The unreasonable effectiveness of quantum theory: a logical inference approach*

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Introduction

- Quantum theory describes a vast number of different experiments very well

WHY ?

- Can we give an explanation that goes beyond "because that is just the way Nature is" ?
  - W. Heisenberg: What we observe is not nature itself, but nature exposed to our method of questioning
  - N. Bohr: It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature

Establishing a bridge between sensory experience and concepts…

- Aristotle: There is nothing in our intelligence that has not passed by the senses
- Face the fact that we cannot know "everything" and deal with this uncertainty from the start
- Describing data of laboratory experiments is a problem of inference, not a problem of deduction
- Can we construct a mathematical model for the cognitive processes that humans use to infer a descriptive theory from observed data?

Plausible reasoning (G. Polyá, R.T. Cox, E.T. Jaynes, …)

- Polyá: Patterns of plausible reasoning
  - Introduce the concept of a plausibility that a proposition *A* is true conditional on proposition *Z* being true
- Logical inference (inductive logic, rational reasoning):
  1. *Plausibilities are represented by real numbers*
  2. *Plausibilities must exhibit agreement with rationality*
  3. *All rules relating plausibilities must be consistent*

Plausible reasoning → logical inference

- It follows (by calculation) that the plausibility $P(A|Z)$ that a proposition A (B) is true given that proposition Z is true must satisfy the rules*

a. $0 \le P(A|Z) \le 1$

b. $P(A|Z) + P(\bar{A}|Z) = 1 \quad ; \quad \bar{A} = \text{NOT } A$

c. $P(AB|Z) = P(A|BZ)P(B|Z) \quad ; \quad AB = A \text{ AND } B$

  - The rules (a-c) are unique.
  - Given the same data, any reasoning yielding a result that is at odds with the one obtained through rules (a-c) necessarily violates plausible reasoning

*R.T. Cox, E.T. Jaynes

Logical inference

- Algebra of logical inference

a. $0 \leq P(A|Z) \leq 1$

b. $P(A|Z) + P(\bar{A}|Z) = 1 \quad ; \quad \bar{A} = \text{NOT } A$

c. $P(AB|Z) = P(A|BZ)P(B|Z) \quad ; \quad AB = A \text{ AND } B$

- Extension of Boolean logic, applicable to situations in which there is uncertainty about some but not all aspects

  - Probability theory is nothing but common sense reduced to calculation (Laplace, 1819)
  - Kolmogorov's probability theory is an example which complies with the rules of rational reasoning

  - Bayesian analysis
  - Quantum theory

Subjective and objective

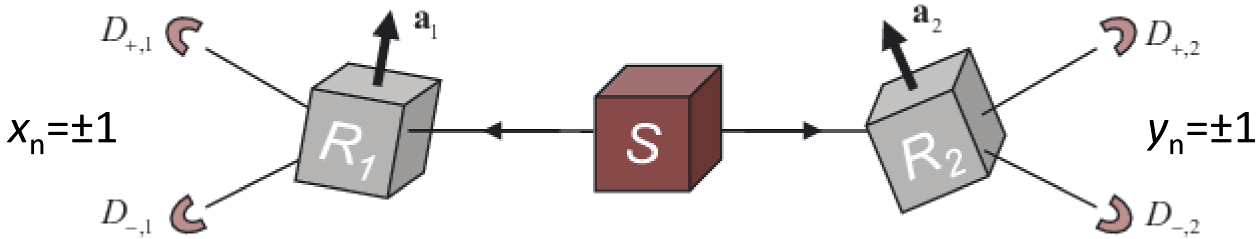
- Plausibility
  - Is an intermediate mental construct to carry out logical inference, inductive logic, rational reasoning
    1. May express a degree of believe (subjective)
    2. May be used to describe phenomena independent of individual subjective judgment ➔ this is our goal

Physics is to be regarded not so much as the study of something a priori given, but rather as the development of methods of ordering and surveying human experience. In this respect our task must be to account for such experience in a manner independent of individual subjective judgment and therefore objective in the sense that it can be unambiguously communicated in ordinary human language (N. Bohr, "XV. The Unity of Human Knowledge," in *Complementarity Beyond Physics (1928 –1962)*)

