Neutron Optical Studies of Fundamental Phonemana in Quantum Mechnics

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I. Introduction: neutron interferometer & polarimeter

II. Uncertainty relation for error-disturbance

V. Summary

The neutron

Particle

Feels four-forces

Wave

$m = 1.674928(1) \times 10^{-27}$ kg

$s = \frac{1}{2}\hbar$

$\mu = -9.6491783(18) \times 10^{-27}$ J/T

$\tau = 887(2)$ s

$R = 0.7$ fm

$\alpha = 12.0(2.5) \times 10^{-4}$ fm^3

u - d - d - quark structure

CONNECTION

de Broglie

$$\lambda_B = \frac{h}{m.v}$$

Schrödinger

$$H\psi(\bar{r},t) = i\hbar\frac{\delta\psi(\bar{r},t)}{\delta t}$$

&

boundary conditions

$$\lambda_c = \frac{h}{m.c} = 1.319695(20) \times 10^{-15} \text{ m}$$

For thermal neutrons
= 2 Å, 2000 m/s, 20meV

$$\lambda_B = \frac{h}{m.v} = 1.8 \times 10^{-10} \text{ m}$$

$$\Delta_c = \frac{1}{2\delta k} \cong 10^{-8} \text{ m}$$

$$\Delta_p = v.\Delta t \cong 10^{-2} \text{ m}$$

$$\Delta_d = v.\tau = 1.942(5) \times 10^{6} \text{ m}$$

$$0 \leq \chi \leq 2\pi\ (4\pi)$$

m ... mass, s ... spin, μ ... magnetic moment, τ ... β-decay lifetime, R ... (magnetic) confine-ment radius, α ... electric polarizability; all other measured quantities like electric charge, magnetic monopole and electric dipole moment are com-patible with zero

$-\mu B$ ________________

μB ________________

two level system

λ_c ... Compton wavelength, λ_B ... deBroglie wavelength, Δ_c ... coherence length, Δ_p ... packet length, Δ_d ... decay length, δk.... momentum width, Δt ... chopper opening time, v ... group velocity, χ ... ... phase.

Neutrons in quantum mechanics

Particle and wave properties

p = mv = h/λ

(L. De Broglie)

Schroedinger equation

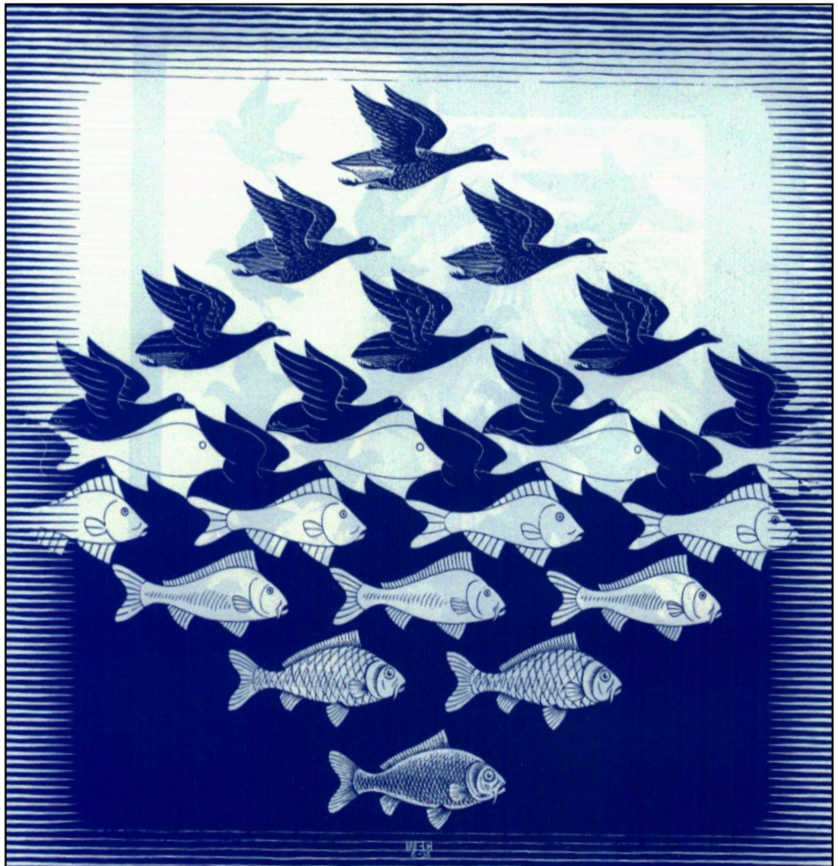
$$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = H\Psi(\vec{r},t)$$

