# Elements of Physics for the 21st century

Werner A Hofer
Department of Physics
The University of Liverpool

www.wernerhofer.eu

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# Internal structures of electrons and photons: The concept of ex

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**Abstract**

The theoretical foundations of quantum
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seem to be related to the existing infinity


*PACS*: 03.65.Bz; 03.70; 03.75; 14.60.Cd

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RESEARCH ARTICLE

2012

# Solving the Einstein–Podolsky–Rosen puzzle: The origin of non-locality in Aspect-type experiments

**Werner A. Hofer**

*Department of Physics, University of Liverpool, L69 3BX Liverpool, UK*

*E-mail: whofer@liverpool.ac.uk*
*Received May 2, 2012; accepted June 7, 2012*

So far no mechanism is known, which could connect the two measurements in an Aspect-type experiment. Here, we suggest such a mechanism, based on the phase of a photon's field during propagation. We show that two polarization measurements are correlated, even if no signal passes from one point of measurement to the other. The non-local connection of a photon pair is the result of its origin at a common source, where the two fields acquire a well defined phase difference. Therefore, it is not actually a non-local effect in any conventional sense. We expect that the model and the detailed analysis it allows will have a major impact on quantum cryptography and quantum computation.

**Keywords** entanglement, Bell inequalities, coincidence measurements, Einstein–Podolsky–Rosen paradox

**PACS numbers** 03.65.Yd, 03.67.-a

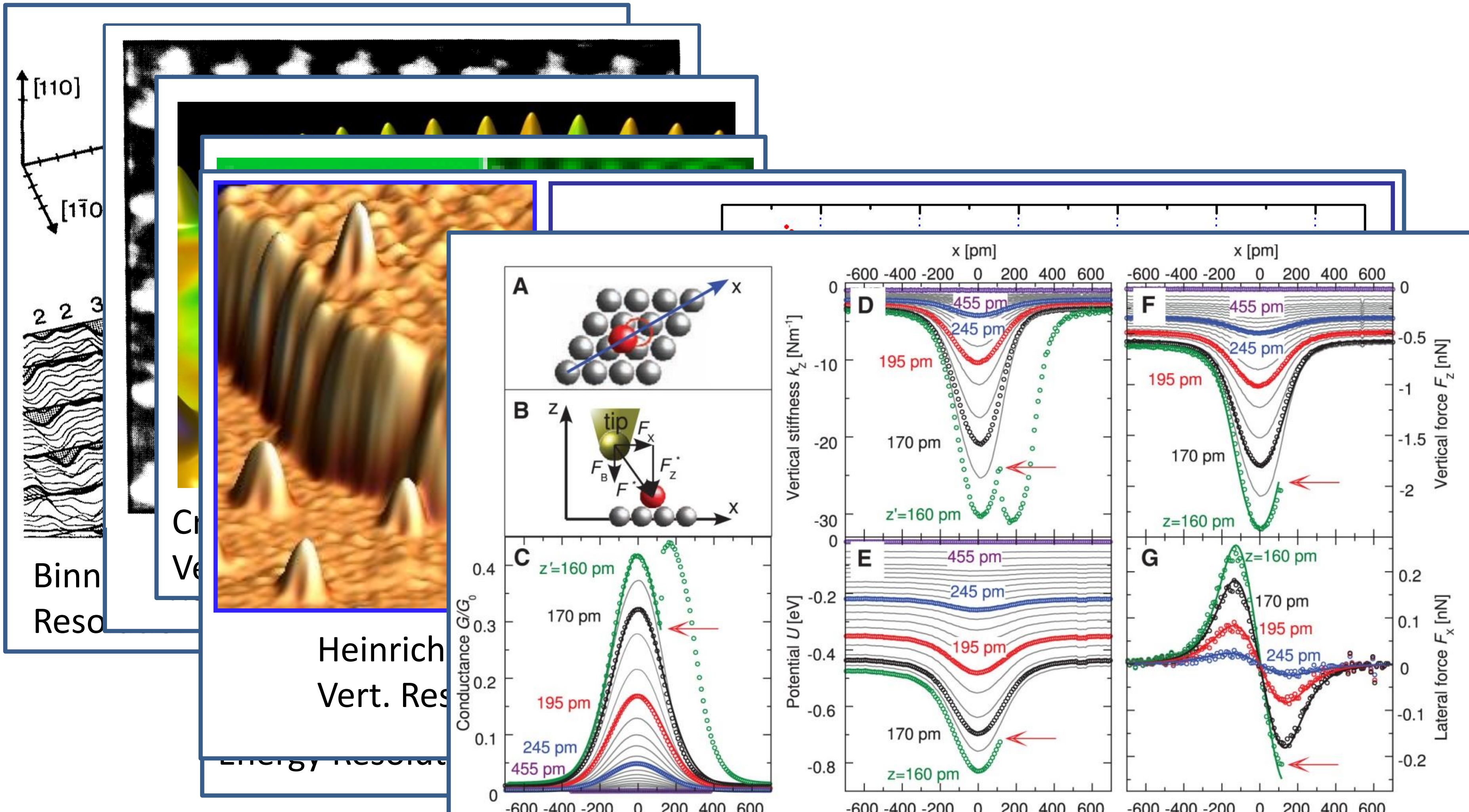
# Overview

- The problem
- Densities and wavefunctions
- Experiments:
  - Accelerations of electrons
  - Stern Gerlach experiments
  - Double slit interferometry
  - Bell-type experiments
- Towards a nuclear model based on densities

$$\Delta p \cdot \Delta x \geq \frac{\hbar}{2}$$

- Implies:
  - That electrons are point particles ?
  - That their electrostatic energy is infinite ?
  - That wavefunctions do not have physical reality ?
  - That measurements cannot be arbitrarily precise

