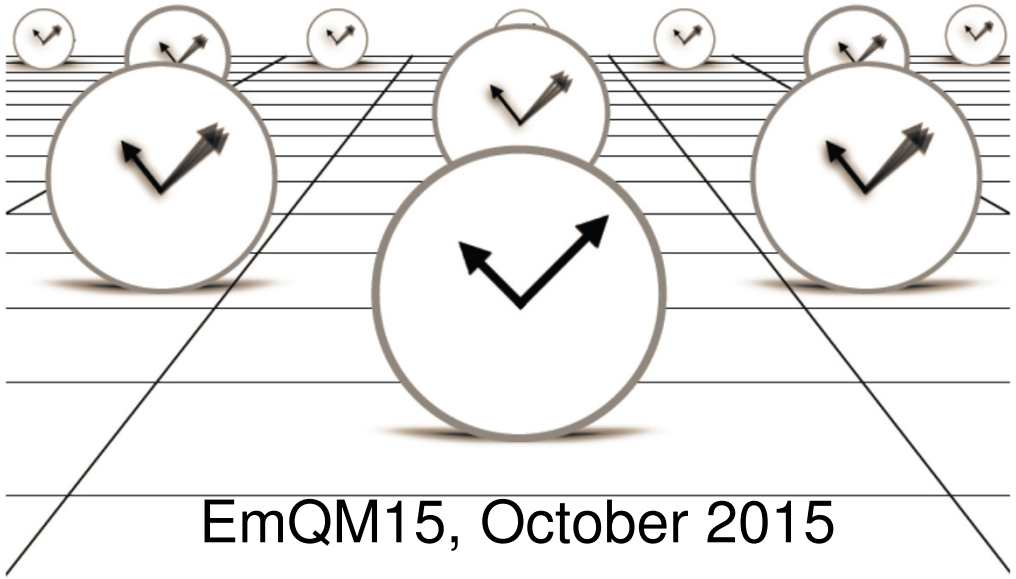
Faculty of Physics, University of Vienna &
Institute for Quantum Optics and Quantum Information, Vienna

Quantum Clocks and Time

Esteban Castro, Flaminia Giacomini, Časlav Brukner



$$|\psi(t)\rangle = |x_1(t)\rangle + |x_2(t)\rangle$$

Time is a parameter

$$H(t)\,|\,\psi(t)\rangle = i\hbar\frac{d}{dt}\,|\,\psi(t)\rangle$$

We have no theory for …

$$|\psi(t)\rangle = |x_1(t)\rangle + |x_2(t)\rangle$$

$$|\phi(t)\rangle = |y_1(t)\rangle + |y_2(t)\rangle$$

Which „t“?

We have no theory for …

$$|\psi(t)\rangle = |E_1\rangle + e^{-i\hbar\frac{\Delta E t}{\hbar}}|E_2\rangle$$

$$|\phi(t)\rangle = |E_1\rangle + e^{-i\hbar\frac{\Delta E t}{\hbar}}|E_2\rangle$$

Which „t“?

Outline

- What is a clock? **Operational** time
- **Complementarity** between precision of time measurements at „nearby“ points
- Entanglement between clocks via time dilation: **limits on time measurements**
- The general relativistic notion of time is recovered in the **classical limit of clocks**

What is a clock?

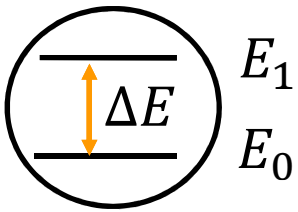
„Single tick“ of the clock:

For given Hamitonian $\widehat{H}_{clock}$ find initial state $|\psi(0)\rangle$ to have minimal $t_\perp$ such that

$$|\langle\psi(0)|\psi(t_\perp)\rangle| \approx 0$$

Orthogonalization time: $t_\perp$

$\widehat{H}_{clock}$



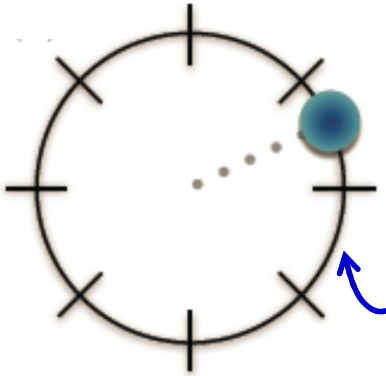
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{-iE_0 t/\hbar}|E_0\rangle + e^{-iE_1 t/\hbar}|E_1\rangle)$$

$$|\psi(t_\perp)\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle - |E_1\rangle)$$

$$t_\perp = \frac{\hbar\pi}{\Delta E}$$

Clocks in the classical limit



$$|\theta = \frac{\pi}{2}, \phi\rangle$$

$$\hat{H}_{clock} = \Delta E\ (j\ \hat{\mathbb{I}} - \hat{Z})$$

$$\hat{Z} = \sum_{m=-j}^{j} m|m\rangle\langle m|$$

POVM elements: $\frac{2\pi}{R}$ bins

$$M_k = \frac{2j+1}{4\pi}\int_0^{\pi} d\theta \sin\theta \int_{(k-1)R}^{kR} d\phi |\theta,\phi\rangle\langle\theta,\phi|$$

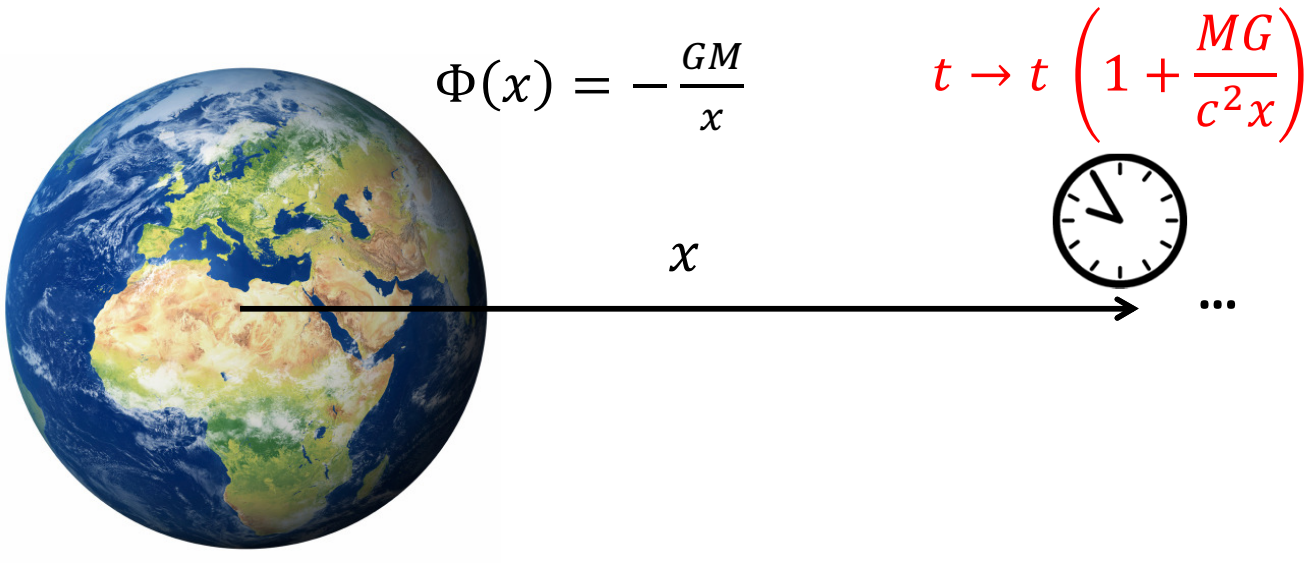
$$p_k = \mathrm{Tr}\,(\rho M_k) = \frac{2j+1}{4\pi}\int_0^{\pi} d\theta \sin\theta \int_{(k-1)R}^{kR} d\phi Q_\rho(\theta,\phi) \qquad Q_\rho(\theta,\phi) = \langle\theta,\phi|\rho|\theta,\phi\rangle$$