(E. Schrödinger)

Uncertainity

Δx Δp ≥ h/4π

(W. Heisenberg)



M.C.Escher, 1938

Neutron interferometry

Neutrons

$m = 1.67 \times 10^{-27}$ kg

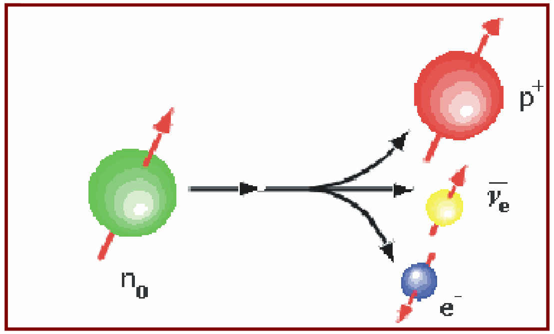
$s = \frac{1}{2}\hbar$

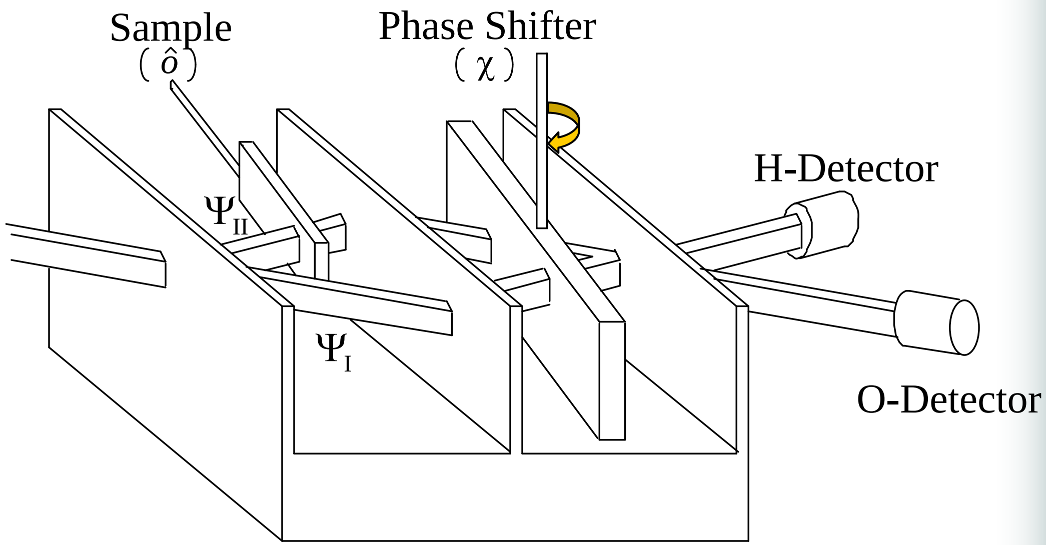
$\mu = -9.66 \times 10^{-27}$ J/T

$\tau = 887$ s

$R = 0.7$ fm

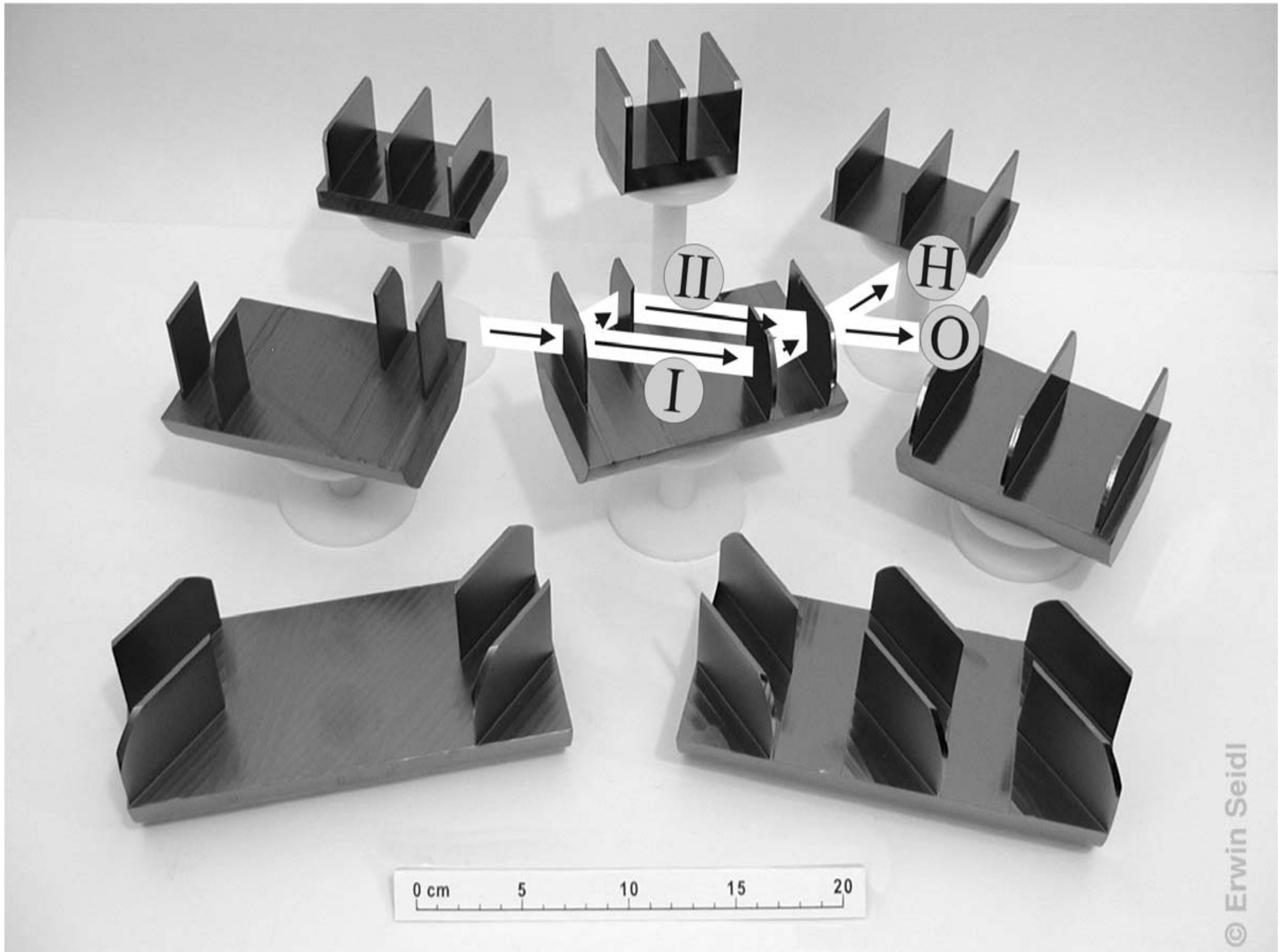