# Introducing: Scanning probe microscopy

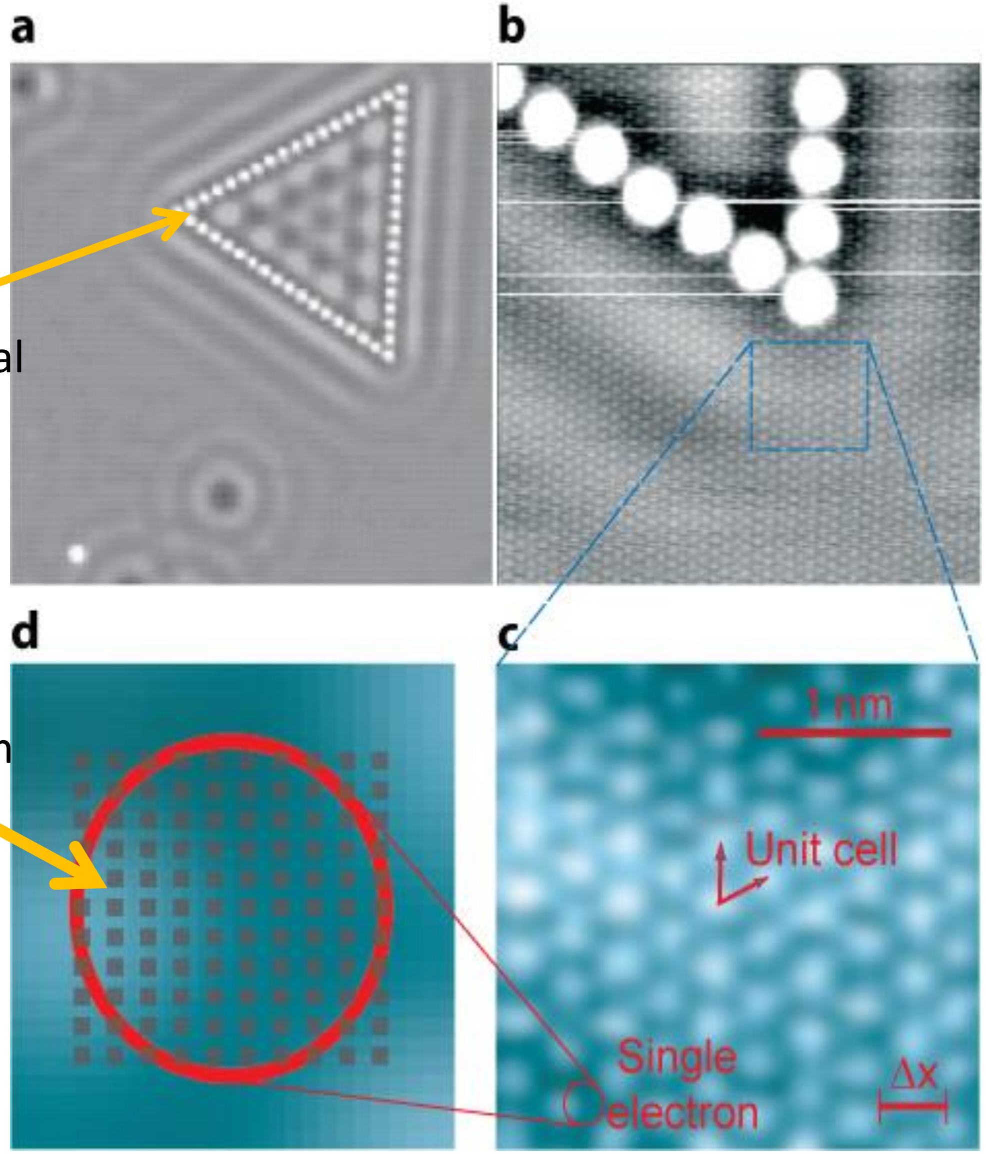



What the scanning probe measures in all these experiments is the two dimensional distribution of the density of electron charge.

# The Ag(111) surface with adatoms

Quantum Corral
Ag on Ag(111)

Pixel: ~ 20pm




## Numbers:

Temperature [1]: 5K

Distance of atoms:
290pm

Wigner-Seitz radius:
106pm

Band energy at $E_F$:
80meV

Maximum energy ΔE:
80meV

Lateral resolution:
20pm

Vertical precision [2]:
0.05pm


[1] Rieder, Phil. Trans. Roy.Soc. A 362, 1207 (2004)    [2] Gawronski, Science 319, 930 (2008)

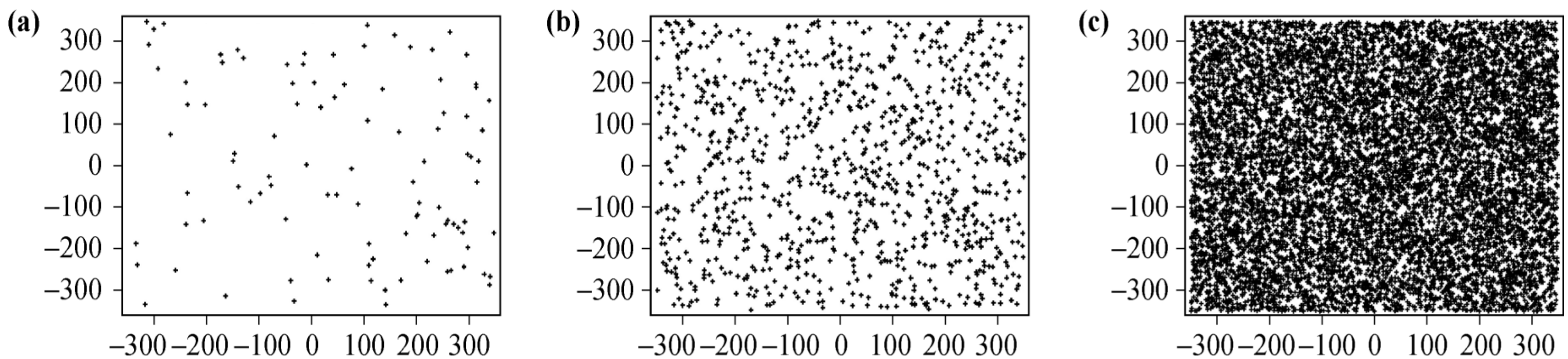
# Maximum energy and local uncertainty

- The maximum available energy for an electron at the Fermi level is the band energy of 80meV:

$$\Delta p = p(80meV) = \sqrt{2mE} = 1.53\times10^{-25} kgms^{-1}$$

$$\Delta x = \Delta x(\Delta p) \geq \frac{\hbar}{2\cdot\Delta p} = 3.48\times10^{-10} m = 348\, pm$$

- Statistical distribution of measurements of point-like electrons with this local uncertainty:




100 1000 10000

# Statistical analysis of experiments

- Standard deviation of a measurement of x-coordinate:

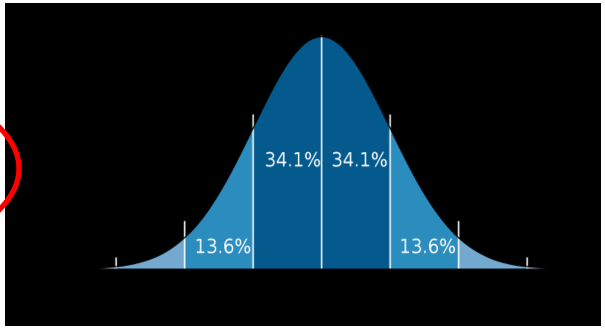
$$\sigma_x = \sqrt{\frac{1}{N}\sum_{i=1,\cdots,N}\left(x_i - \langle x\rangle\right)^2}, \quad \langle x\rangle = \frac{1}{N}\sum_{i=1,\cdots,N} x_i \ (1)$$

- Maximum standard deviation:

0.1pm

$$\frac{\Delta z}{z_0} \leqslant 0.3\% \quad \Rightarrow \quad \Delta x(\mathrm{STM}) \geqslant 3\sigma(\Delta E)$$

30pm




- Result for surface state electron on Ag(111):

$$\Delta x(STM) \approx 10\,pm \qquad 3\sigma(\Delta E) \approx 800\,pm$$

Distribution last slide: σ = 280pm

$$\Rightarrow \Delta x(STM) << 3\sigma(\Delta E)$$

Contradiction!

