not Fundamental.***

- *Viewing QM as Emergent: Proposals of **sub-level theories***
  S. L. **Adler**'s book and recent papers, **'t Hooft**'s papers

Excellent Review by A. Bassi et al, Rev. Mod. Phys. 85, 471- 527 (2013)

- Recall: Consider a wave function composed of 2 Gaussian packets located at +L0 and –L0

Hu Paz Zhang PRD 92
Paz Habib Zurek PRD 93

$$\Psi(x,t=0)=\Psi_1(x)+\Psi_2(x)\ ,$$

- where

$$\Psi_{1,2}(x)=N\exp\left[-\frac{(x\mp L_0)^2}{2\delta^2}\right]\exp(\pm iP_0x)\ ,$$

$$N^2\equiv\frac{\bar{N}^2}{\pi\delta^2}=\frac{1}{2\pi^2\delta^2}\left[1+\exp\left[-\frac{L_0^2}{\delta^2}-\delta^2P_0^2\right]\right]^{-1}$$

Decoherence in QBM models:

1 HO System– nHO bath

$$S[x,q_n]=\int_0^t ds\left[\frac{1}{2}M(\dot{x}^2-\Omega_0^2x^2)+\sum_n\frac{1}{2}m_n(\dot{q}_n^2-\omega_n^2q_n^2)\right.$$

$$\left.-\sum_n C_n x q_n\right], \qquad (3)$$

$$\Psi(x,t=0)=\Psi_1(x)+\Psi_2(x),$$

where

$$\Psi_{1,2}(x)=N\exp\left[-\frac{(x\mp L_0)^2}{2\delta^2}\right]\exp(\pm iP_0x),$$

$$N^2\equiv\frac{\bar{N}^2}{\pi\delta^2}=\frac{1}{2\pi^2\delta^2}\left[1+\exp\left[-\frac{L_0^2}{\delta^2}-\delta^2P_0^2\right]\right]^{-1}$$