u–d–d quark structure





$$I = |\Psi_I + e^{i\chi} \cdot \hat{o} \cdot \Psi_{II}|^2$$

Neutron interferometer family



Two-particle vs. two-space entanglement

2-Particle Bell-State

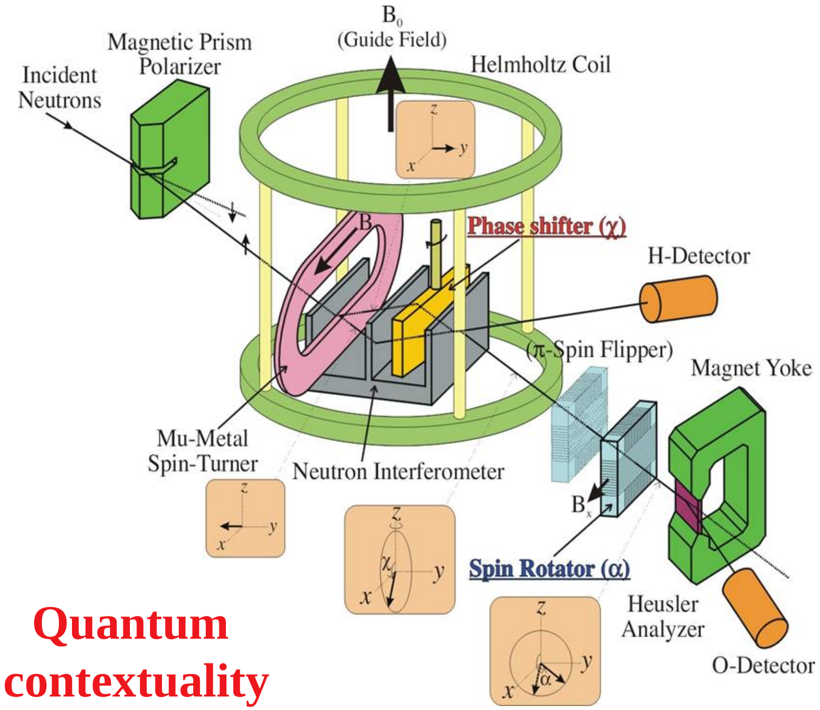
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{I}\otimes|\downarrow\rangle_{II} + |\downarrow\rangle_{I}\otimes|\uparrow\rangle_{II}\}$$

