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Physics on the Boundary between Classical and Quantum Mechanics

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Approaches towards understanding quantum mechanics:

? Explain QM as an approximation to a deeper, underlying theory

or

? Accept QM as it is, but explain how it relates to physical reality

or

? Explain how QM can emerge as an exact statistical description of an underlying theory

Such an underlying deterministic theory (with “hidden variables”) must be *entirely different* from what we are used to.

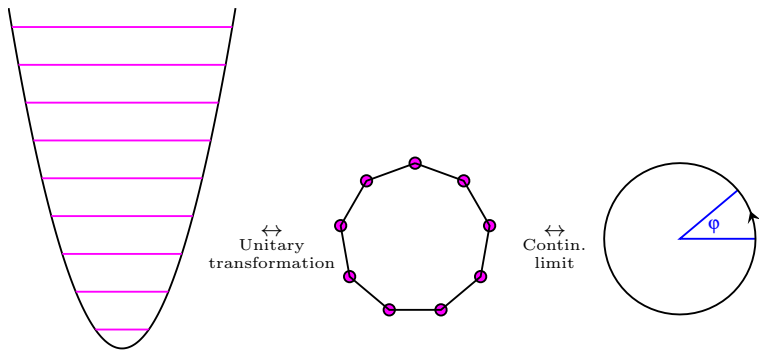
Difficulties: missing bounds on Lorentz boosts, divergences in space-time curvature, black hole information ...

To set the scene: **we have specific models that show classical and quantum behavior at the same time.**

Examples:

- quantum harmonic oscillator =
classical point moving around circle
- chiral Dirac fermion =
infinite plane moving with speed of light
- the bulk of a quantized superstring =
string on a lattice in transverse space

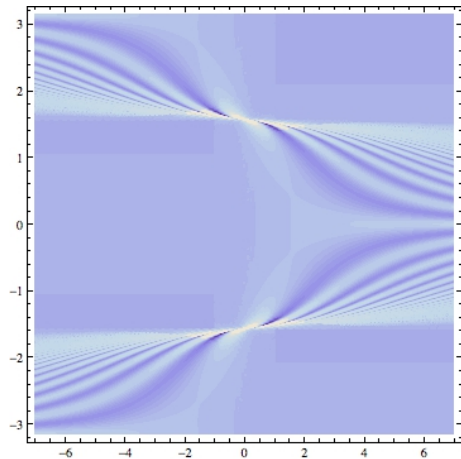
The harmonic oscillator.



Quantum Oscillator

\leftrightarrow

Classical periodic system



Mapping of
quantum oscillator
states $|x\rangle$ onto
periodic system
states $|\varphi\rangle$

$$\langle \varphi | x \rangle = \sum_{n=0}^{\infty} \frac{H_n(x) e^{-\frac{1}{2}x^2}}{2^{(n+1)/2} \pi^{3/4} \sqrt{n!}} e^{in\varphi}$$

Massless chiral fermions ($c = 1$). Let

$$H = \vec{\sigma} \cdot \vec{p}, \quad \hat{p} = \pm \frac{p}{|p|}, \quad s = (\hat{p} \cdot \vec{\sigma}), \quad r = \frac{1}{2}(\hat{p} \cdot \vec{x} + \vec{x} \cdot \hat{p}).$$

$$\frac{d}{dt} \vec{x} = -i[\vec{x}, H] = \vec{\sigma}, \quad \frac{d}{dt} \vec{p} = 0,$$

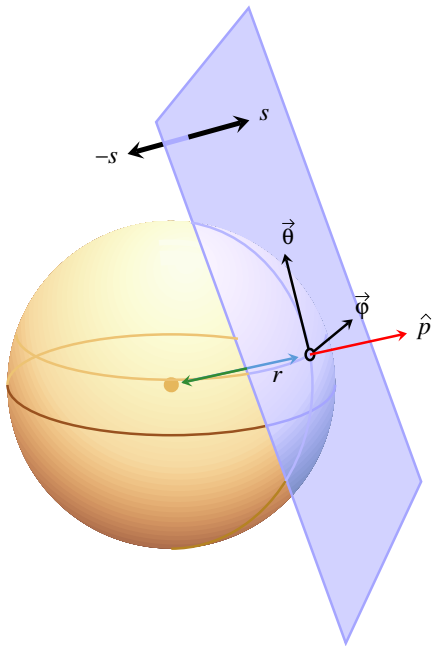
$$\frac{d}{dt} \sigma_i = 2\varepsilon_{ijk} p_j \sigma_k; \quad \frac{d}{dt} \hat{p} = 0;$$