# Result of the statistical analysis:

- Under the assumptions that
  1. The electron is a point-particle
  2. The uncertainty of location is related to the uncertainty of er
- We find
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Front. Phys., 2012, 7(2): 218–222
DOI 10.1007/s11467-012-0246-z

## Heisenberg, uncertainty, and the scanning tunneling microscope

**Werner A. Hofer**

*Department of Physics, University of Liverpool, L69 3BX Liverpool, UK*

*E-mail: whofer@liverpool.ac.uk*

*Received January 9, 2012; accepted February 10, 2012*

We show by a statistical analysis of high-resolution scanning tunneling microscopy (STM) experiments, that the interpretation of the density of electron charge as a statistical quantity leads to a conflict with the Heisenberg uncertainty principle. Given the precision in these experiments we find that the uncertainty principle would be violated by close to two orders of magnitude, if this interpretation were correct. We are thus forced to conclude that the density of electron charge is a physically real, i.e., in principle precisely measurable quantity.

**Keywords** scanning tunneling microscope, electron charge, density functional theory, uncertainty relations

**PACS numbers** 31.10.+z, 71.15.Mb

# Wavefunctions and densities

Erwin Schrödinger
Born Vienna 1887

Walter Kohn
Born Vienna 1923

1. A system is fully defined by its wavefunction of electrons.
2. A system is fully defined by its density of electron charge.

# One principle: the density of electrons is real

$$\rho(z,t) = \frac{\rho\ldots}{2\ldots}$$

$$\lim_{v_{el}\to 0} \rho = \rho\ldots$$

$$\mathcal{E} = \mathbf{e}_1 \mathcal{E}_0 \ldots$$

$$\mathcal{H} = \mathbf{e}_2 \mathcal{H}_0 \ldots$$

Note: the vector … y
variations of d… e
velocity of the e… s
spin density.

Found Phys (2011) 41: 754–791
DOI 10.1007/s10701-010-9517-0

## Unconventional Approach to Orbital-Free Density Functional Theory Derived from a Model of Extended Electrons

**Werner A. Hofer**

Received: 16 March 2010 / Accepted: 2 November 2010 / Published online: 12 November 2010


**Abstract** An equation proposed by Levy, Perdew and Sahni (Phys. Rev. A 30:2745, 1984) is an orbital-free formulation of density functional theory. However, this equation describes a bosonic system. Here, we analyze on a very fundamental level, how this equation could be extended to yield a formulation for a general fermionic distribution of charge and spin. This analysis starts at the level of single electrons and with the question, how spin actually comes into a charge distribution in a non-relativistic model. To this end we present a space-time model of extended electrons, which is formulated in terms of geometric algebra. Wave properties of the electron are referred

# Wavefunctions ψ

$$\mathcal{S} = \mathcal{E}\mathcal{H} = \mathbf{e}_1\mathbf{e}_2\mathcal{E}_0\mathcal{H}_0 \sin^2\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right).$$

$$\mathcal{S} = i\mathbf{e}_3 S,$$

$$S = \mathcal{E}_0\mathcal{H}_0 \sin^2\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right),$$

$$S = S_0 \sin^2\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right), \quad S_0 \equiv \mathcal{E}_0\mathcal{H}_0.$$

Spin component

$$\psi = \rho^{1/2} + S^{1/2}\mathbf{e}_1\mathbf{e}_2 = \rho^{1/2} + iS^{1/2}\mathbf{e}_3.$$

$$\psi^\dagger = \rho^{1/2} + S^{1/2}\mathbf{e}_2\mathbf{e}_1 = \rho^{1/2} - iS^{1/2}\mathbf{e}_3.$$

General form of a wavefunction

$$\psi^\dagger\psi = \rho + S = \rho_0 = constant$$

Born rule

1. The group velocity is equal to the velocity of the electron (de Broglie)
2. The frequency of the wave is proportional to the kinetic energy (Planck)
3. The total energy is the energy of its inertial mass (classical mechanics)

# Many-body physics

Many-body Wavefunction

$$\Psi = \rho^{1/2} + i\mathbf{e}_S S^{1/2},$$

Schrodinger Equation

$$\left[-\frac{1}{2}\nabla^2 + v_{ext} + v_{\mathit{eff},0} + i\mathbf{e}_v v_i\right]\Psi = \mu\Psi,$$

Density

$$\rho_0 = \Psi^\dagger \Psi,$$

Spin density

$$\Lambda(\mathbf{r}) = \mathbf{e}_S S^{1/2},$$

Bivector potential

$$\Pi(\mathbf{r}) = \mathbf{e}_v v_i\,,$$

Many body interactions

$$v_0(\mathbf{r}) = v_{ext} + v_{\mathit{eff},0},$$

Coupled Schrodinger equations

$$\left[-\frac{1}{2}\nabla^2 + v_0(\mathbf{r}) - \mu\right]\rho^{1/2}(\mathbf{r}) = \Pi(\mathbf{r})\cdot\Lambda(\mathbf{r}),$$

$$\mathbf{r}) \times \Lambda(\mathbf{r}).$$

Advantage: the many-body wavefunction in this case has only 4 instead of 3N variables

# Energy changes

- What happens if an electron accelerates (decelerates) in a static field?
  1. Its velocity will change
  2. Its density distribution will change
  3. Its field components will change
  4. The external field will change due to energy transfer
- Comprehensive description:

'Local' Ehrenfest theorem

$$\mathbf{f} = -\nabla\phi = \rho_0 \frac{d\mathbf{v}_{el}}{dt}.$$

- Internal changes:

$$\rho + S = \rho_0 = constant.$$

$$\dot{S} + \dot{\rho} = 0 \quad \rightarrow \quad \frac{d}{dt}(\psi_S^\dagger \psi_S)v = \rho_0 \frac{dv_{el}}{dt}.$$

Energy is shifted from mass components to field components

**Wavelength changes because of energy redistribution**

# Electrons in static magnetic fields

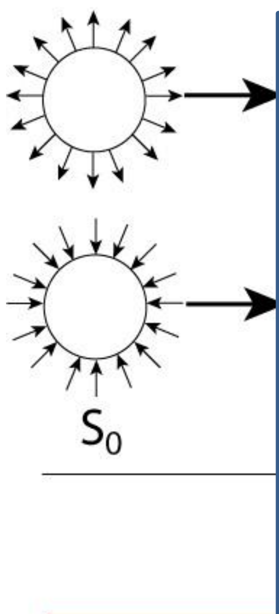
$$\rho_0 \frac{d\mathbf{v}}{dt} = \rho_0 \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right).$$