Q-function $>$ 0

For R $\gg \sqrt{j}$ and $j \gg 1$ the POVM noninvasive, **evolution of a classical spin**

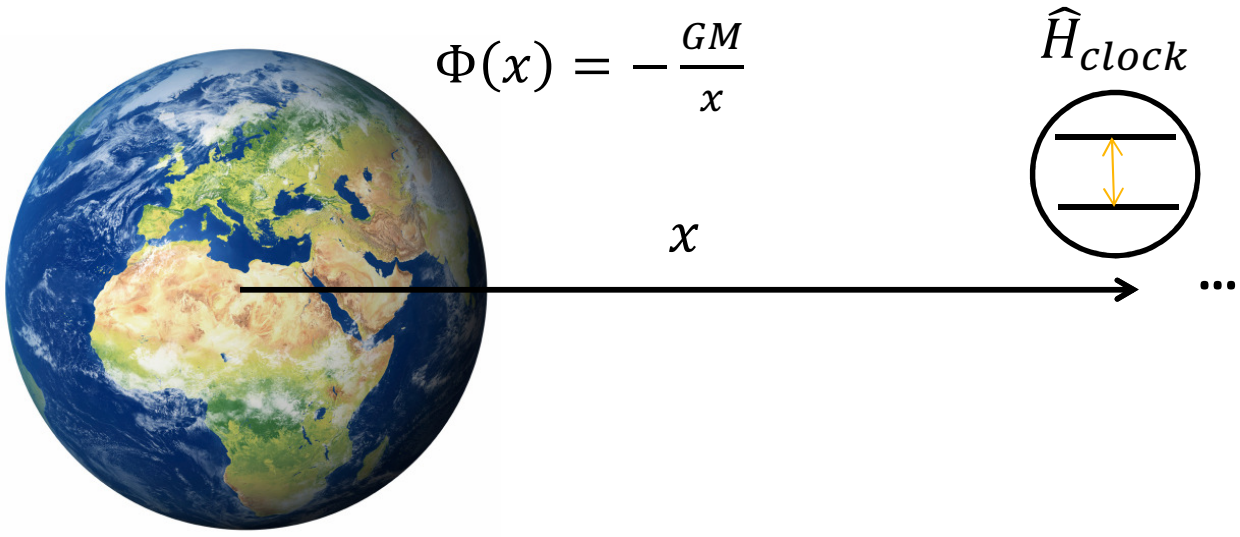
J. Kofler and Č. Brukner, Phys. Rev. Lett. **99**, 180403 (2007)

Gravitational time dilation



post-Newtonian background $g_{00} = -\left(1 + \frac{2\Phi(x)}{c^2}\right), g_{ij} = \delta_{ij}\left(1 - \frac{2\Phi(x)}{c^2}\right)$

Gravitational time dilation



post-Newtonian background $g_{00} = -\left(1 + \frac{2\Phi(x)}{c^2}\right), g_{ij} = \delta_{ij}\left(1 - \frac{2\Phi(x)}{c^2}\right)$

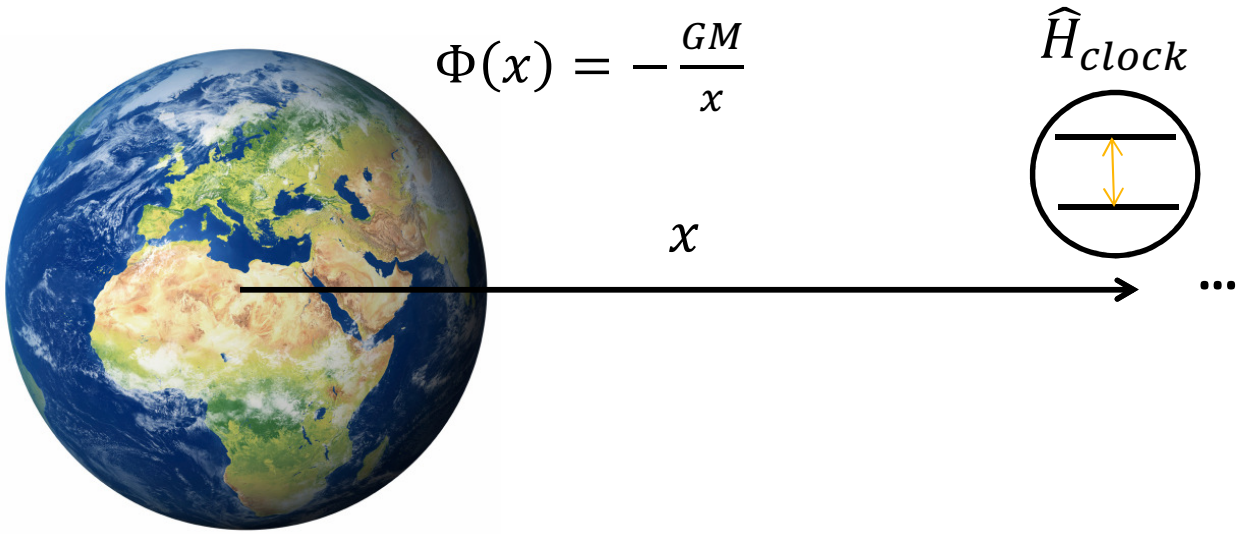
$$\widehat{H} = \widehat{M}c^2 + \frac{\hat{p}^2}{2\,\widehat{M}} + \widehat{M}\Phi(\hat{x}) - \frac{\hat{p}^4}{8\,\widehat{M}^3c^2} + \frac{\widehat{M}\Phi^2(\hat{x})}{2c^2} + \frac{3}{2\,\widehat{M}c^2}\left(\Phi(\hat{x})\hat{p}^2 + [\hat{p}\Phi(\hat{x})]\hat{p} + \frac{1}{2}[\hat{p}^2\Phi(\hat{x})]\right)$$

$$\widehat{M} = m + \frac{\widehat{H}_{clock}}{c^2}$$

Mass-energy equivalence

Lämmerzahl, C. A Hamilton operator for quantum optics in gravitational fields, Phys.Lett. A **203**, 12-17 (1996)