$$\frac{d}{dt} (\hat{p} \cdot \vec{\sigma}) = \pm 2\varepsilon_{ijk} (p_i/|p|) p_j \sigma_k = 0,$$

$$\frac{d}{dt} (\hat{p} \cdot \vec{x}) = \hat{p} \cdot \vec{\sigma}, \quad \text{where } \hat{p} = \pm \frac{p}{|p|}, \quad \text{and so}$$

$$\frac{d}{dt} \hat{p} = 0, \quad \frac{d}{dt} s = 0, \quad \frac{d}{dt} r = s = \pm 1.$$

The neutrino sheet



$$\frac{d}{dt}\hat{p} = 0, \quad \frac{d}{dt}s = 0, \quad \frac{d}{dt}r = s = \pm 1.$$

This is a sheet, moving with speed $s = \pm 1$ in orthogonal direction.
 Mapping neutrino states \longleftrightarrow orthonormal sheet states:

$$\text{neutrino} \langle \vec{p}, \alpha | \hat{p}, p_r, s \rangle_{\text{sheet}} = p_r \delta^3(\vec{p} - \hat{p} p_r) \chi_{\alpha}^s(\hat{p}),$$

$$\text{neutrino} \langle \vec{x}, \alpha | \hat{p}, r, s \rangle_{\text{sheet}} = \frac{i}{2\pi} \delta'(r - \hat{p} \cdot \vec{x}) \chi_{\alpha}^s(\hat{p}).$$

Another example of a result:

$$x_i^{\text{neutrino}} = \hat{p}_i \left(r - \frac{i}{p_r} \right) + \varepsilon_{ijk} \hat{p}_j L_k^{\text{ont}} / p_r + \frac{1}{2p_r} \left(-\varphi_i s_1 + \theta_i s_2 + \frac{\hat{p}_3}{\sqrt{1-\hat{p}_3^2}} \varphi_i s_3 \right).$$

Here $s_3 = s$, $\vec{\theta}$ and $\vec{\varphi}$ are orthogonal vectors in sheet,
 s_1 and s_2 are spin flip operators.

Real numbers and integers

Imagine that, in contrast to appearances, the real world, at its most fundamental level, were *not* based on real numbers at all. We here consider systems where *only* integers describe what happens at a deeper level. Can one understand *why* our world *appears* to be based on real numbers?

A *mapping* exists of

<i>deterministic</i>		quantum physics
(or quantum) physics	onto	on N real observables
of a set of		q_i with N associated
$2N$ integers Q_i, P_i		momenta p_i

Canonical Variables. Our mapping replaces quantum operator sets p_i and q_i (with usual commutation relations) by sets of universally commuting integers P_i and Q_i .

Operators

Define $\epsilon \equiv e^{2\pi} = 535.5$

Consider a Hilbert space spanned by the states

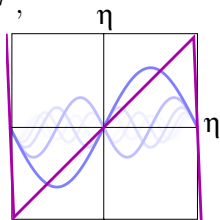
$|Q\rangle$, $Q = -\infty, \dots, -2, -1, 0, 1, 2, \dots, \infty$.