Free electrons: Lorentz forces

$$\mathbf{S} = \mathbf{e}_S \cdot S \qquad \frac{d\mathbf{e}_s}{dt} = \text{const} \cdot \mathbf{e}_S \times \left( \mathbf{u} \times \frac{d\mathbf{B}}{dt} \right)$$

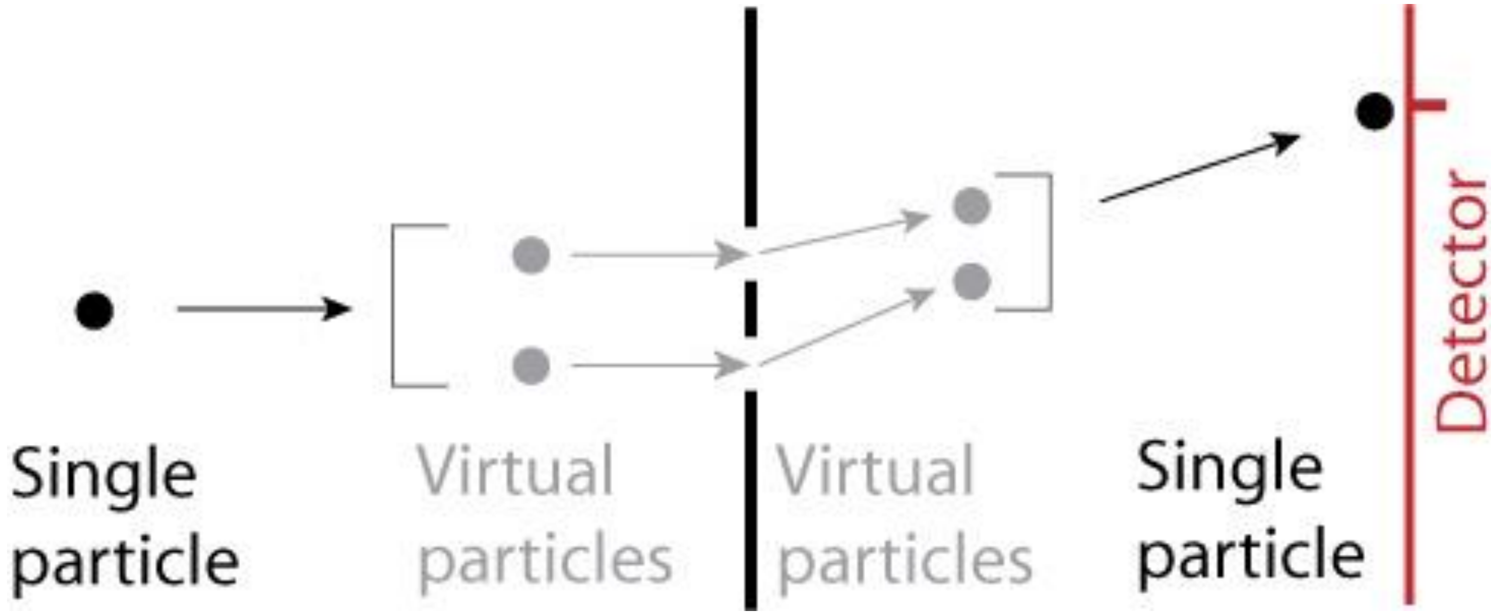
Constrained trajectory: rotation of spin vectors

Stern-Gerlach experiments:




- Standard model:
  - Spin is isotropic
  - Measurement breaks spin isotropy
  - No process to explain symmetry breaking
- New model:
  - Spin is isotropic
  - Measurement induces spin components aligned to field
  - Measurement measures induced spin components

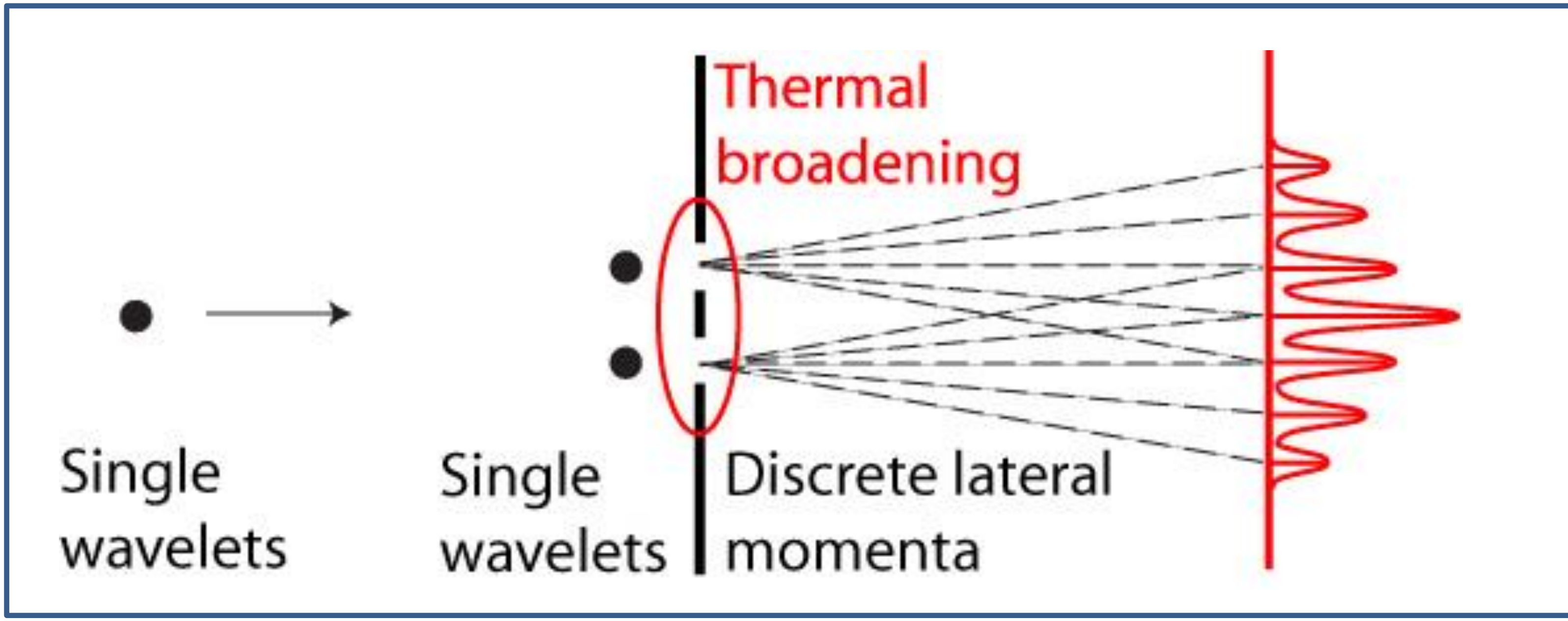
# Double slit experiments




- Conventional model (Feynman path integrals)
  - A single particle splits into virtual particles
  - Each virtual particle passes one slit

The problem: No physical process is known which could account for the creation of virtual particles and their recombination after the interferometer.