I, II represent 2-Particles

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{s}\otimes|\text{I}\rangle_{p} + |\downarrow\rangle_{s}\otimes|\text{II}\rangle_{p}\}$$

s, p represent 2-Spaces, e.g., spin & path



Quantum contextuality

Violation of Bell-like inequality

$S' \equiv E'(\alpha_1,\chi_1) + E'(\alpha_1,\chi_2) - E'(\alpha_2,\chi_1) + E'(\alpha_2,\chi_2)$

$= \mathbf{2.051 \pm 0.019} \quad > \mathbf{2}$

Hasegawa et al., Nature2003, NJP2011

Kochen-Specker-like contradiction 1

$$E_x \cdot E_y = 0.407 \xleftrightarrow{63\%} E' \equiv \langle \hat{X}_1\hat{Y}_2 \cdot \hat{Y}_1\hat{X}_2 \rangle = -0.861$$

Hasegawa et al., PRL2006/2009

Tri-partite entanglement (GHZ-state)

$$\left|\Psi_{\text{Neutron}}\right\rangle = \{|\Psi_{\text{I}}\rangle\otimes|\uparrow\rangle\otimes|\Psi(E_0)\rangle + (e^{i\chi}|\Psi_{\text{II}}\rangle)\otimes(e^{i\alpha}|\downarrow\rangle)\otimes(e^{i\gamma}|\Psi(E_0+\hbar\omega_r)\rangle)\}$$

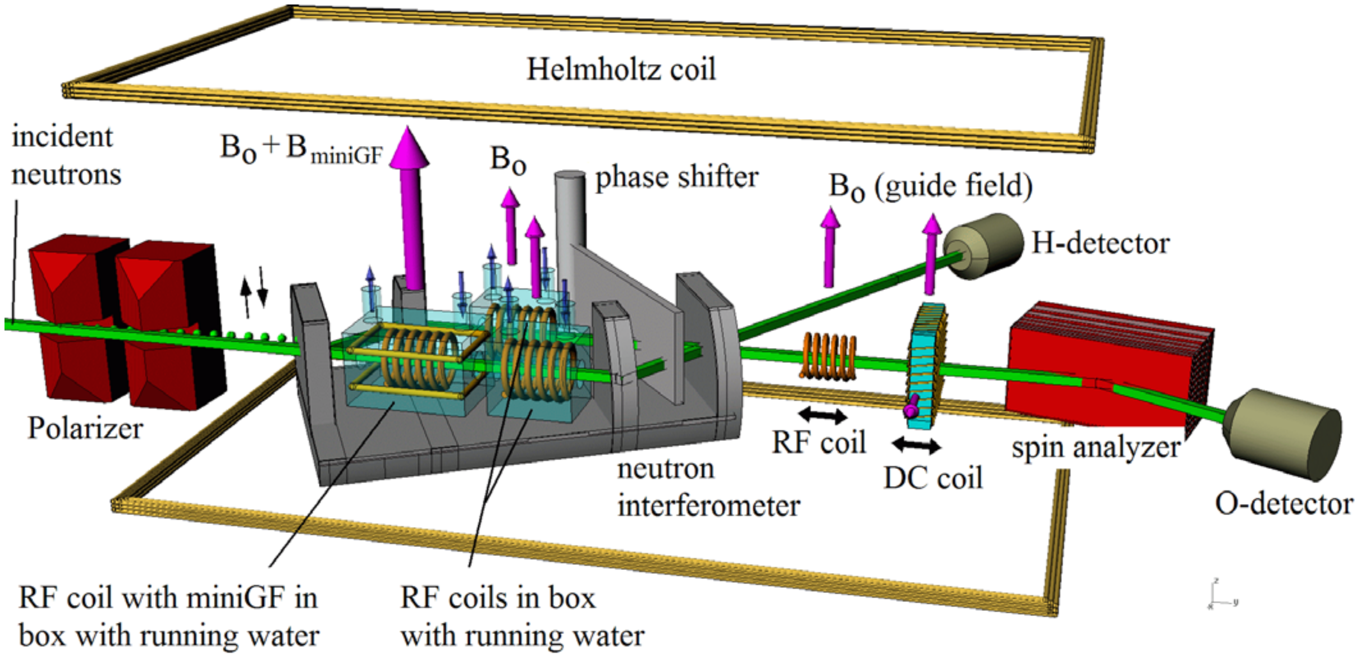
$$M_{Measured} = 2.558 \pm 0.004 \quad > 2$$