Introduce the *operator* η , on the interval $-\frac{1}{2} < \eta < \frac{1}{2}$, defined by:

$\epsilon^{iN\eta}|Q\rangle = |Q + N\rangle$, and Fourier transform the function η

$$\eta = \sum_N \epsilon^{iN\eta} \int_{-\frac{1}{2}}^{\frac{1}{2}} \eta d\eta \epsilon^{-iN\eta} = \sum_{N \neq 0} \frac{i(-1)^N}{2\pi N} \epsilon^{iN\eta},$$

$$\langle Q_1 | \eta | Q_2 \rangle = \frac{i}{2\pi} (1 - \delta_{Q_1 Q_2}) \frac{(-1)^{Q_1 - Q_2}}{Q_1 - Q_2}.$$



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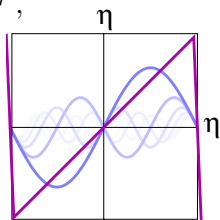
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$$Q_1 [|\eta Q, Q\rangle | Q_2\rangle = \frac{i}{2\pi} (\delta_{Q_1 Q_2} - (-1)^{Q_2 - Q_1}) = \frac{i}{2\pi} (\mathbb{I} - |\psi\rangle\langle\psi|).$$

Find that $[\eta, Q] = \frac{i}{2\pi} (\mathbb{I} - |\psi\rangle\langle\psi|)$. $|\psi\rangle$ is an *edge state*

$$\begin{array}{cccccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ -2 & -1 & 0 & 1 & 2 & 3 & & \end{array} = \text{circle with tick on left} \quad (\text{Fourier duality})$$

$$\text{circle with tick on left} = \text{horizontal line with tick on right}$$

$$\left(\begin{array}{cccccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ -2 & -1 & 0 & 1 & 2 & 3 & & \end{array} \right) \otimes \text{horizontal line with tick on right} = \text{horizontal line}$$

Make real number operators $-\infty < q < \infty$ as follows: $q = Q + \eta P$

There is a unitary transformation of states from one basis to another: $\langle Q, \eta P | \psi \rangle = \langle q | \psi \rangle$.

$$\text{Then transform } \langle Q, \eta P | \psi \rangle = \sum_{P=-\infty}^{\infty} \epsilon^{-iP\eta P} \langle Q, P | \psi \rangle = \langle q | \psi \rangle$$

$$\text{Alternatively, find the } p \text{ basis: } \langle q | p \rangle = \epsilon^{ipq}$$

Matrix elements

(mathematical detail is skipped here: make the mapping $P \leftrightarrow Q$ symmetric)

In Hilbert space $\{|Q, P\rangle\}$, we have

$$q = Q + a_Q \quad , \quad p = P + a_P \quad ,$$
$$\langle Q_1, P_1 | a_Q | Q_2, P_2 \rangle = \frac{(-1)^{P+Q+1} i P}{2\pi(P^2 + Q^2)}$$
$$\langle Q_1, P_1 | a_P | Q_2, P_2 \rangle = \frac{(-1)^{P+Q} i Q}{2\pi(P^2 + Q^2)} .$$

From these:

$$[q, p] = \frac{i}{2\pi}(1 - |\psi_{\text{edge}}\rangle\langle\psi_{\text{edge}}|), \quad \text{with} \quad \langle Q, P | \psi_{\text{edge}} \rangle = (-1)^{Q+P}$$

How does this work in QFT?

In ordinary QFT, the splitting $\phi(\vec{x}, t) \rightarrow Q(x, t) + \eta_P(x, t)$ does not survive the field equations, because splitting numbers into “integer part” and “fractional part” is **non-linear!** It does work with real numbers, if the field equations just *interchange* them. In 1+1 dimensions, we have *left movers* and *right movers*:

Free massless bosons in 1 + 1 dimensions

$$(\partial_x^2 - \partial_t^2)\phi(x, t) = (\partial_x + \partial_t)(\partial_x - \partial_t)\phi(x, t) = 0 \rightarrow \\ \phi(x, t) = \phi_L(x + t) + \phi_R(x - t) .$$

$$[\phi(x, t), p(y, t)] = \frac{i}{2\pi}\delta(x - y) ; \quad H = \pi \int dx (p(x)^2 + (\partial_x \phi)^2) .$$

Temporary: put x and t on a lattice.

$$\phi_{x,t} \equiv \phi(x, t) ; \quad [\phi_{x,t}, p_{x',t}] = \frac{i}{2\pi}\delta_{x,x'} .$$