# Model due to Duane and Lande [1,2]




- Duane-Lande model:
  - Single wavelets interact with the slit system
  - The interaction spectrum of the interferometer is discrete
  - Wavelets acquire discrete lateral momenta

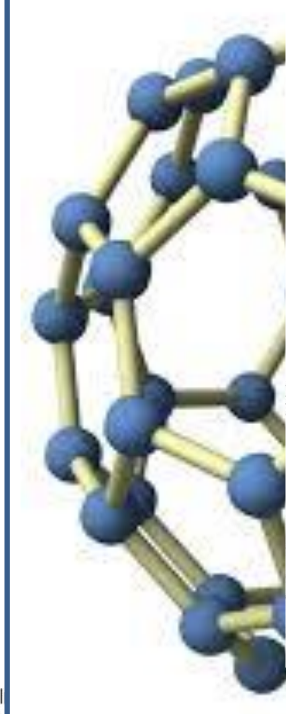
Note: The slit environment is composed of atoms in a regular crystal lattice. Such a system always has a discrete interaction spectrum depending on the chemical element and the crystal symmetry.

  - Perform interference experiments at 4K

[1] W. Duane, Proc. Nat. Acad. Sci. 9, 158 (1923);
[2] A Lande, From Dualism to Unity in Quantum Physics, CUP (1960)

# Large molecules

BERNINGER *et al.*



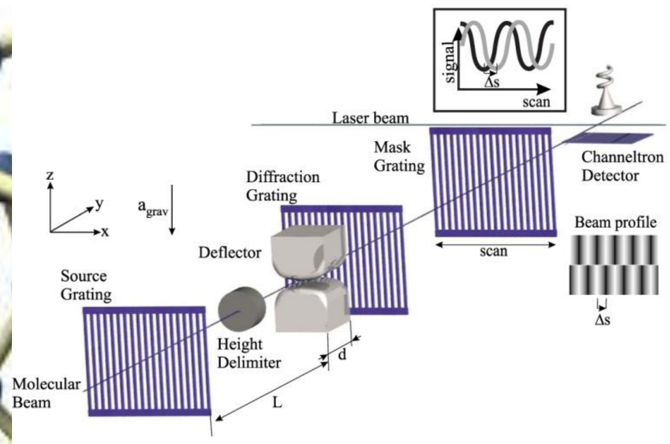



FIG. 1. (Color online) Experimental setup. The three gratings are used for coherence preparation, diffraction, and detection. Path-dependent matter-wave phase shifts in the external field lead to a deflection of the beam profile at the position of the mask grating. The lateral shift $\Delta s$ of the interference fringes at the detector is directly proportional to the scalar molecular polarizability $\alpha$.

**Figure 2** | (open circl… zeroth and first-order maxima can be clearly seen. Details of the theory are discussed in the text. **b**, The molecular beam profile without the grating in the path of the molecules.

Quantum fiction: wave properties of large molecules[1]

PHYSICAL REVIEW A **76**, 013607 (2007)

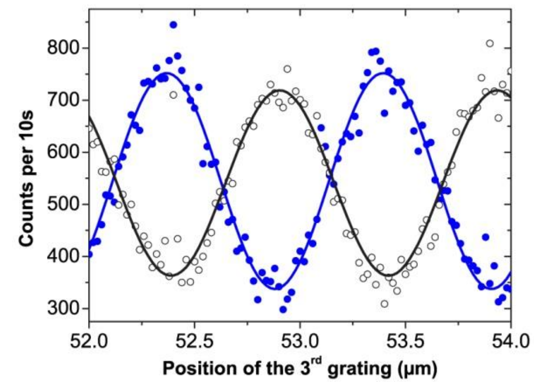



FIG. 2. (Color online) Deflection of a $C_{60}$ beam with $\bar{v}=117$ m/s and $\sigma_v=8\%$. A phase shift of $\Delta\phi=\pi$ is obtained at a voltage of 6 kV (full circles). The open circles represent the reference at $U=0$ kV.

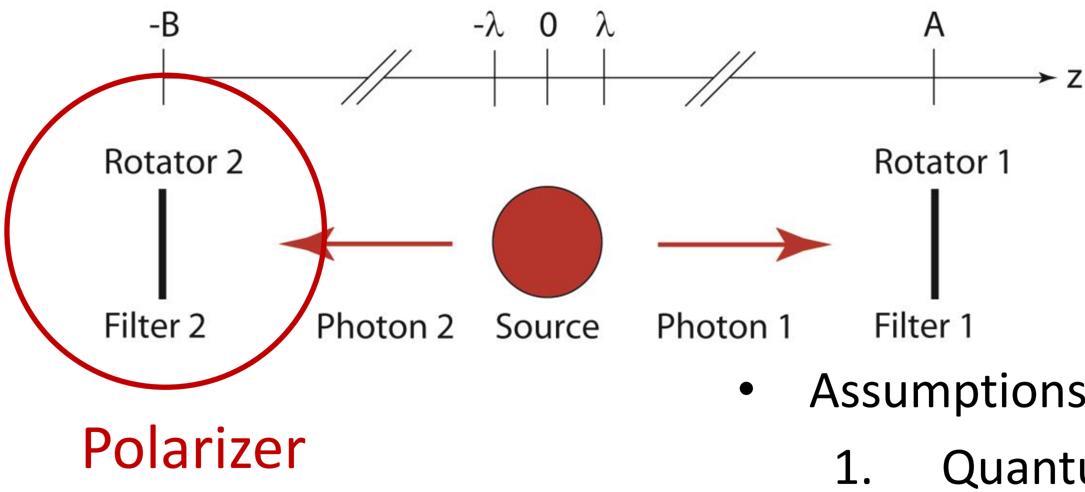
To monitor and numerically compensate for drifts, an additional reference point (with $U=0$ kV) is always included before and after each high-voltage deflection scan. From the interference curves thus obtained we extract the voltage de-

Quantum fact: fringes due to interactions between molecule and slit environment[2]

[1]Zeilinger group: Nature 401, 680 (1999)

[2]Berninger et al.: PRA 76, 013607 (2007)

# Aspect-type experiments




- Assumptions:
  1. Quantum theory is complete
  2. Correlation due to common source

$$
\begin{aligned}
R(z_1) &= \exp(\mathbf{e}_1\mathbf{e}_2)\,\mathbf{e}_3 z_1\, 2\pi/\lambda \\
R(z_2) &= \exp-(\mathbf{e}_1\mathbf{e}_2)\,\mathbf{e}_3 z_2\, 2\pi/\lambda \\
R(\varphi_1) &= \exp i z_1\, 2\pi/\lambda = e^{i\varphi_1} \\
R(\varphi_2) &= \exp -i\,(z_2\, 2\pi/\lambda + \Delta) = e^{-i(\varphi_2+\Delta)}
\end{aligned}
$$