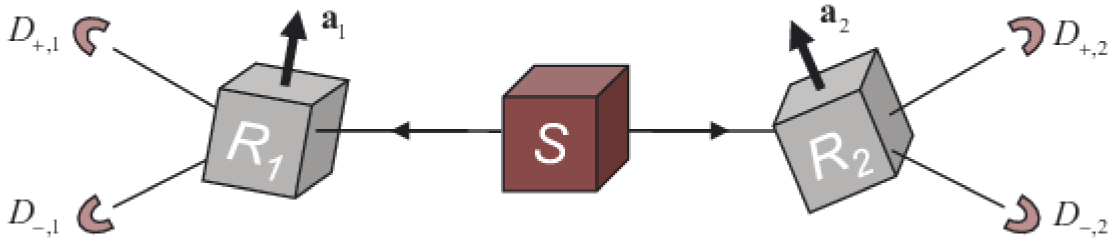
Logical inference: application

- Einstein-Podolsky-Rosen-Bohm experiment

- Seems to be the simplest nontrivial model for demonstrating how the logical inference yields results known from quantum theory without using concepts of quantum theory such as wavefunctions, spin-1/2, Hilbert space, …



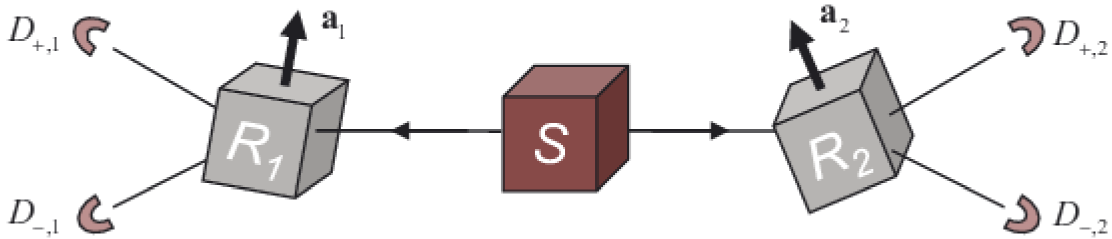
- *S*: source sends signals to routers R_1 and R_2
  - No assumption about nature (correlation) between two signals
- R_i : router decides to send signal to either detector D_{+i} or D_{-i}
  - Decision depends on the orientation $\mathbf{a}_i$ relative to laboratory reference frame
- $D_{\pm i}$: detectors operate with 100% efficiency
  - *n*-th signal produces $\{x_n=\pm1, y_n=\pm1\}$



- We assign a plausibility to observe an event $\{x, y\}$ and call it $P(x, y|\mathbf{a}_1, \mathbf{a}_2, Z)$
- We assume that $P(x, y|\mathbf{a}_1, \mathbf{a}_2, Z)$ does not change if we rotate both routers in the same way

$$P(x, y \mid \mathbf{a}_1, \mathbf{a}_2, Z) = P(x, y \mid \mathbf{a}_1 \cdot \mathbf{a}_2, Z) = P(x, y \mid \theta, Z)\ ; \cos\theta = \mathbf{a}_1 \cdot \mathbf{a}_2$$

  – *To simplify things, consider the case* $\langle x \rangle = \langle y \rangle = 0$



- Based on all other (future or later) events, it is impossible to say with some certainty what the particular event will be (independence)
- Straightforward application of the algebra of logical inference: the plausibility to observe counts $\{n_{++}, n_{-+}, n_{+-}, n_{--}\}$ is given by

$$P(n_{++}, n_{-+}, n_{+-}, n_{--} \mid \theta, N, Z) = N! \prod_{x,y=\pm 1} \frac{P(x, y \mid \theta, Z)^{n_{xy}}}{n_{xy}!}$$

Reproducible and robust experiments

- We require that the observed frequencies are
  - Reproducible
    - repeating the experiment on another day/location/… yields similar results
  - Robust with respect to small changes in θ ($\cos\theta = \mathbf{a}_1 \cdot \mathbf{a}_2$)
    - Criterion for a "good" experiment: if the conditions change smoothly, so should the frequencies

How to express robustness?