Gravitational time dilation



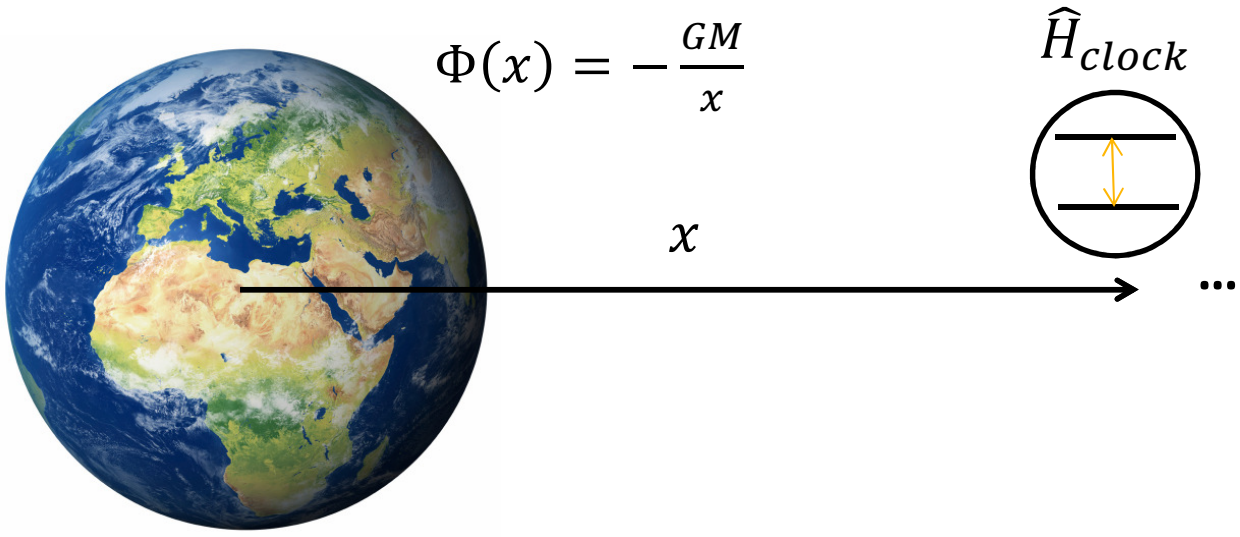
post-Newtonian background $g_{00} = -\left(1 + \frac{2\Phi(x)}{c^2}\right), g_{ij} = \delta_{ij}\left(1 - \frac{2\Phi(x)}{c^2}\right)$

$$\hat{H} = mc^2 + \hat{H}_{clock} + \frac{\hat{p}^2}{2m}\left(1 - \frac{\hat{H}_{clock}}{mc^2}\right) + \left(m + \frac{\hat{H}_{clock}}{c^2}\right)\phi(\hat{x})$$

Slow particles, weak fields (order $1/c^2$)

Lämmerzahl, C. A Hamilton operator for quantum optics in gravitational fields, Phys.Lett. A **203**, 12-17 (1996)

Gravitational time dilation



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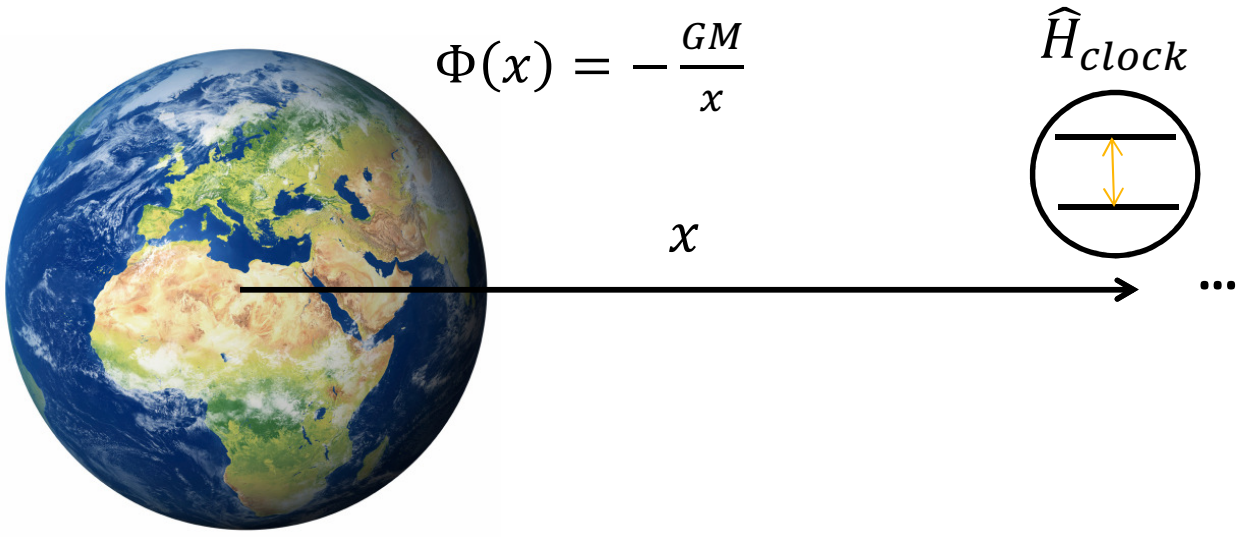
$$\hat{H} = mc^2 + \hat{H}_{clock} + \frac{\hat{p}^2}{2m}\left(1 - \frac{\hat{H}_{clock}}{mc^2}\right) + \left(m + \frac{\hat{H}_{clock}}{c^2}\right)\phi(\hat{x})$$

Special relativistic time dilation

Gravitational time dilation

I. Pikovski, M. Zych, F. Costa and Č.B, Nature Physics **11**, 668–672 (2015)

Gravitational time dilation



post-Newtonian background $g_{00} = -\left(1 + \frac{2\Phi(x)}{c^2}\right), g_{ij} = \delta_{ij}\left(1 - \frac{2\Phi(x)}{c^2}\right)$

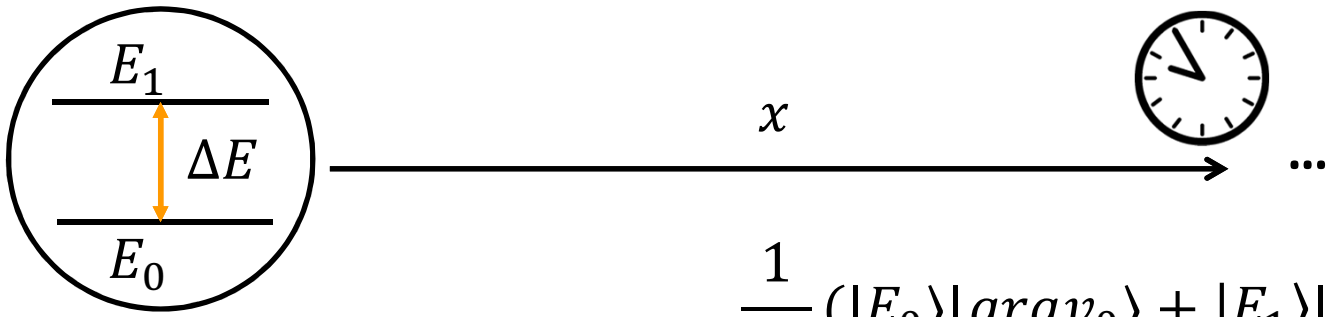
Static, massless clocks: $\hat{H} = \hat{H}_{clock}\left(1 + \frac{\Phi(\hat{x})}{c^2}\right)$

Complementarity relation

Time dilation uncertainty

$$|\psi_{clock}\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$

$$\Delta t = t\left(1 + \frac{\Delta E G}{c^2 x}\right)$$



$$\frac{1}{\sqrt{2}}(|E_0\rangle|grav_0\rangle + |E_1\rangle|grav_1\rangle)$$

Orthogonalization time:

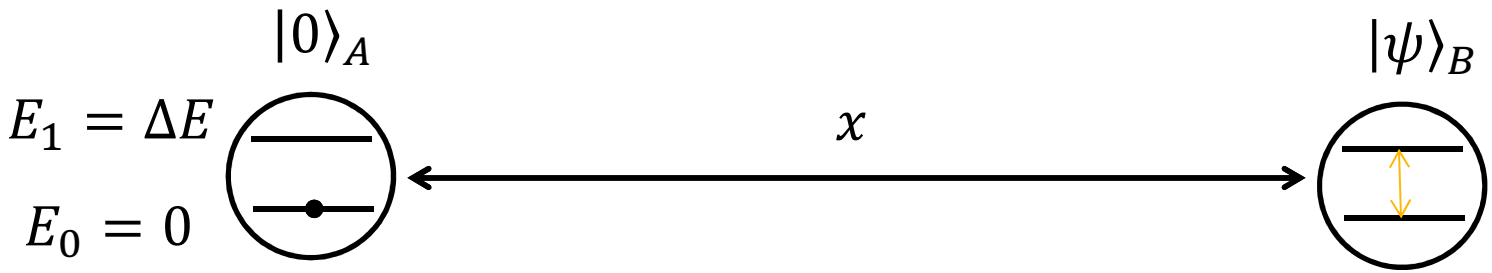
$$t_\perp = \frac{\hbar\pi}{\Delta E}$$

$$\Delta\Phi(x) = \frac{G\Delta E}{c^4 x}$$

Complementarity between precisions of time measurements at "nearby" points

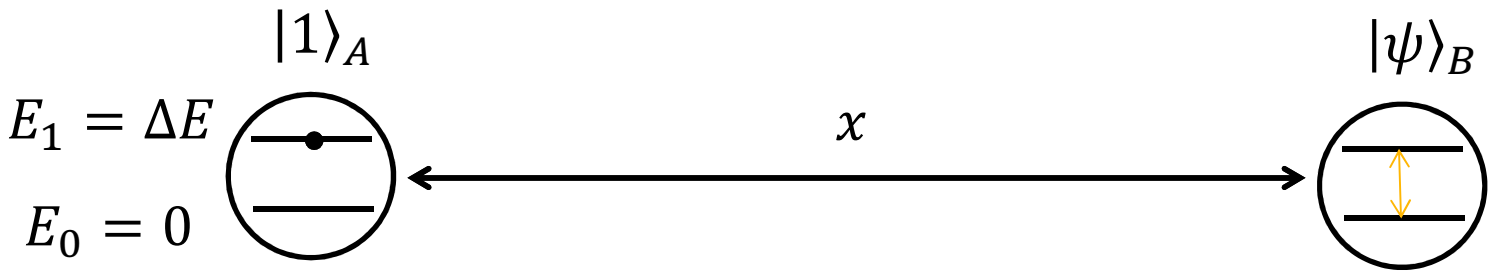
$$t_\perp \Delta t = \frac{\hbar\pi G t}{c^4 x}$$

2 clocks



$$|\psi\rangle_B \to e^{-\frac{it}{\hbar}\widehat{H}_B}|\psi\rangle_B \quad \text{for} \quad |0\rangle_A$$

2 clocks



$$|\psi\rangle_B \to e^{-\frac{it}{\hbar}\widehat{H}_B}|\psi\rangle_B \quad \text{for} \quad |0\rangle_A$$

$$|\psi\rangle_B \to e^{-\frac{it}{\hbar}\widehat{H}_B\left(1-\frac{G\Delta E}{c^4\hat{x}}\right)}|\psi\rangle_B \quad \text{for} \quad |1\rangle_A$$

$$|0\rangle_A \to |0\rangle_A$$

$$|1\rangle_A \to e^{-\frac{it}{\hbar}\widehat{H}_A}|1\rangle_A$$

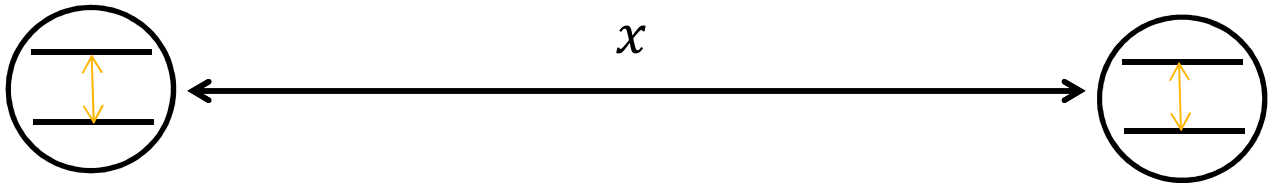
$$|\Psi\rangle = \sum_i c_i |\psi_i^{class.-like}\rangle$$

2 clocks

$$|\psi_{in}\rangle = (\alpha|0\rangle_A + \beta|1\rangle_A)|\psi\rangle_B$$

$E_1 = \Delta E$

$E_0 = 0$



$$|\psi_{in}\rangle \to \alpha|0\rangle_A e^{-\frac{it}{\hbar}\hat{H}_B}|\psi\rangle_B + \beta e^{-\frac{it}{\hbar}\Delta E}|1\rangle_A e^{-\frac{it}{\hbar}\hat{H}_B\left(1-\frac{G\Delta E}{c^4\hat{x}}\right)}|\psi\rangle_B = |\psi_{fin}\rangle$$

$$|\psi_{fin}\rangle = e^{-\frac{it}{\hbar}(\hat{H}_A+\hat{H}_B-\frac{\mathrm{G}}{\mathrm{c}^4\,\hat{x}}\hat{H}_A\hat{H}_B)}\,|\psi_{in}\rangle$$

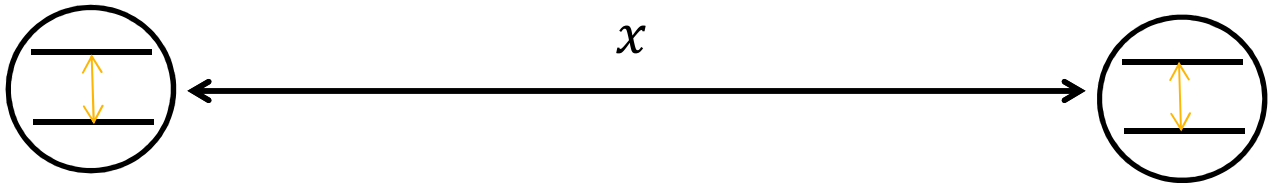
$$\hat{H}_{AB} = \hat{H}_A + \hat{H}_B - \frac{G}{c^4\hat{x}}\hat{H}_A\hat{H}_B$$

Newtonian gravitational interaction

2 clocks

$E_1 = \Delta E$

$E_0 = 0$



$$\hat{H}_{AB} = m_A c^2 + m_B c^2 - \frac{G}{\hat{x}} m_A m_B$$

$$m_i = \frac{\hat{H}_i}{c^2}$$

Mass-energy equivalence

$$\hat{H}_{AB} = \hat{H}_A + \hat{H}_B - \frac{G}{c^4 \hat{x}} \hat{H}_A \hat{H}_B$$