Rotation

$$
p(\varphi_i) = (\Re\,(R(\varphi_i)))^2
$$

$$
p\,(\varphi_1, \varphi_2) = [\Re\,[R(\varphi_1)\cdot R(\varphi_2)]]^2
$$

Filtering

Polarizer

$$
\begin{aligned}
C^{++} &= C^{--} = \cos^2(\varphi_1 - \varphi_2) \\
C^{+-} &= C^{-+} = 1 - \cos^2(\varphi_1 - \varphi_2)
\end{aligned}
$$

$$
E(\varphi_1, \varphi_2) = 2\cos^2(\varphi_1 - \varphi_2) - 1 = \cos 2\,(\varphi_1 - \varphi_2)
$$

Coincidences

Aspect 1982

# Experimental results and Bell inequalities

$$\varphi_1 = 0, \varphi_1' = 45, \varphi_2 = 22.5, \varphi_2' = 67.5,$$

$S(\varphi_1,$

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RESEARCH ARTICLE

## Solving the Einstein–Podolsky–Rosen puzzle: The origin of non-locality in Aspect-type experiments

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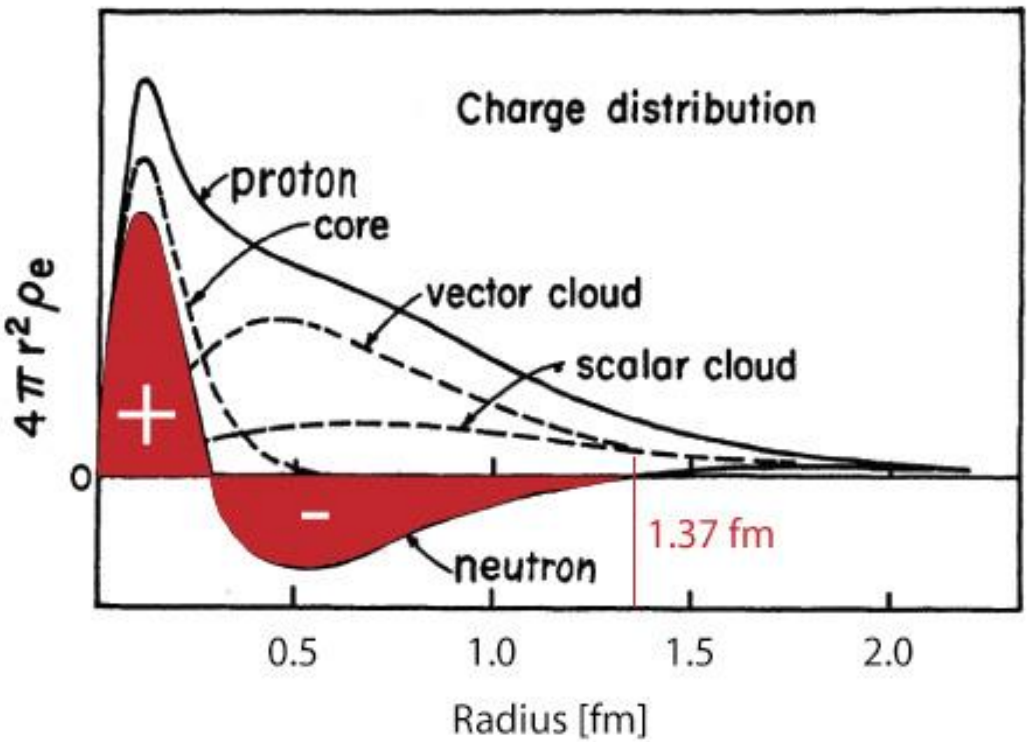
**Keywords** entanglement, Bell inequalities, coincidence measurements, Einstein–Podolsky–Rosen paradox

**PACS numbers** 03.65.Yd, 03.67.-a

# Electrons and neutrons

$n^0 \rightarrow p^+ + e^- + 0.782$ MeV

Scattering on neutrons[1]:




Question: Is there a high-density phase of electrons in nuclei?

[1] Littauer et al. Phys. Rev. Lett. 7, 144 (1961)

# Energetics

$$W_e^0 = \frac{1}{2}\int_{\infty}^{r_e} \frac{1}{\epsilon_0}|\mathbf{E}|^2 dV = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_e} \approx 11eV$$

Electrostatic field:
hydrogen electron

$$W_e^n = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n} \approx 0.960MeV$$

Electrostatic field:
neutron electron (1.5fm)

The electrostatic field energy of a high density electron is sufficient to explain the excess mass of a neutron

$$\rho(r) = \rho_0 e^{-2\alpha r} \qquad \psi(r) = \sqrt{\rho_0}e^{-\alpha r}$$

$$\left(-\frac{\hbar^2}{2m}\Delta - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}\right)\psi(r) = E\psi(r)$$

Density and wavefunction

Schrödinger equation

$$\left(\frac{-\hbar^2\alpha^2}{2m} + \frac{2\hbar^2\alpha}{2mr} - \frac{e^2}{4\pi\epsilon_0 r}\right)\psi(r) = E\psi(r)$$

$$\frac{2\hbar^2\alpha}{2mr} - \frac{e^2}{4\pi\epsilon_0 r} = 0 \qquad \Rightarrow \qquad \alpha = \frac{me^2}{4\pi\epsilon_0\hbar^2}$$

Characteristic equation

Solution

# Length scales and energy scales

$$\alpha = \frac{2.101 \times 10^{-58}}{\hbar^2} = (\hbar = 6.672 \times 10^{-34})$$
$$= 1.89 \times 10^{10} \left[m^{-1}\right]$$

Length scale in atomic physics: depends on Planck's constant

$$\hbar_n = x\hbar \qquad \alpha_n = \frac{1.89 \times 10^{10}}{x^2} \left[m^{-1}\right]$$

Constants in nuclear environment ($a_0$= Bohr radius)

$$\psi(r) = \sqrt{\rho_0} e^{-\alpha_n r}$$

$$a_n = x^2 \cdot a_0 \qquad E_n = \frac{E_0}{x^2}$$

The decay length, the unit of length and the unit of energy all scale differently in a nuclear environment

$$\left(-\frac{1}{2}\nabla^2 - \frac{1}{r}\right)\psi(r) = E\psi(r)$$

Schrödinger equation: nuclear units

$$W_n = \frac{e^2}{4\pi\epsilon_o a_0}\left(-\frac{1}{2x^2} + \frac{a_0}{r_n}\right) = \left(\frac{a_0}{r_n} - \frac{1}{2x^2}\right) \times 27.211[\text{eV}]$$