Hasegawa et al., PRA2010

W- and GHZ- states in a single neutron system

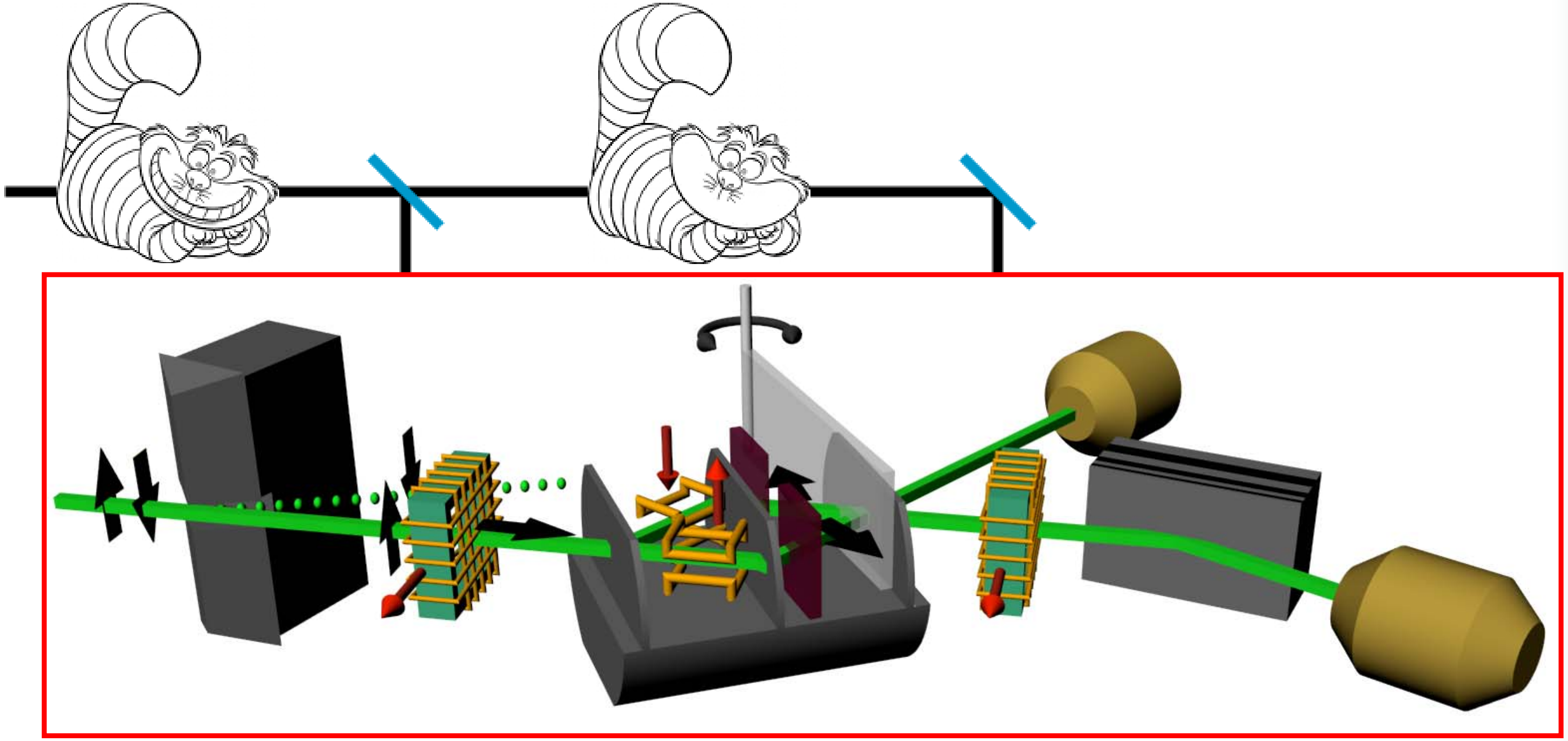
W-state: $$|\Psi\rangle_W = \frac{1}{\sqrt{3}} \cdot |\mathrm{II} \downarrow \hbar\omega\rangle + \frac{1}{\sqrt{3}} \cdot |\mathrm{I} \uparrow \hbar\omega\rangle + \frac{1}{\sqrt{3}} \cdot |\mathrm{I} \downarrow 2\hbar\omega\rangle$$

GHZ-state: $$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}} \cdot |\mathrm{II} \downarrow \hbar\omega\rangle + \frac{1}{\sqrt{2}} \cdot |\mathrm{I} \uparrow 0\rangle$$