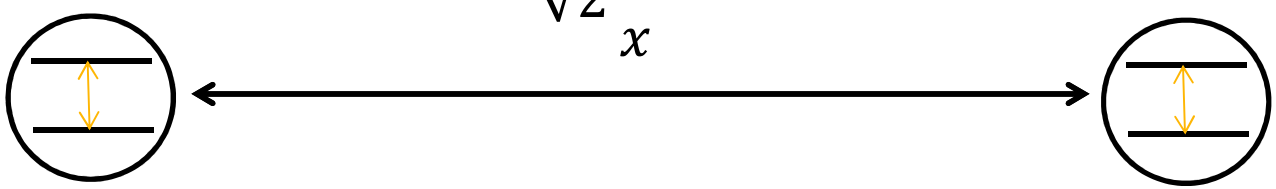
Newtonian gravitational interaction

Entanglening clocks via time dilation

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)^{\otimes 2}$$

$E_1 = \Delta E$

$E_0 = 0$



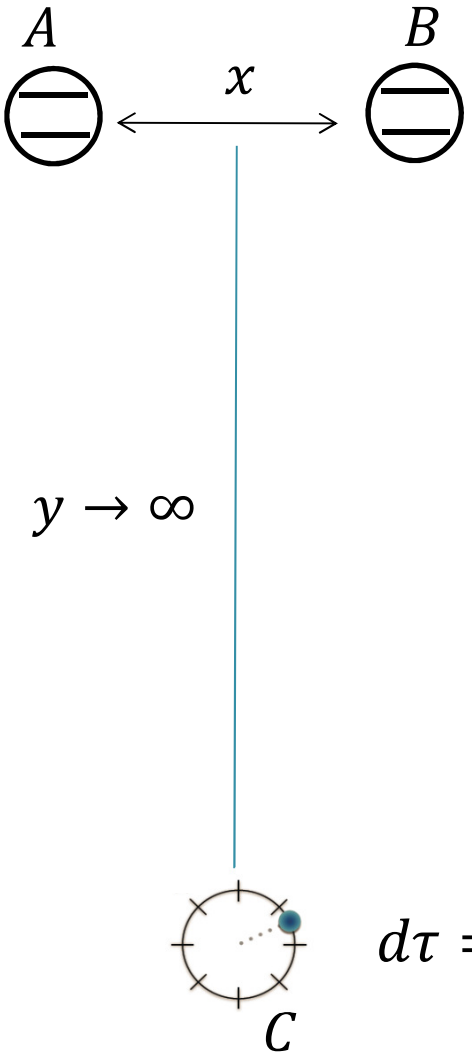
Entangled Clocks:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}|0\rangle[\frac{1}{\sqrt{2}}(|0\rangle + e^{-\frac{it}{\hbar}\Delta E}|1\rangle)] + \frac{1}{\sqrt{2}}e^{-\frac{it}{\hbar}\Delta E}\,|1\rangle[\frac{1}{\sqrt{2}}(|0\rangle + e^{-\frac{it}{\hbar}\Delta E(1-\frac{G\Delta E}{c^4 x})}|1\rangle)]$$

The rate at which time runs in one clock is correlated to the value of the energy of the other clock.

$$t_{ent} = \frac{\pi\hbar c^4 x}{G(\Delta E)^2} = \frac{\pi\xi}{\epsilon^2} \quad [\text{in } t_P \text{ units}] \qquad \xi = \frac{x}{l_p}, \epsilon = \frac{\Delta E}{E_p}$$

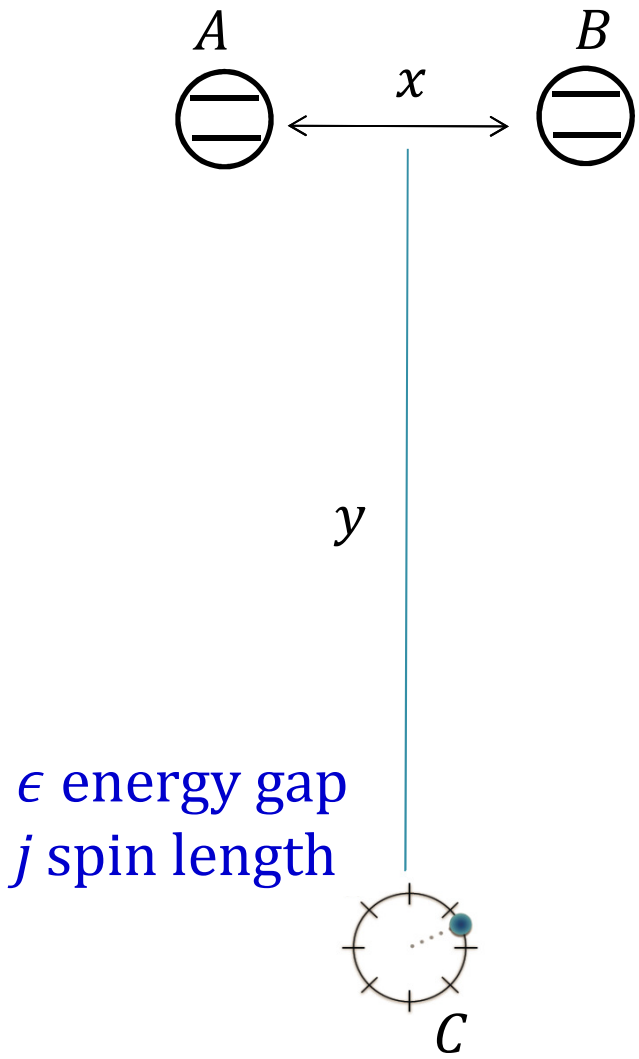
Whose time is **t**?



$$|\psi_{fin}(t)\rangle = e^{-\frac{it}{\hbar}\hat{H}_{AB}}|\psi_{in}\rangle$$

$$d\tau = dt\left(1 + \frac{\Phi(y)}{c^2}\right) \xrightarrow{y\to\infty} dt$$

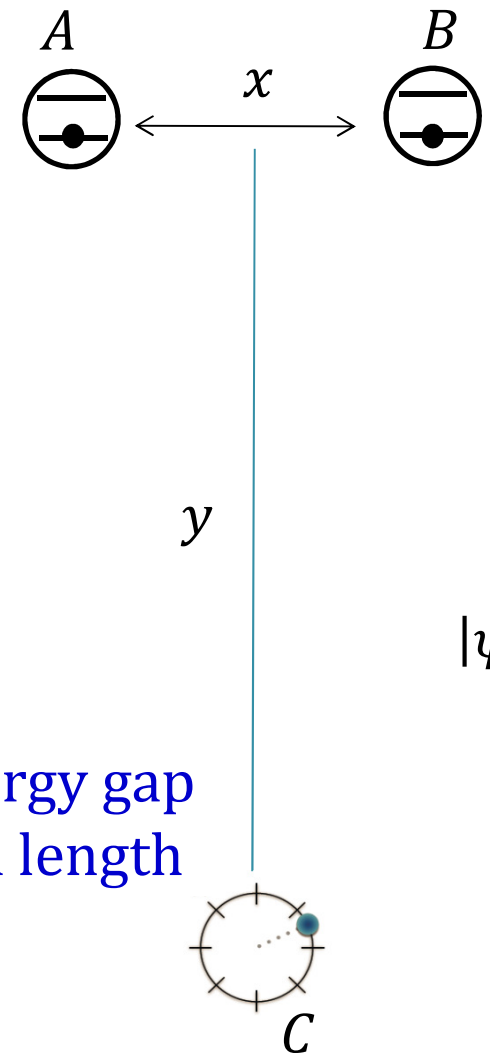
Whose time is **t**?



ϵ energy gap
j spin length

$$|\psi_{fin}(t)\rangle = e^{-\frac{i\boldsymbol{t}}{\hbar}\hat{H}_{BC}}|\psi_{in}\rangle$$

Whose time is **t**?