Total energy

The total energy depends on the neutron radius and the scaling factor x

# Fine structure constant and unit energy

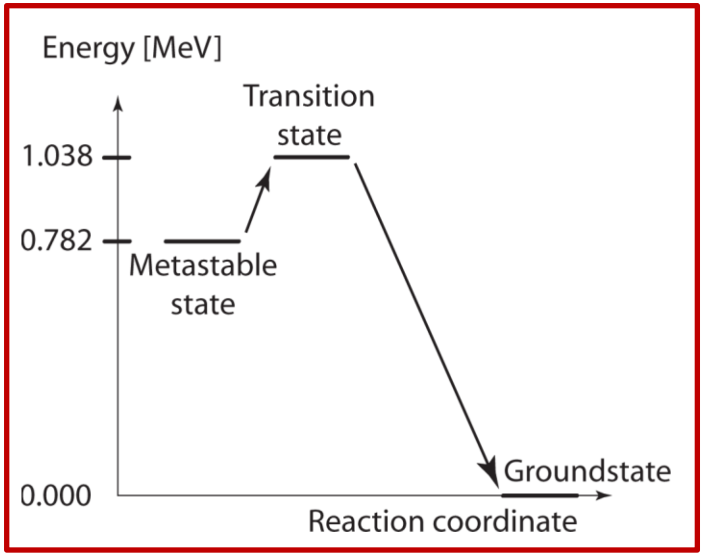
$r_n$ = 1.37 fm:

$$x^2 = \frac{1}{18779} = \alpha_f^2$$

$\alpha_f$ equal to the scaling factor of nuclear units.

$$a_n = \frac{a_0}{18779} = 2.81 \quad [\mathrm{fm}]$$
$$E_n = E_0 \times 18779 = 0.511 \quad [\mathrm{MeV}] = m_e c^2$$

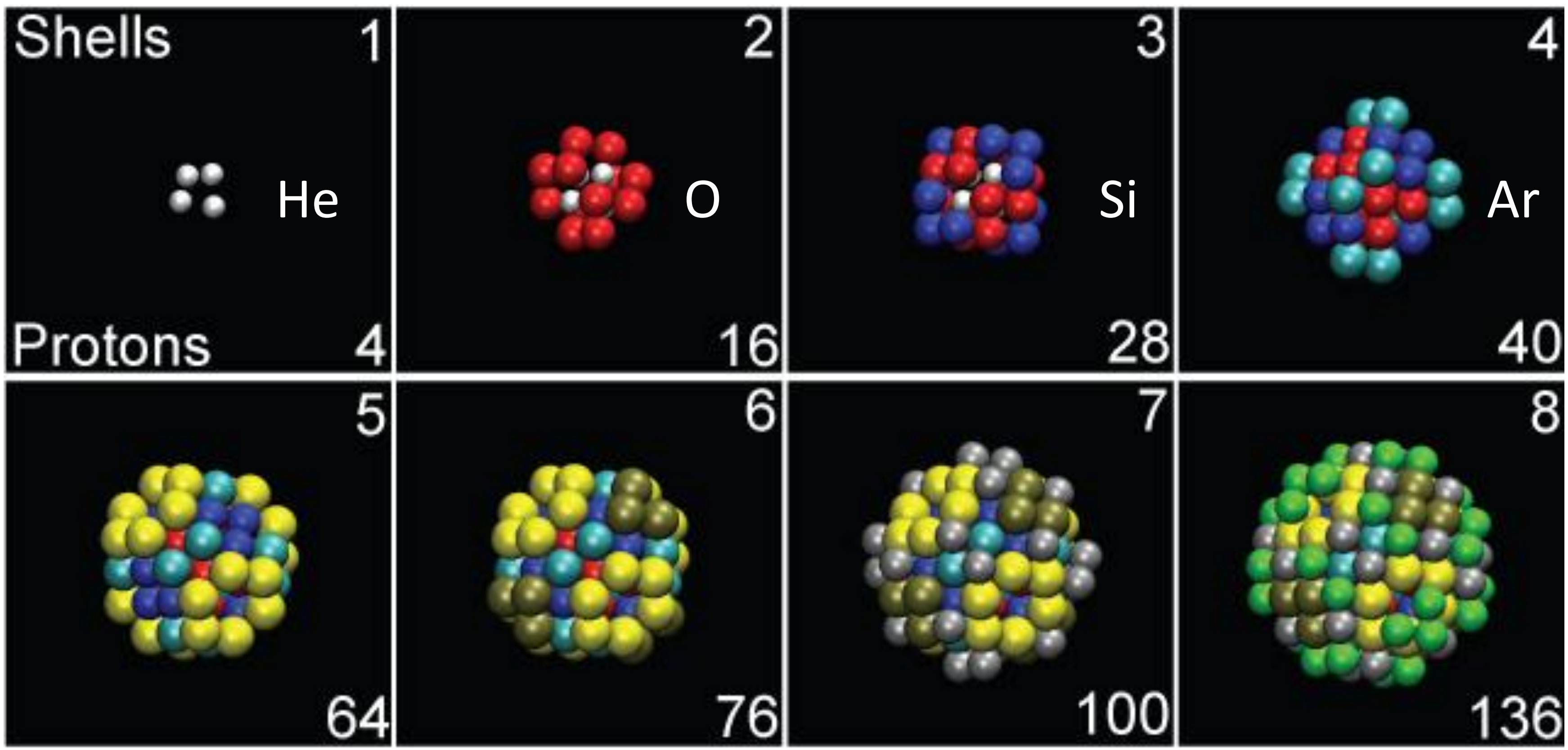
Energy unit equal to the rest mass of electrons.




Neutron:
Energetics

# Nuclear shell model

- If neutrons are composite entities then
  - Nuclei are composed of protons and electrons
  - Protons are immersed in negative charge
  - Inter-proton distances are equal



# Six things to remember

1. The uncertainty relations are violated by up to two orders of magnitude in thousands of experiments every single day.

2. Wavefunctions themselves are not real, but their components, mass and spin densities, are real.

3. Rotations in space generate complex numbers, which are not described in a Gibbs vector algebra.

4. Double slit interference experiments most likely show two features: a discrete interaction spectrum with the slit system and a thermal broadening due to environmental conditions.

5. Based on the experimental neutron radius we are led to conclude that the fine structure constant describes the nuclear energy scale.

6. Closed shell nuclei could be due to the geometrical arrangement of nuclear protons.

# Three things I have learned

- Mathematics is not physics
  - Mathematical models must be based on a sound understanding of physical processes
  - Proper mathematicians will invent a new reality if the existing one contradicts their theorems
- It is easy to come up with complicated models
  - It is much more difficult to develop simple ones
- If it’s weird, it is probably wrong


Thank you for your attention

# Electrons in electrostatic fields

$$\frac{v_{el}^2}{2} = \omega_{el} = \omega_{ph} - \phi_m .$$

Photoelectric effect: (Hertz effect, 1887): the kinetic energy of an electron subjected to electromagnetic fields of frequency $\omega_{ph}$ is reduced by the electrostatic interaction within the metal $\phi_m$.

$$\omega_{el}(\phi_m) = \omega_{el}^0 - \phi_m .$$

Physical process: the frequency of an electron is reduced in the presence of an electrostatic field.