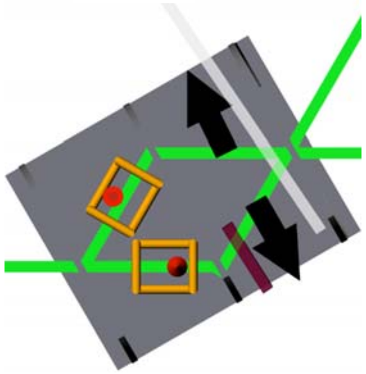
D. Erdösi et al. New J. Phys. 15 (2013) 023033

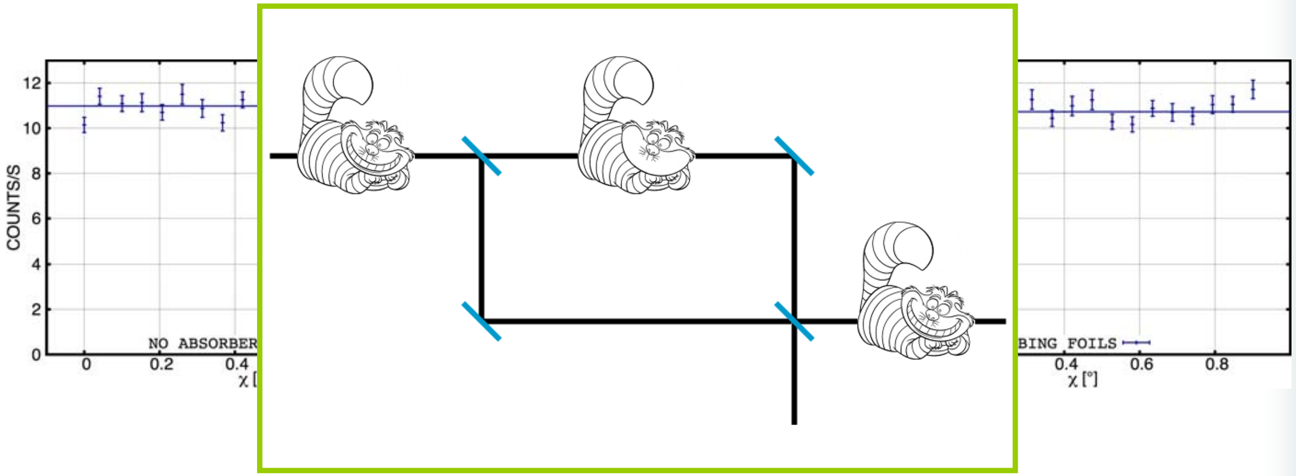
Cheshire Cat 1: paradoxical behavior of neutrons



T. Denkmayr et al., to be published

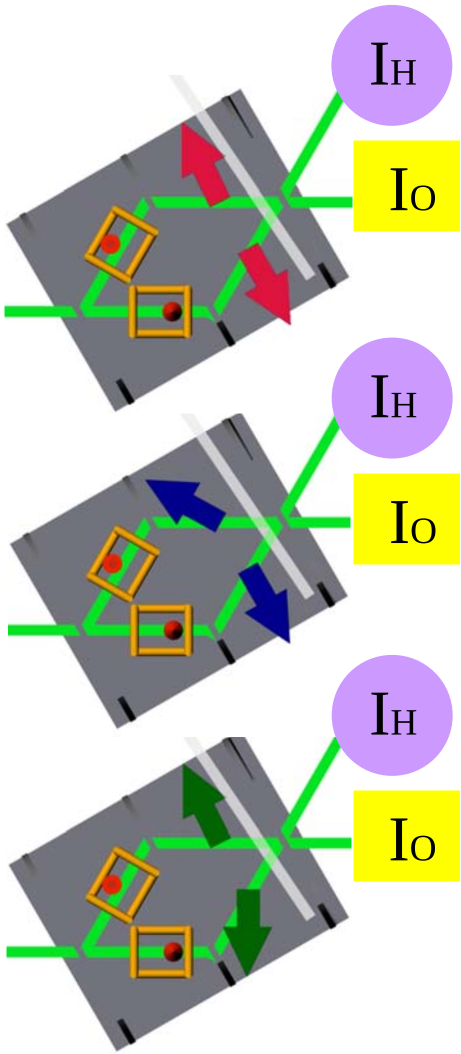
Cheshire Cat 2: neutron(cat) in upper path