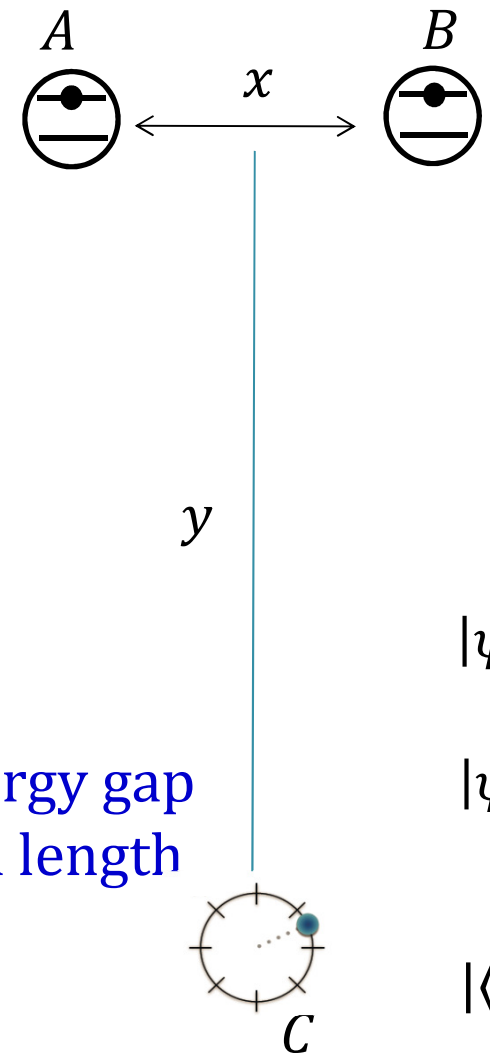
$$|\psi_{fin}(t)\rangle = e^{-\frac{it}{\hbar}\hat{H}_{BC}}|\psi_{in}\rangle$$

$$|\psi_{00}(t)\rangle_C = |\theta = \frac{\pi}{2}, \phi = -\frac{t\epsilon}{\hbar}\rangle$$

ϵ energy gap

j spin length

Whose time is **t**?



$$|\psi_{fin}(t)\rangle = e^{-\frac{it}{\hbar}\hat{H}_{BC}}|\psi_{in}\rangle$$

$$|\psi_{00}(t)\rangle_C = |\theta = \frac{\pi}{2}, \phi = -\frac{t\epsilon}{\hbar}\rangle$$

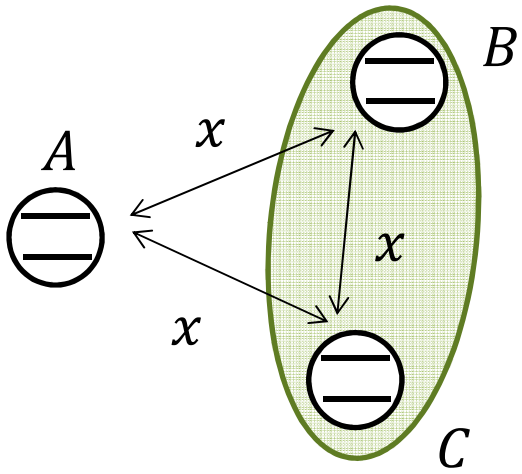
$$|\psi_{11}(t)\rangle_C = |\theta = \frac{\pi}{2}, \phi = -\frac{t\epsilon}{\hbar}\left(1 - \frac{2G\Delta E}{c^4 y}\right)\rangle$$

ϵ energy gap

j spin length

$$|\langle\psi_{00}|\psi_{11}\rangle|^2 \geq 1 - \delta \Longrightarrow y \geq \frac{2\sqrt{2}jG\Delta E\epsilon t}{\hbar c^4\sqrt{\delta}}$$

3 clocks

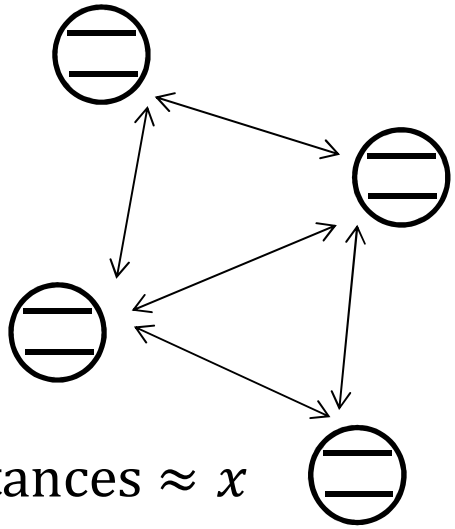


$$|\psi_{in}\rangle = (\alpha|0\rangle_A + \beta|1\rangle_A)|\psi\rangle_{BC}$$

$$|\psi_{in}\rangle \to |0\rangle_A e^{-\frac{it}{\hbar}\widehat{H}_{BC}}|\psi\rangle_{BC} + \beta e^{-\frac{it}{\hbar}\Delta E}|1\rangle_A e^{-\frac{it}{\hbar}\widehat{H}_{BC}\left(1-\frac{G\Delta E}{c^4 x}\right)}|\psi\rangle_{BC} = |\psi_{fin}\rangle$$

$$\widehat{H}_{ABC} = \widehat{H}_A + \widehat{H}_B + \widehat{H}_C - \frac{G}{c^4\hat{x}}(\widehat{H}_A\widehat{H}_B + \widehat{H}_A\widehat{H}_C + \widehat{H}_B\widehat{H}_C)$$

N+1 Clocks

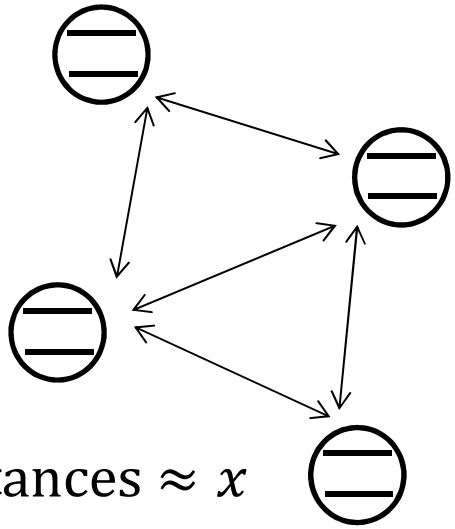


All distances $\approx x$

Iteratively …

$$\hat{H} = \sum_{i=1}^{N+1} \hat{H}_i - \frac{G}{c^4 \hat{x}} \sum_{i<j}^{N+1} \hat{H}_i \, \hat{H}_j$$

N+1 Clocks



All distances $\approx x$

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)^{\otimes N+1}$$

$$\hat{H} = \sum_{i=1}^{N+1} \hat{H}_i - \frac{G}{c^4 \hat{x}} \sum_{i<j}^{N+1} \hat{H}_i \hat{H}_j$$

Reduced density matrix:

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & \left[\frac{1}{2}\left(1 + e^{-i\frac{\tau\epsilon^2}{\xi}}\right)\right]^N \\ \left[\frac{1}{2}\left(1 + e^{i\frac{\tau\epsilon^2}{\xi}}\right)\right]^N & 1 \end{pmatrix}$$

$$\epsilon = \frac{\Delta E}{E_P}, \tau = \frac{t}{t_P}, \xi = \frac{x}{l_P}$$

Visibility: $V \approx e^{-\left(\frac{\sqrt{N}\tau\epsilon^2}{2\xi}\right)^2}$

Decoherence time:

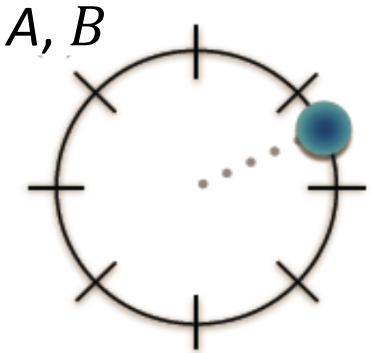
$$\tau_{dec} \approx \frac{2\hbar c^2 x}{\sqrt{N} G \Delta E^4}$$

$$N = 10^{23}, \Delta E = 10GeV, x = 10^{-15}m$$

$$\tau_{dec} = 80s$$

Schwarzschield radius $= 10^{-45}m$

Gravitational time dilation as the classical limit



$$\hat{H}_{AB} = \hat{H}_A + \hat{H}_B - \frac{G}{c^4 x}\hat{H}_A\hat{H}_B$$

$$\hat{H}_A = \Delta E\,(j_A\,\hat{\mathbb{I}} - \hat{Z}), \hat{H}_B = \Delta E\,(j_B\,\hat{\mathbb{I}} - \hat{Z}),$$

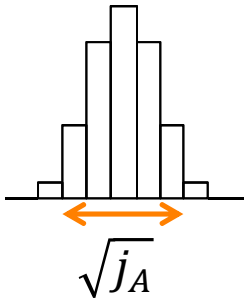
$$|\theta = \pi/2, \phi = 0\rangle_A \otimes \left|\theta = \frac{\pi}{2}, \phi = 0\right\rangle_B$$

$$\Downarrow$$

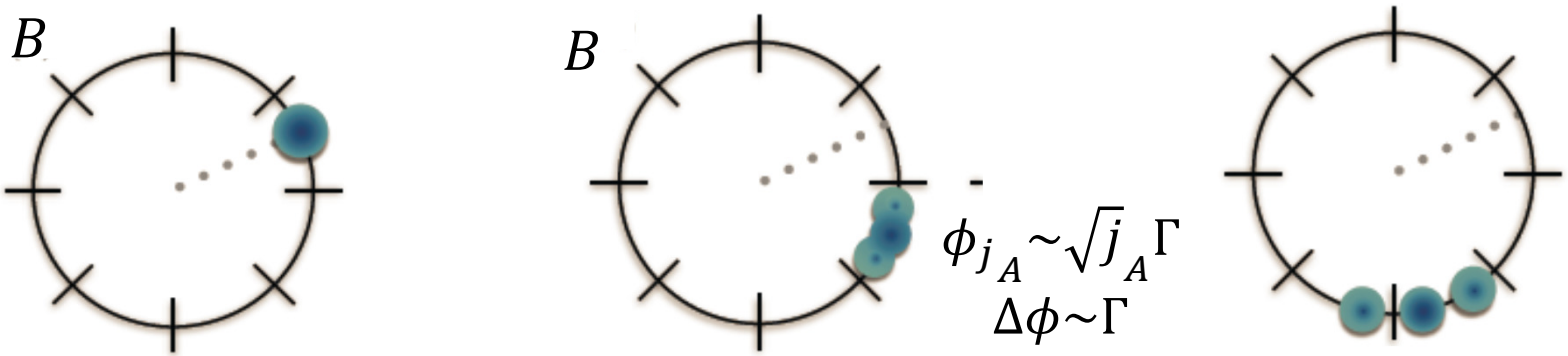
$$\rho_B = \frac{1}{4^{j_A}}\sum_{k=0}^{2j_A}\binom{2j_A}{k}\left|\theta = \frac{\pi}{2}, \phi_k\right\rangle\left\langle\theta = \frac{\pi}{2}, \phi_k\right| \qquad \phi_k = -\frac{t\Delta E}{\hbar}\left(1 - \frac{Gk\Delta E}{c^4 x}\right)$$

Average Phase $\phi_{j_A} = -\frac{t\Delta E}{\hbar}\left(1 - \frac{G j_A \Delta E}{c^4 x}\right)$

Phase Difference $\Delta\phi = \frac{2\sqrt{2j_A}Gt(\Delta E)^2 t}{\hbar c^4 x}$



Gravitational time dilation as the classical limit

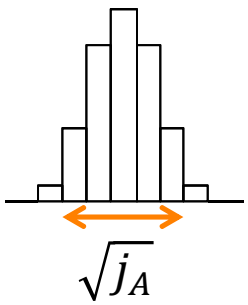


For $t = \Gamma t^*, \Gamma \ll 1$ **For $t > t^*$**

$$t^* = \frac{\hbar c^4 x}{G\sqrt{2j_A}(\Delta E)^2}$$

Average Phase $\phi_{j_A} = -\frac{t\Delta E}{\hbar}\left(1 - \frac{G j_A \Delta E}{c^4 x}\right)$

Phase Difference $\Delta\phi = \frac{2\sqrt{2j_A} G t (\Delta E)^2 t}{\hbar c^4 x}$



Master equation

$$\frac{d\hat{\rho}_B}{dt} = \frac{i}{\hbar}\underbrace{\left[\hat{H}_B\left(1+\frac{Gj_A\Delta E}{c^4x}\right),\hat{\rho}_B\right]}_{\text{Unitary part: Time-dilation part in the field of A}} - \underbrace{\left(\sqrt{\frac{j_A}{2}}\frac{G\Delta E}{2c^4x}\right)^2\int_0^t ds[\hat{H}_B,[\hat{H}_B,\hat{\rho}_B]_s]}_{\text{Decoherence part: proportional to the variance of the energy of A}}$$

$$[\hat{H}_B,\hat{\rho}_B]_s = e^{-\frac{is}{\hbar}\hat{H}_B}[\hat{H}_B,\hat{\rho}_B]e^{\frac{is}{\hbar}\hat{H}_B}$$

Following the derivation from :
I. Pikovski, M. Zych, F. Costa and Č.B, Nature Physics **11**, 668–672 (2015)

Summary

Measuring time by physical clocks

- Complementarity between local and time in “nearby” regions
- Gravitationally interacting quantum clocks
- Decoherence implies a limit on measurability of time (far away from the regime where BH forms)
- Gravitational time dilation as the classical limit of the model

Thank you!

CoQuS Complex Quantum Systems

FWF Der Wissenschaftsfonds.