Pointer Basis:
Interaction Hamiltonian

left: xq
right: pp

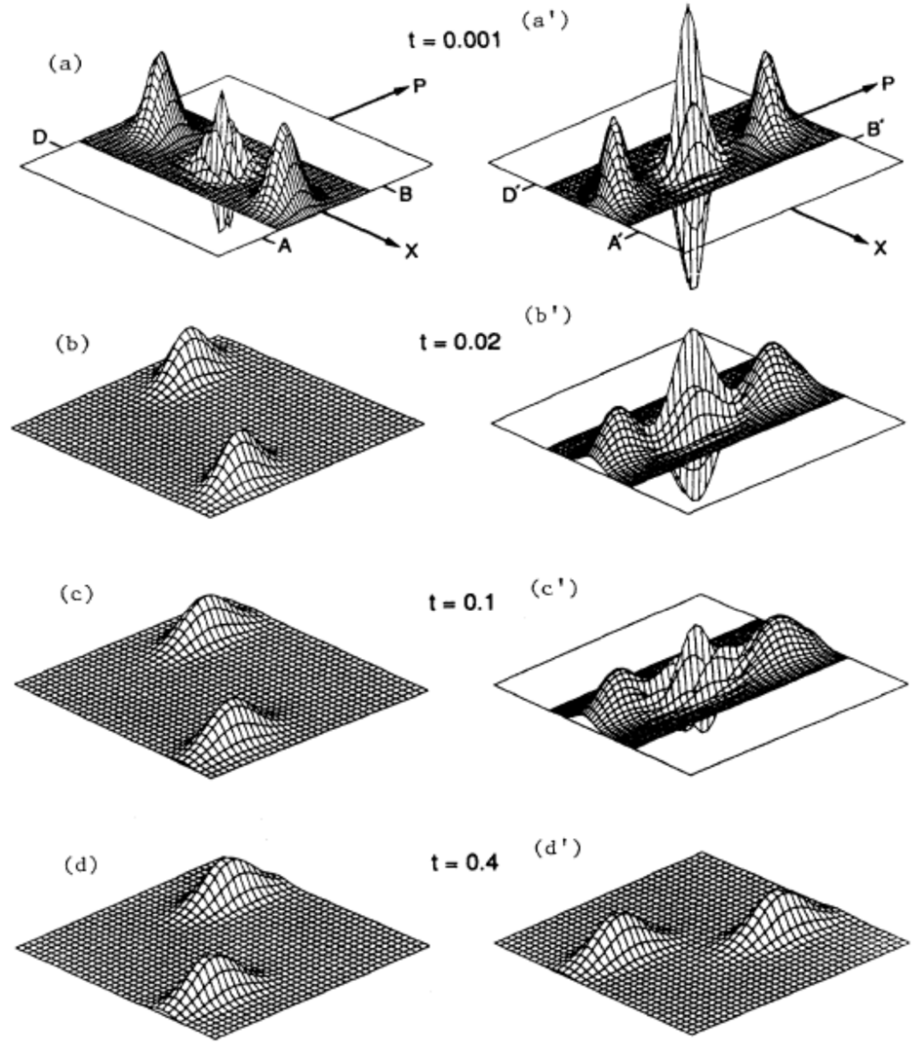


FIG. 2. The time evolution of initial conditions A and A'. The oscillations disappear faster in the first case since the environment can distinguish between the two peaks. In the second case, the interference is damped over a dynamical time scale.

Gravitational Cat State:

a consequence of the intrinsic conflicts of Q + G

Penrose (1996) "**On gravity's role in quantum state reduction**". *Gen. Rel. Grav.* **28**, 581-600 [*just read the letters in red below*:]

Addresses the question of the *stationarity* of a quantum system which consists of a linear superposition |ψ> = |α> + |β> of two well-defined states |α > and |β >, each of which would be stationary on its own, and where we assume that each of the two individual states has the same energy E

$$\mathrm{i}\frac{\partial|\alpha\rangle}{\partial t} = E|\alpha\rangle, \quad \mathrm{i}\frac{\partial|\beta\rangle}{\partial t} = E|\beta\rangle.$$

Just QM alone: If gravitation is ignored, then the quantum superposition |ψ> = a|α> +b |β> would also be stationary, with the same energy *E* and this is the normal supposition.

$$\mathrm{i}\frac{\partial|\psi\rangle}{\partial t} = E|\psi\rangle,$$

With Gravity: However, when the gravitational fields of the mass distributions of the states are taken into account, we must ask what the Schroedinger operator $\partial/\partial t$ actually *means* in such a situation.

Let us consider that each of the stationary states $|\alpha >$ and $|\beta >$ takes into account whatever the correct quantum description of its gravitational field might be, in accordance with Einstein's theory.

Then, to a good degree of approximation, there will be a classical spacetime associated with each of $|\alpha>$ and $|\beta>$, and the operator $\partial/\partial t$ would correspond to the action of the Killing vector representing the time displacement of stationarity, in each case.

Stationary state makes demand of spacetime properties.

Clash between QM and GR

Now, the problem that arises here is that these two Killing vectors are *different* from each other. They could hardly be the same, as they refer to time symmetries of two different spacetimes.

It could only be appropriate to identify the two Killing vectors with one another if it were appropriate to identify the two different spacetimes with each other point-by-point.

But such an identification would be at variance with the principle of general covariance, a principle which is fundamental to Einstein's theory. According to standard quantum theory, unitary evolution requires that there be a Schr¨odinger operator that applies to the superposition just as it applies to each state individually; and its action on that superposition is precisely the superposition of its action on each state individually.

There is thus a certain tension between the fundamental principles of these two great theories, and one needs to take a position on how this tension is to be resolved.

Penrose's position is (provisionally) to take the view that an *approximate* pointwise identification may be made between the two spacetimes, and that this corresponds to a slight error in the identification of the Schr¨odinger operator for one spacetime with that for the other. This error corresponds, in effect, to a slight ***uncertainty in the energy*** of the superposition.

One can make a reasonable assessment as to what this energy uncertainty E_G might be, at least in the case when the amplitudes *a* and *b* are about equal in magnitude.

This estimate (in the Newtonian approximation) turns out to be the gravitational self-energy of the difference between the mass distributions of the two superposed states. This energy uncertainty E_G is taken to be a fundamental aspect of such a superposition and, in accordance with Heisenberg's uncertainty principle, the reciprocal hbar/E_G is taken to be a measure of the *lifetime* of the superposition (as with an unstable particle).

The two decay modes of the superposition $|\psi\rangle = a\,|\alpha\rangle + b\,|\beta\rangle$ would be the individual states $|\alpha\rangle$ and $|\beta\rangle$, with relative probabilities $|a|^2 : |b|^2$.