We have: $\phi(x, t + a) + \phi(x, t - a) = \phi(x - a, t) + \phi(x + a, t) .$

How to *map* this model one-to-one on the cellular automaton:

$$Q(x, t + a) + Q(x, t - a) = Q(x - a, t) + Q(x + a, t) ,$$

where Q are integers.

$$p(x, t) = \frac{1}{2}a^L(x + t) + \frac{1}{2}a^R(x - t) .$$

$$a^L = p + \partial_x \phi ; \quad a^R = p - \partial_x \phi .$$

$$\text{Now,} \quad H = \frac{1}{2}(p^2 + (\partial_x \phi)^2) = \frac{1}{4}(a^{L2} + a^{R2}) ,$$

$$[a^L, a^R] = 0 ; \quad [a^L(x), a^L(y)] = [a^R(y), a^R(x)] = \frac{i}{\pi} \partial_x \delta(x - y) ;$$

Our cellular automaton will be on a lattice: $(x, t) \in \mathbb{Z}$. Therefore, replace commutator by

$$[\phi(x), p(y)] = \frac{i}{2\pi} \delta_{x,y} \quad (1)$$

$$[a^L(x), a^L(y)] = \pm \frac{i}{2\pi} \quad \text{if } y = x \pm 1 \quad .$$

Replace **real valued** operators $a^{L,R}(x)$ by **integer** valued operators $A^{L,R}(x)$ and their associated operators $\eta_A^{L,R}(x)$:

$$a^L(x) = A^L(x) + \eta_A^L(x - 1) .$$

This splitting survives the evolution law:

a^L , A^L , and η_A^L all move to the left, and a^R , A^R , and η_A^R move to the right.

Use the *quantum hamiltonian* (its space-lattice version) to describe the evolution of this classical automaton. $H = H^L + H^R$.

In *momentum space* :

$$H^L = \frac{1}{2} \int_0^{1/2} dk a^L(k) a^L(-k) M(k) \quad ; \quad M(\kappa) = \frac{\pi \kappa}{\sin(2\pi \kappa)} .$$

This hamiltonian turns $a^L(x)$ into a pure left-mover, and $a^R(x)$ into a right-mover:

$$A^L(x+t) = Q(x, t+1) - Q(x-1, t)$$

The (super) string is a 1+1 dimensional theory.

Here, the quantized field is the set of (super) string coordinates. They are now replaced by the integer valued left- and right-movers $A^{L,R}(x \pm t)$.

Re-inserting the units gives a surprise: these coordinates form a discrete lattice with lattice length a that is independent of the lattice chosen on the world sheet. Even if you send the world sheet to a continuum, the space-time lattice length a is

$$a = 2\pi\sqrt{\alpha'}$$

Furthermore, as we will see later, the string constant ρ is not freely adjustable.

Fermions

A fermionic system can be handled the same way. Assume a Majorana fermionic field ψ_A with $\psi_A = \psi_A^\dagger$, $A = 1, 2$ (or, $A = L, R$). Dirac equation: $(\gamma_+ \partial_- + \gamma_- \partial_+) \psi = 0$.

$$\text{One finds that } \psi_A^\mu(x, t) = \begin{pmatrix} \psi_L^\mu(x+t) \\ \psi_R^\mu(x-t) \end{pmatrix} .$$

The corresponding classical theory now has **Boolean degrees of freedom**, $\sigma(x, t) = \pm 1$, obeying the equations:

$$\sigma(x, t+1) = \sigma(x-1, t) \sigma(x+1, t) \sigma(x, t-1) .$$

This also splits up into left- and right-movers:

$$\sigma(x, t) = \sigma_L(x+t) \sigma_R(x-t) .$$

Superstring theories contain $D - 2$ independent bosonic fields (coordinates) and $D - 2$ Majorana fermion species. All these can be mapped onto **deterministic** models processing integers as well as ± 1 's (Boolean variables) **classically**.

So-far, we only handled strings of infinite length. We need to add: (periodic) **boundary conditions**, **interactions**, and **constraints**. The constraints give us the remaining two *longitudinal* coordinates, needed to investigate *Lorentz invariance*.

The constraints only need to be imposed on the quantum side of the theory, as is done in superstrings.

As is standard in Superstring theory, this restricts us to $D = 10$.