- Hypothesis H_0: given θ we observe the data
- Hypothesis H_1: given $\theta+\varepsilon$ we observe the data
  - N is fixed and the same for both hypotheses H_0 and H_1
- The evidence Ev(H_1/H_0) is given by

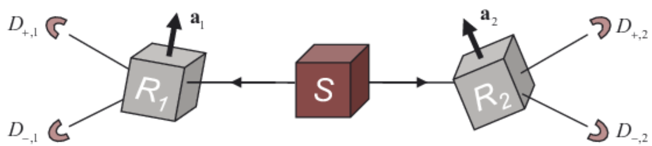
$$\mathrm{Ev}(H_1/H_0) = \ln\frac{P(n_{++},n_{-+},n_{+-},n_{--}\,|\,\theta+\varepsilon,N,Z)}{P(n_{++},n_{-+},n_{+-},n_{--}\,|\,\theta,N,Z)} = \sum_{x,y=\pm1} n_{xy}\ln\frac{P(x,y\,|\,\theta+\varepsilon,Z)}{P(x,y\,|\,\theta,Z)}$$

- Observed frequencies should be robust with respect to small changes in ε and for all θ ➔ we should minimize $|\mathrm{Ev}(H_1/H_0)|$ for all possible θ and ε simultaneously ➔ global optimization problem
  - Solutions for which $P(x,y|\theta,Z)$ independent of θ are uninformative and therefore discarded.

Solve the optimization problem

- Robust experiment ➔ the plausibility of the individual event must be equal to the frequency of observation
- Frequency n_{xy}/N is "objective" knowledge ➔ independent of personal judgment

- Robust experiment ➔ $$P(x, y \mid \theta, Z) = \frac{1 - xy\, \mathbf{a}_1 \cdot \mathbf{a}_2}{4}$$
  - The same as the expression of the probability of two S=1/2 particles in the singlet state
    - no quantum-theoretical concepts in this derivation
    - Expressions for e.g. the triplet state follow in the same manner



Connect to the wavefunction formalism

- Trivial rearrangement of the events (x, y) and $P(x, y|\theta, Z)$ as 4x4 matrices + separation into "system" and "magnets" + some basic linear algebra
  ➔ the "quantum" representation

$$P(x, y\,|\,\mathbf{a}_1, \mathbf{a}_2, Z) = \mathbf{Tr}\,\hat{\rho}\left(1 + x\mathbf{a}_1 \cdot \boldsymbol{\sigma}_1\right)\left(1 + y\mathbf{a}_2 \cdot \boldsymbol{\sigma}_2\right)$$

$$\hat{\rho} = \left(\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right)\left(\frac{\langle\uparrow\downarrow| - \langle\downarrow\uparrow|}{\sqrt{2}}\right)$$

- No quantum-theory concepts in this derivation

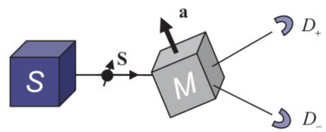
General strategy

- Data ➔ robust experiment ➔ include constraints ➔ solve global optimization problem for functional in $P(\ldots|\ldots)$
  - the observed data are assumed to be independent events
  - In the absence of uncertainty, motion complies with classical mechanics (≈ Bohr's correspondence principle)
- Ways to connect to "quantum" equations
  1. separation principle ➔ linear algebra ➔ density matrix, …
  2. non-linear equations ➔ transformation ➔ quadratic form $F = \int \Phi^{\dagger}\mathbf{Q}\Phi$ ⇔ Schrödinger-like equations

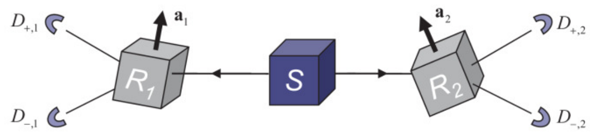
- ~~Quantization rules, Born rule, particle-wave duality, measurement problem, wave function collapse, … , magic, interpretation~~

Successful derivations so far

- Stern-Gerlach experiment



- Einstein-Podolsky-Rosen-Bohm experiment (e.g. singlet state)



- Time-independent Schrödinger equation
- Time-dependent Schrödinger equation for particle in electromagnetic field
- Pauli equation, S=1/2
- Klein-Gordon equation
- ...???

What quantum theory seems to be

- Quantum theory is logical inference, that is common sense reasoning, applied to reproducible and robust experimental data.
  - ➔ quantum theory is a phenomenological theory which can be derived from a set of simple general principles in a way that is independent of any (strictly speaking, unknown) “more microscopic” level of description
  - As a solution of an optimization problem quantum theory is “the” probabilistic description, hard to beat…

➔ WHY it is so powerful

- A parallel with Einstein’s view on thermodynamics: a theory of principles based on empirically observed properties of phenomena, independent of a particular underlying model rather than a constructive theory, an attempt to build a picture of complex phenomena out of some relatively simply propositions

Thank you