Consequence: Modification of Schrodinger equation

$$-\frac{1}{2}\nabla^2 \psi_S = \frac{v_{el}^2}{2}\psi_S = \omega_0 \psi_S,$$

$$i\frac{\partial}{\partial t}\psi_S = (\omega_0 - \phi_m)\psi_S,$$

$$i\frac{\partial \psi_S}{\partial t} = \left[-\frac{1}{2}\nabla^2 + V\right]\psi_S .$$

# Energy and wavelength

$$E_{field} = \frac{1}{2}\epsilon_0 \mathcal{E}^2 + \frac{1}{2}\mu_0 \mathcal{H}^2$$
$$= \left(\frac{1}{2}\epsilon_0 \mathcal{E}_0^2 + \frac{1}{2}\mu_0 \mathcal{H}_0^2\right)\cos^2\left(\frac{2\pi}{\lambda}z - 2\pi\nu t + \phi\right).$$

$$\phi = \frac{\pi}{2} \quad \Rightarrow \quad E_{field} = \left(\frac{1}{2}\epsilon_0 \mathcal{E}_0^2 + \frac{1}{2}\mu_0 \mathcal{H}_0^2\right)\sin^2\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right)$$

$$E_{kin} = \frac{1}{4}\rho_0 v_{el}^2\left[1 + 2\cos^2\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right) - 1\right].$$

$$E_{tot} = \frac{1}{4}\rho_0 v_{el}^2 + \frac{1}{4}\rho_0 v_{el}^2\left[2\cos^2\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right) + 2\sin^2\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right) - 1\right]$$
$$= \frac{1}{2}\rho_0 v_{el}^2.$$

Total energy density

$$v_g = \frac{d\omega}{dk} = \frac{d(mv_{el}^2/2\hbar)}{d(mv_{el}/\hbar)} = v_{el}.$$

Group velocity

1. The group velocity is equal to the velocity of the electron (de Broglie)
2. The frequency of the wave is proportional to the kinetic energy (Planck)
3. The total energy is the energy of its inertial mass (classical mechanics)

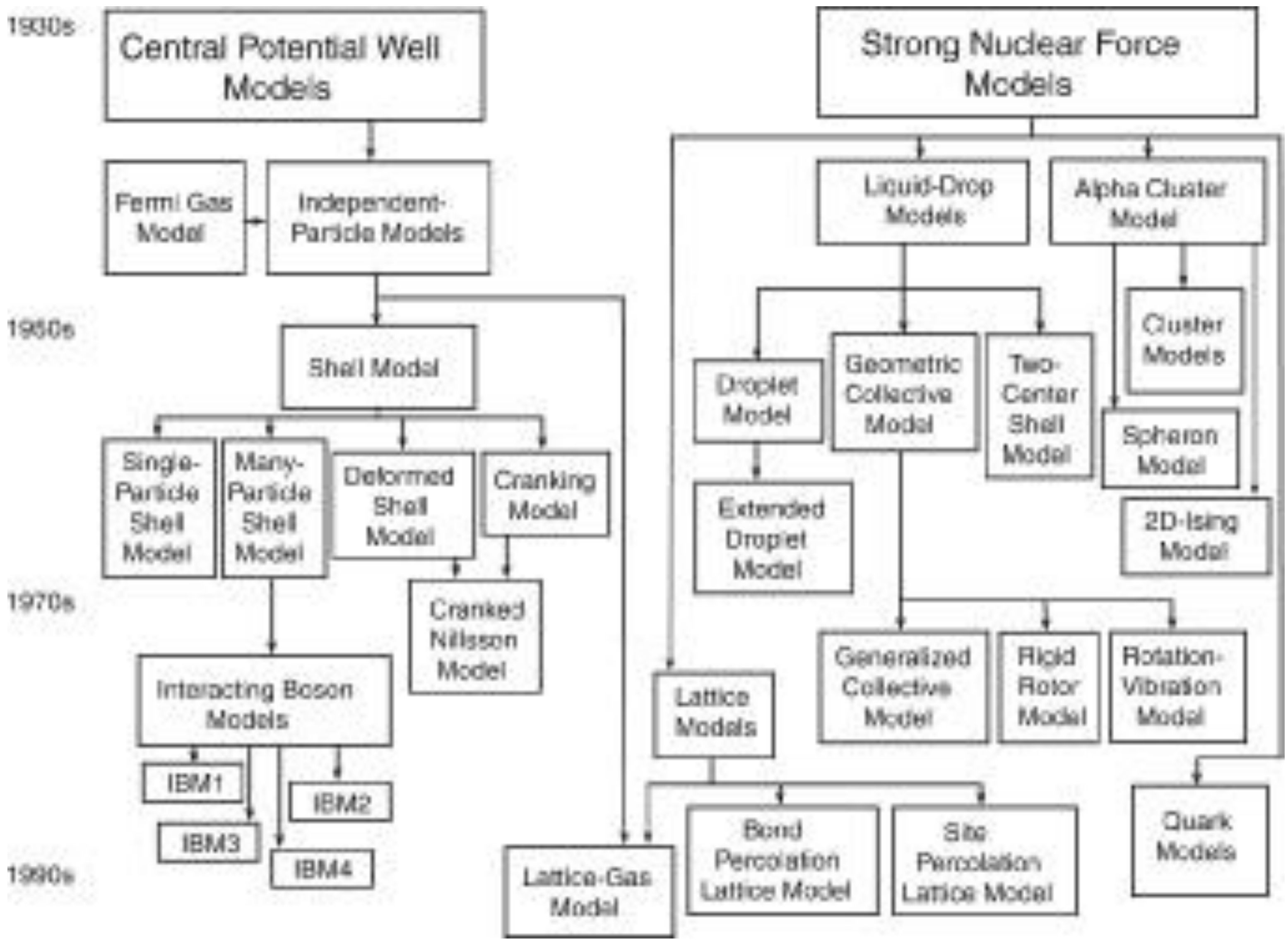
# If the density of electron charge is a statistical quantity, then

- A measurement of location is a measurement of a statistical ensemble.
- And this statistical ensemble must comply with the uncertainty relations:

J. L. Park and H. Margenau, Int. J. Theor. Phys., 1968, 3(3): 211

See in particular page 213, "Many gedankenexperiments have been designed to illustrate Heisenberg's famous law; unfortunately, the false impression is often conveyed that his principle, which is actually a theorem about standard deviations in collectives of measurement results, imposes restrictions on *measur-ability*." italics in the original text.

# Existing nuclear models[1]




[1]Norman D. Cook, Models of the Atomic Nucleus, Springer (2010)