FQXi

quantumfoundations.org

Proper distance

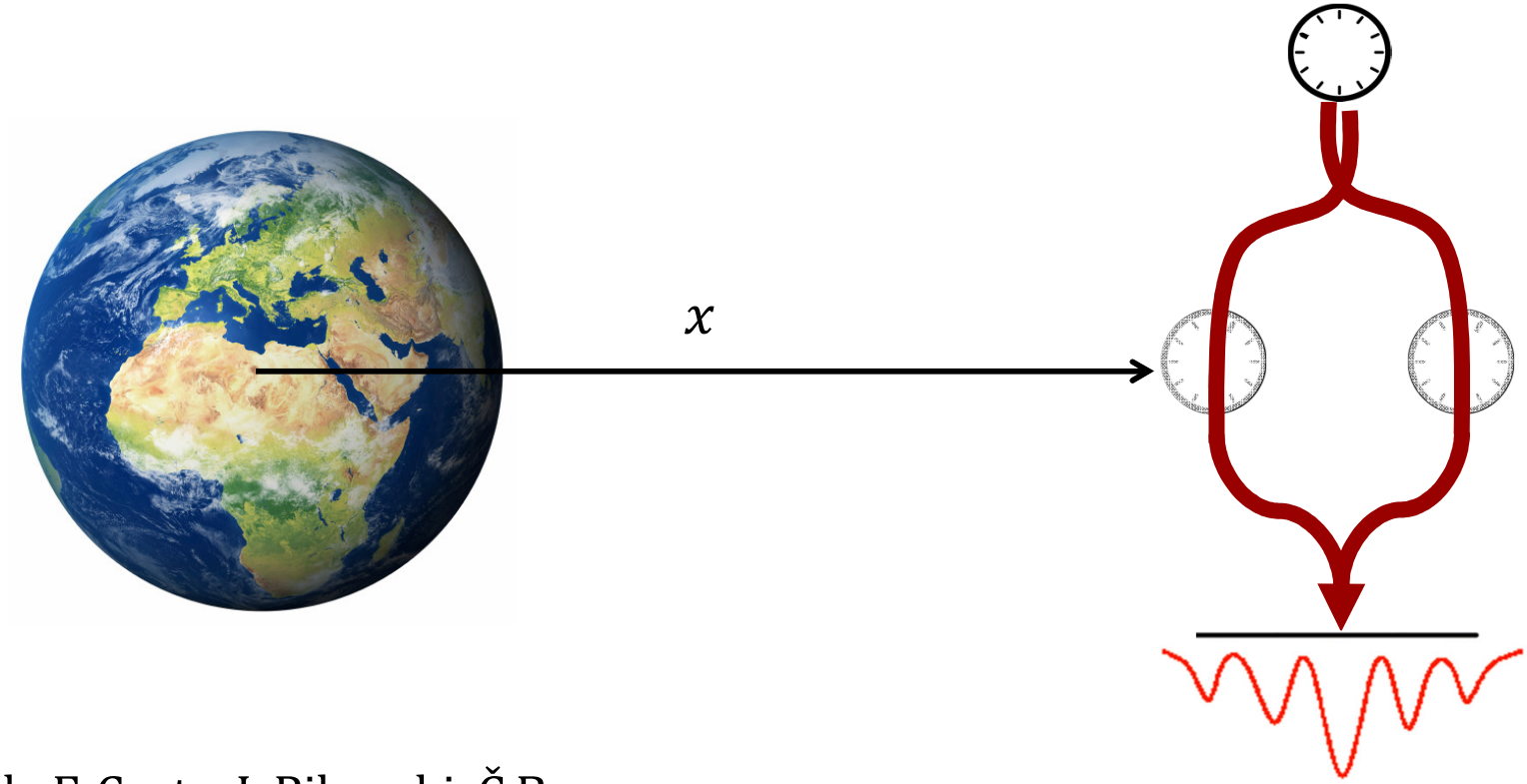
$$ds^2 = -\left(1 + \frac{\Phi}{c^2}\right)c^2 dt^2 + \delta_{ij}\left(1 - \frac{2\Phi(x)}{c^2}\right)dx^i dx^j \qquad \Phi = -\sum_i \frac{G\Delta E}{c^2|x - x_i|}$$

$$S_{AB} = \int_{x_A}^{x_B} \sqrt{1 + \frac{2G\Delta E}{c^4(x - x^i)}}\, dx \approx r(1 - \frac{\gamma}{2r}\log\frac{\gamma}{4r})$$

$$r = x_B - x_A, \qquad \gamma = 2G\Delta E/c^4$$

$$t \to t\left(1 - \frac{G\Delta E}{c^4 r}\right) \qquad r \to r(1 - \frac{\gamma}{2r}\log\frac{\gamma}{4r}) \qquad \sim O(\frac{1}{c^8})$$

Interference of clocks



M. Zych, F. Costa, I. Pikovski, Č.B.,
Nature Communication **2**:505 (2011)

M. Zych, F. Costa, I. Pikovski, T. C. Ralph and Č.B.,
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Nature Physics **11**, 668–672 (2015)

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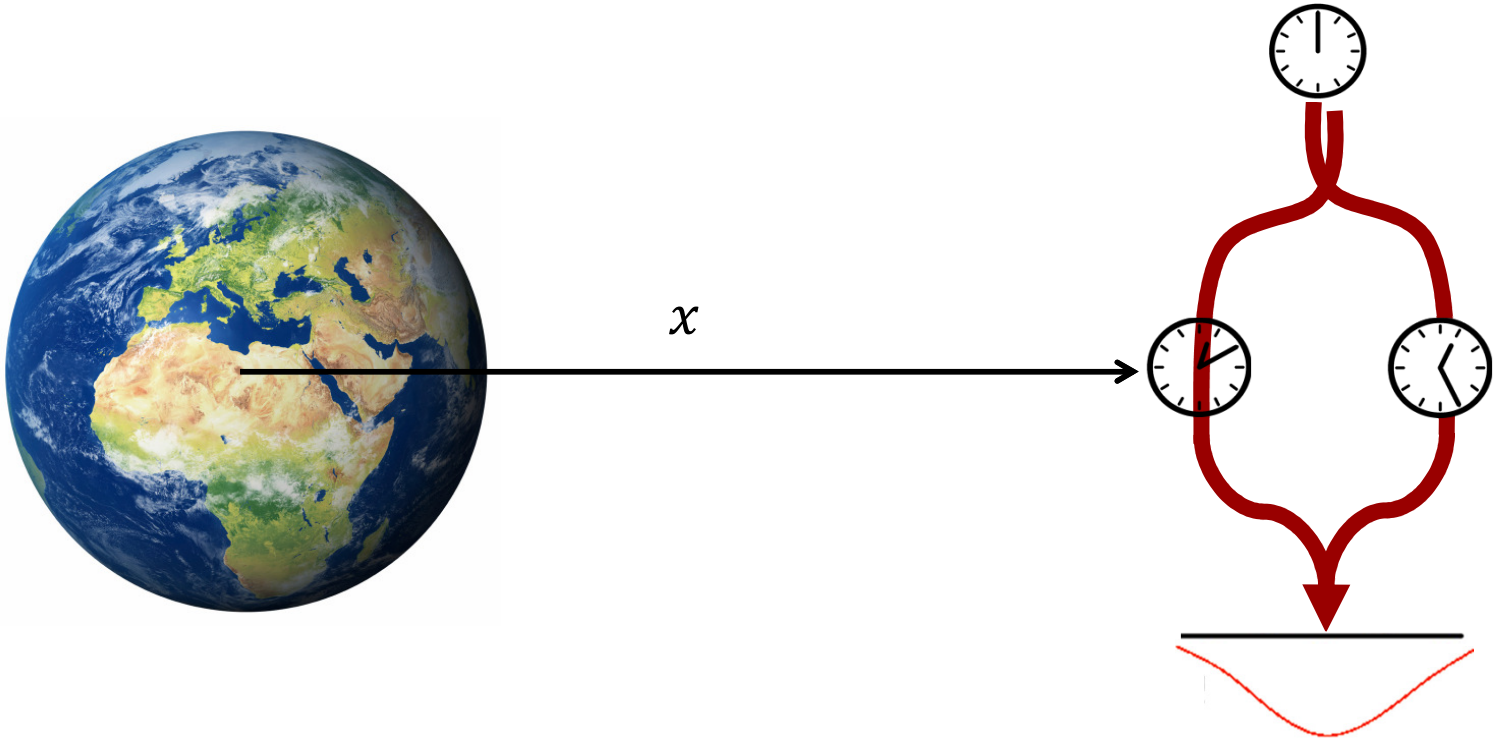

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