If the sampling function g is a characteristic function of some set (i.e., if $g^2 = g$), then $\hat{P}_{\mathbf{r}}$ is a projection operator. Here, we will consider Gaussian functions of the form $g(\mathbf{r}) = e^{-\frac{\mathbf{r}^2}{2s_x^2}}$, where s_x is the width of the sampling. In this case, $\hat{P}_{\mathbf{r}}$ is an approximate projector. The corresponding smearing function is

$$f(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}s_x^3} e^{-\frac{\mathbf{r}^2}{2s_x^2}} \tag{25}$$

and corresponds to $\ell = \sqrt{2\pi} s_x$.

The correlation functions of the mass density become

$$\langle \mu_s(\mathbf{r}, t) \rangle = \frac{m}{\ell^3} \langle \phi | \hat{P}_{\mathbf{r},t} | \phi \rangle \tag{26}$$

$$\langle \mu_s(\mathbf{r}, t) \mu_s(\mathbf{r}', t') \rangle = \frac{m^2}{\ell^6} \langle \phi | \hat{P}_{\mathbf{r}t} \hat{P}_{\mathbf{r}'t'} | \phi \rangle, \tag{27}$$

where $\hat{P}_{\mathbf{r}t} = e^{i\hat{H}t} \hat{P}_{\mathbf{r}} e^{-i\hat{H}t}$ is the Heisenberg-picture evolution of $\hat{P}_{\mathbf{r}}$.

The expectation value of the smeared mass density is proportional to the probability of a position measurement at time t. The two point correlation function is proportional to the decoherence functional

$$\mathcal{D}(\mathbf{r}, t; \mathbf{r}', t') := \langle \phi | \hat{P}_{\mathbf{r}t} \hat{P}_{\mathbf{r}'t'} | \phi \rangle \tag{28}$$

for a pair of histories one corresponding to a position record $\mathbf{r}$ at time t and the other to a position record $\mathbf{r}'$ at time t'. As explained by the decoherent histories approach to quantum

Decoherence Functional

1.3 Relation to the decoherence functional

The identification of the expectation values (13) does not correspond to the c
functions of a physical process, because in realistic systems the mass dens
defined at a sharp spacetime point but smeared in finite spacetime region.
experiments, the particles under consideration (atoms) have a finite size d
meaningless to talk about mass densities at scales smaller than d, unless
detailed knowledge of the particle's internal state.

For this reason, rather than the exact mass density function, we consider
mass density function

$$\hat{\mu}_\delta(\mathbf{r}, t) = \int d\mathbf{r}' f(\mathbf{r} - \mathbf{r}')\hat{\mu}(\mathbf{r}, t),$$

where $f(\mathbf{r})$ is a Gaussian smearing function

$$f(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}\delta^3} e^{-\frac{\mathbf{r}^2}{2\delta^2}}$$

and δ is the length scale of the smearing region.

We define the positive operator

$$\hat{P}_{\mathbf{r}} = \int d\mathbf{r}' f(\mathbf{r} - \mathbf{r}') |\mathbf{r}'\rangle\langle\mathbf{r}|, \tag{19}$$

that (modulo a multiplicative constant) represents an approximate measurement of position with a width δ around $\mathbf{r}$.

The correlation functions of the mass density become

$$\langle\mu_s(\mathbf{r}, t)\rangle = \langle\phi|\hat{P}_{\mathbf{r},t}|\phi\rangle \tag{20}$$

$$\langle\mu_s(\mathbf{r}, t)\mu_s(\mathbf{r}', t')\rangle = \langle\phi|\hat{P}_{\mathbf{r}t}\hat{P}_{\mathbf{r}'t'}|\phi\rangle, \tag{21}$$

where $\hat{P}_{\mathbf{r}t} = e^{i\hat{h}t}\hat{P}_{\mathbf{r}}e^{-i\hat{h}t}$ is the Heisenberg-picture evolution of $\hat{P}_{\mathbf{r}}$.

We observe that the expectation value of the smeared mass density is proportional to the probability of a position measurement at time t, and the two-point correlation function is proportional to the **decoherence functional** between two histories, one corresponding to a position record $\mathbf{r}$ at time t and the other to a position record $\mathbf{r}'$ at time t'.