In **Superstring Theory**, both bosons and fermions obey gauge conditions and constraints, which should determine ψ_A^\pm in terms of $a_{L,R}^{\text{tr}}$, and so also $\sigma_{L,R}^0$ and $\sigma_{L,R}^{D-1}$ should be determined by the transverse $\sigma_{L,R}^a$

The quantum – classical mapping in string theory is not free of problems:

How do the *longitudinal* modes $A_{L,R}^{\pm}(x, t)$ behave in the deterministic model?

String theory wants us to pick a gauge such as $A_{L,R}^{+}(x, t) = 1$.
Then

$$A_{L,R}^{-}(x, t) = \sum_{i=1}^{D-2} A_{L,R}^{i,2}(x, t) \rightarrow A_{L,R}^{-}(x, t) \geq 1 ,$$

which does allow us to use X^{+} as our time coordinate, but violates Lorentz invariance.

How to do this better: is there a better gauge? What is then our time coordinate?

We conclude: *What we know is that the bulk of a superstring is a deterministic, classical system.*

Bell's inequalities

Theorem (Bell):

In any deterministic theory intended to reproduce quantum behavior, (for instance when Einstein-Podolsky-Rosen photons are observed through two spacelike separated filters, \vec{a} and \vec{b}), one will have to allow superluminal signals between \vec{a} and \vec{b} .

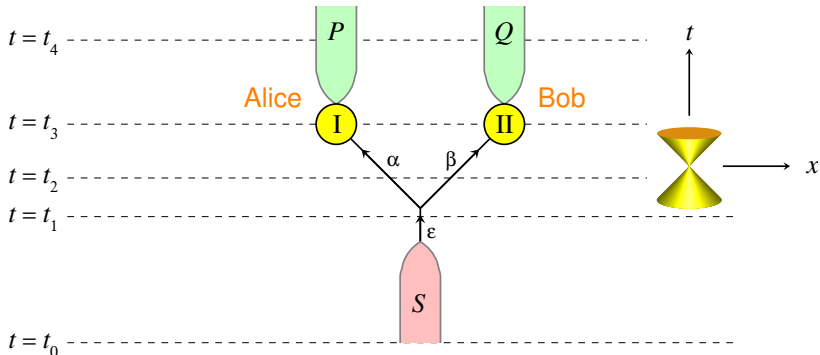
... since we should be allowed to modify the settings \vec{a} and/or \vec{b} any time, at free will...

But there is no “free will” in a deterministic theory (Super-determinism).

Theorem: even so, you cannot avoid Bell's inequalities!

unless you accept “ridiculous fine-tuning”, or “conspiracy”

Today's claim: we never need actual signals going backwards in time or faster than light. ... but the problem is more basic than Bell's. What we do need ...



In the Bell experiment, at $t = t_0$, one must demand that those degrees of freedom that later force Alice and Bob to make their decisions, and the source that emits two entangled particles, have **3 - body correlations of the form**

$$\langle a b c \rangle \propto |\sin(a - b - c)|$$

(or worse)

What we do need is

non-locally correlated vacuum fluctuations.

Vacuum fluctuations are ubiquitous in QFT vacua.

And we need:

a conservation law for the ontological identity of states:

ontological states evolve into ontological states

Issue: This means that photons observed going through a polarization filter are ontological. *Therefore* with that polarization, they always were ontological in the past.

This is *more elementary* than the discussion of entangled states in Bell's inequality.

Is this “conspiracy”? The ontological nature of a physical state is *conserved in time*. If a photon is observed, at late times, to be in a given polarization state, it has been in *exactly the same state* the moment it was emitted by the source. *The conspiracy argument now demands that the “ontological basis” be unobservable!* (as it is in string theory)

Shut up and calculate!

THE END

arXiv: 1204.4926

arXiv: 1205.4107

arXiv: 1207.3612

and to be published.

PS

The ultimate model of nature will be very unconventional

It will have to solve many problems:

- Closure of the Lorentz group
- Black hole information
- Hierarchy

But humanity is smart — we will figure it out . . .

If I am correct:

No measurement problem

Collapse automatic

Born probabilities automatic