1.4 Calculating the projectors

Next, we proceed to an evaluation of the smeared correlation functions (21). It is convenient to work in the Wigner-Weyl representation, in which an operator $\hat{A}$ on the Hilbert space of a particle is represented by a function $F_{\hat{A}}$ on the associated state space.

$$F_{\hat{A}}(\mathbf{x}, \mathbf{p}) = \int d\mathbf{y} \langle\mathbf{x} - \frac{\mathbf{y}}{2}|\hat{A}|\mathbf{x} + \frac{\mathbf{y}}{2}\rangle e^{i\mathbf{p}\cdot\mathbf{y}}. \tag{22}$$

Wigner-Weyl Transform

First, we evaluate the Wigner-Weyl transform of $\hat{P}_{\mathbf{r},t}$, which we denote as $F_{\mathbf{r},t}$. We assume that $\hat{h}$ corresponds to a free particle. Then

$$F_{\mathbf{r},t}(\mathbf{x},\mathbf{p}) = f(\mathbf{r}-\mathbf{x}-\frac{\mathbf{p}t}{m}). \tag{23}$$

Next, we evaluate the Wigner-Weyl transform $F_{\mathbf{r},t;\mathbf{r}',t'}$ of the product $\hat{P}_{\mathbf{r}t}\hat{P}_{\mathbf{r}'t'}$ that appears in Eq. (21). Within a semiclassical approximation,

$$F_{\mathbf{r},t;\mathbf{r}',t'}(\mathbf{x},\mathbf{p}) \simeq F_{\mathbf{r},t}(\mathbf{x},\mathbf{p})F_{\mathbf{r}',t'}(\mathbf{x},\mathbf{p}), \tag{24}$$

which implies that

$$F_{\mathbf{r},t;\mathbf{r}',t'}(\mathbf{x},\mathbf{p}) = \frac{1}{(2\pi\delta^2)^3}\exp\left[-\frac{1}{\delta^2}\left(\mathbf{x}-\frac{\mathbf{r}+\mathbf{r}'}{2}+\frac{\mathbf{p}(t+t')}{2m}\right)^2-\frac{(t-t')^2}{4m^2\delta^2}\left(\mathbf{p}-m\frac{\mathbf{r}-\mathbf{r}'}{t-t'}\right)^2\right] \tag{25}$$

1.5 Evaluation for different initial states

Using the noise kernel to analyze the mass density correlation of a Cat State in a double well potential

Gravitational Effects of Quantum Matter

- What is the gravitational field generated by a quantum distribution of matter?
- **A popular answer**: if $\psi(\mathbf{r})$ is a single-particle's wave function, then $m\,|\psi(\mathbf{r})|^2$ is the mass density $\mu(r)$, and we solve Poisson's equation to obtain the gravitational potential.

$$V_{\mathrm{N}}(\psi, \boldsymbol{r}) = -Gm \int d\boldsymbol{r}' \frac{|\,\psi(\mathbf{r}')\,|^2}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

- This is wrong, or at least this is not what QM states.
- Involves mixing QM with mean field theory.

leads to Newton-Schrodinger (NS) equation

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + V_{\mathrm{N}}(\psi, \boldsymbol{r})\psi,$$

often employed in relation to gravitational decoherence.

The NS equation follows from the non-relativistic limit of semiclassical Einstein (SCE) equations

$$G_{\mu\nu} = 8\pi G\langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle,$$

However, an equation such as the SCE follows from a quantum theory only as a **Hartree approximation**, which does not apply to single-particle states $|\Psi>$.

It presupposes **new physics** in the relation between mass density and gravitational forces.

Hu+CA 14, Bahrami et al 14, Giulini +Grossardt 14

Fluctuating gravitational force

- The mass density (the stress energy tensor in general) is a quantum observable, defined in the Hilbert space of the associated QFT.
- In the Newtonian regime, the gravitational potential V is completely slaved to the mass density via Poisson's equation. If the mass density is represented by an operator, so is the potential.
- This does not necessarily imply quantization of gravity, which usually refers to the true degrees of freedom.
- It means that *any measurement of the gravitational force* involves probabilities that are defined in terms of the **mass density operator.**

Beyond the Newtonian regime, the analogous statement is that the SCE equation is not enough.

Stochastic gravity program: include fluctuations of quantum fields as a part of the total source driving the Einstein equation (Hu, Calzetta, Verdaguer, Roura, 93-now).

The mass density operator

Write the non relativistic limit of a scalar field theory.

Consider a scalar quantum field $\hat{\phi}(\mathbf{r})$ and its conjugate momentum $\hat{\pi}(\mathbf{r})$ expressed in terms of the creation and annihilation operators $\hat{a}_{\mathbf{k}}$ and $\hat{a}^{\dagger}_{\mathbf{k}}$

$$\hat{\phi}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} \left[\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \hat{a}^{\dagger}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}}\right] \quad (13)$$

$$\hat{\pi}(\mathbf{r}) = i \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{\omega_{\mathbf{k}}}{2}} \left[-\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \hat{a}^{\dagger}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}}\right]. \quad (14)$$

For a free field, the Hamiltonian operator is

$$\hat{H} = \int \frac{d^3k}{(2\pi)^3} \omega_{\mathbf{k}} \hat{a}^{\dagger}_{\mathbf{k}} \hat{a}_{\mathbf{k}}, \quad (15)$$

where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$.

In the non-relativistic approximation, we define the fields

$$\hat{\psi}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}, \qquad \hat{\psi}^{\dagger}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \hat{a}^{\dagger}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}}, \quad (16)$$

Non-relativistic fields are simply the creation and annihilation operators in position basis.

We define a *regularized* mass density operator

$$\hat{\mu}_s(\mathbf{r}) = m \int d\mathbf{r}' f(\mathbf{r}' - \mathbf{r}) \hat{\psi}^{\dagger}(\mathbf{r}') \hat{\psi}(\mathbf{r}'), \quad (20$$

using a smearing function $f_s(\mathbf{r})$ that satisfies the conditions

(i) $f_s(\mathbf{r}) \geq 0$.

(ii) $\lim_{s\to 0} f_s(\mathbf{r}) = \delta^3(\mathbf{r})$.

(iii) $\int d^3x f_s(\mathbf{r}) = 1$.

Mass density operator must be regularized (regularization is very simple in Newtonian regime).

s is the smearing length-scale.

$$\hat{H} = m\hat{N} - \frac{1}{2m}\int d\mathbf{r}\hat{\psi}^{\dagger}(\mathbf{r})\nabla^2\hat{\psi}(\mathbf{r}) + \int d\mathbf{r} V(\mathbf{r})\hat{\psi}^{\dagger}(\mathbf{r})\hat{\psi}(\mathbf{r}) - G\int d\mathbf{r}d\mathbf{r}' \frac{\hat{\mu}_s(\mathbf{r})\hat{\mu}_s(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}.$$

Hamiltonian including Newtonian Interaction and external potential

Correlations

Define smearing scale l by $f_s(0) = l^3$.

Define positive operator $\hat{P}_{\boldsymbol{r}} = \int d\boldsymbol{r}' \frac{f_s(\boldsymbol{r}-\boldsymbol{r}')}{f_s(0)} |\boldsymbol{r}'\rangle \langle \boldsymbol{r}'|$

It represents a position sampling of width l.

We assume an one-particle state

$$|\phi\rangle = \int d\mathbf{r}\phi(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})|0\rangle,$$

where $\phi(\mathbf{r})$ is the one-particle wave-function.

In general, correlation functions of the mass density correspond to elements of the decoherence functional for position samplings.

The correlation functions of the mass density become

$$\langle \mu_s(\mathbf{r},t)\rangle = \frac{m}{\ell^3}\langle\phi|\hat{P}_{\mathbf{r},t}|\phi\rangle$$

$$\langle \mu_s(\mathbf{r},t)\mu_s(\mathbf{r}',t')\rangle = \frac{m^2}{\ell^6}\langle\phi|\hat{P}_{\mathbf{r}t}\hat{P}_{\mathbf{r}'t'}|\phi\rangle,$$

where $\hat{P}_{\mathbf{r}t} = e^{i\hat{H}t}\hat{P}_{\mathbf{r}}e^{-i\hat{H}t}$ is the Heisenberg-picture evolution of $\hat{P}_{\mathbf{r}}$.

Stochastic Gravity Program

- ***Review***

  B. L. Hu and E. Verdaguer, "Stochastic gravity: Theory and Applications", in ***Living Reviews in Relativity*** *7 (2004) 3. updated in* ***11*** *(2008) 3 [arXiv:0802.0658]*

- ***Recent work*** *(sample)*

  - *Black hole fluctuations and Backreaction.*

    B.L.Hu, A. Roura, Phys. Rev. D 76 (2007) 124018

  - *Cosmological perturbations:*

    A. Roura and E. Verdaguer, P*hys. Rev. D* (2008), (2009)

  - *Current work by*

    - *Paul Anderson, Jason Bates, Hing Tong Cho and B L Hu*
    - *Markus B. Fröb, Guillem Pérez-Nadal, Albert Roura, Enric Verdaguer*
    - *L. Ford, J T Hsiang, S P Miao, R.Woodard, C H Wu,*