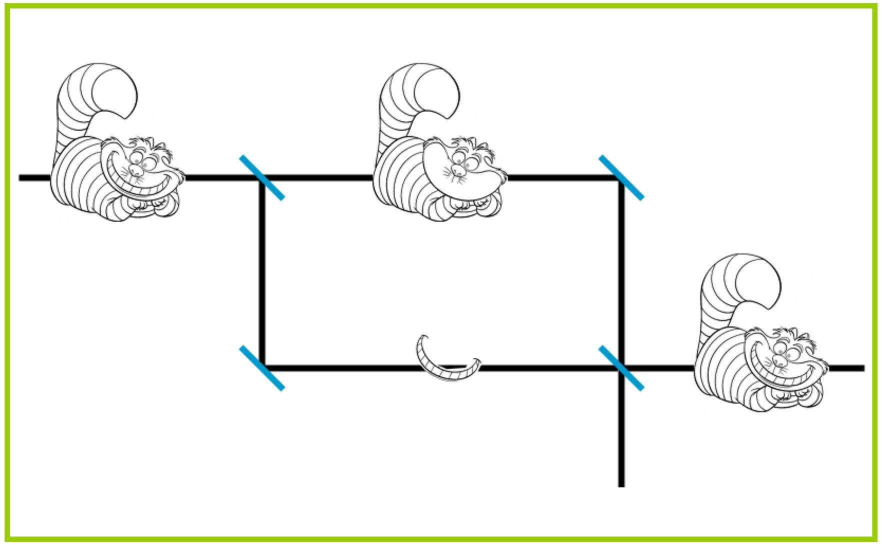


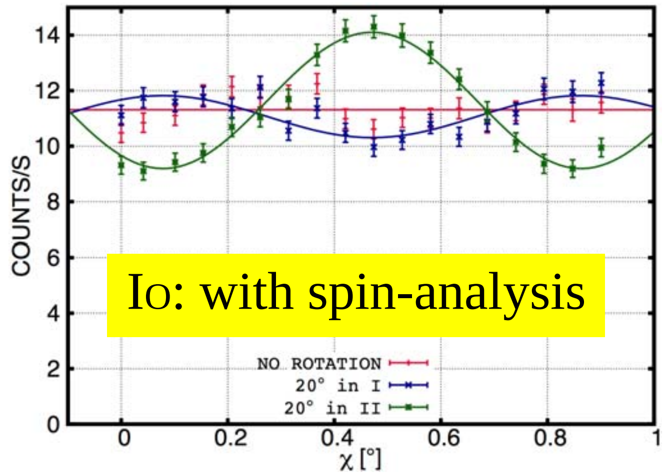


T=1 T=0.8 T=0.6

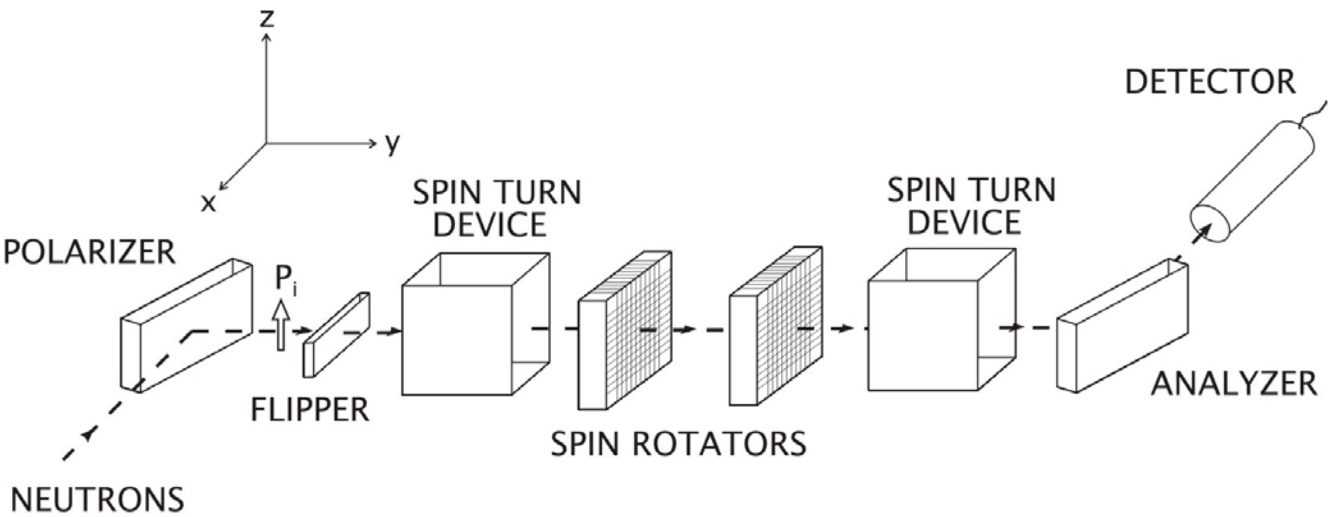
Cheshire Cat: spin(smile) in lower path

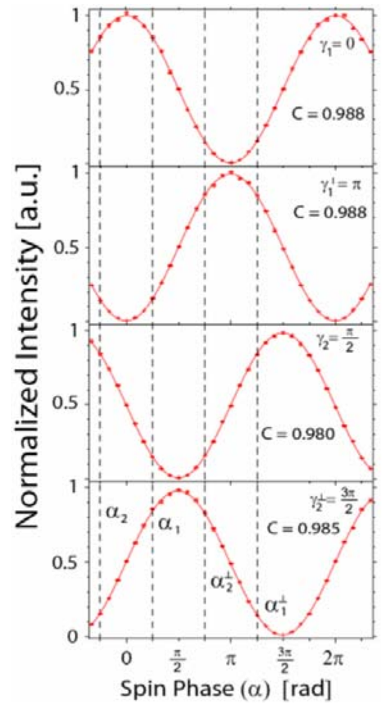




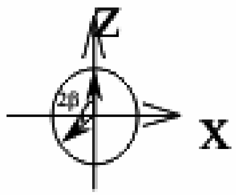


Neutron polarimetry





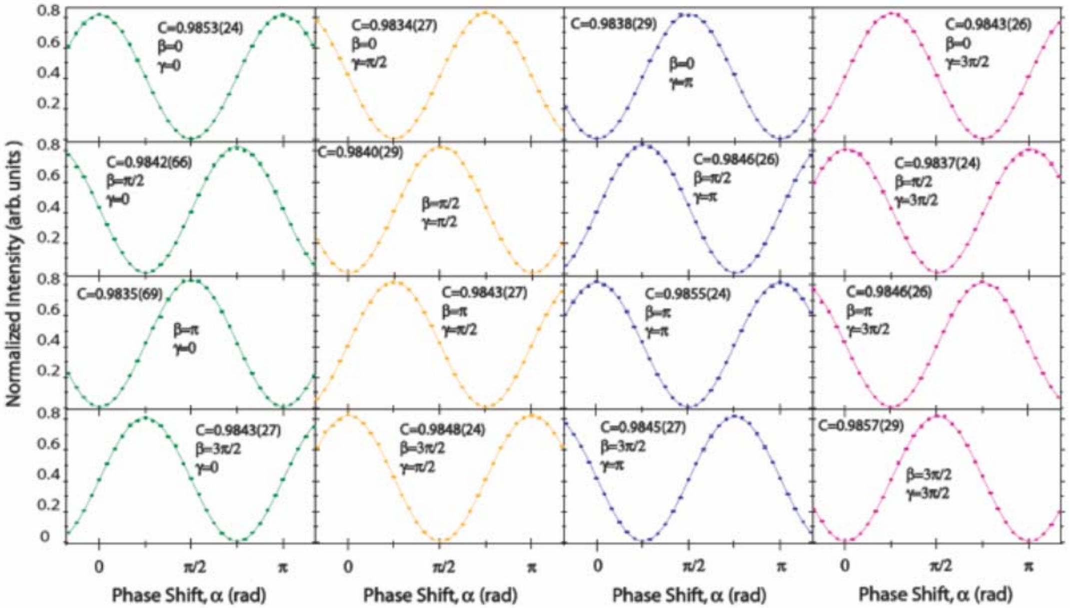
$$\hat{O}|\pm Y\rangle = -e^{\pm i\beta}\cdot|\pm Y\rangle \Rightarrow\Rightarrow$$



$$\hat{O}|+Z\rangle = \hat{O}\left\{\frac{1}{\sqrt{2}}\left(|+Y\rangle + |-Y\rangle\right)\right\} = \frac{1}{\sqrt{2}}\left(e^{i\beta}|+Y\rangle + e^{-i\beta}|-Y\rangle\right) = \frac{e^{i\beta}}{\sqrt{2}}\left(|+Y\rangle + e^{-2i\beta}|-Y\rangle\right)$$

- **Non-commutability of σ_j, PRA (1999)**
- **Bell-Test, PLA (2010)**
- **Leggett-Test, NJP (2012)**
- **GHZ-entanglement, NJP (1012)**
- **Spin-rotation coupling**

Neutron polarimetry: tri-partite entanglement



	α	β	γ	values
$\sigma_x^{(S)}\sigma_x^{(k)}\sigma_x^{(E)}$	$(0;\pi)$	$(0;\pi)$	$(0;\pi)$	0.9843(10)
$\sigma_x^{(S)}\sigma_y^{(k)}\sigma_y^{(E)}$	$(0;\pi)$	$(\pi/2;3\pi/2)$	$(\pi/2;3\pi/2)$	-0.9839(10)
$\sigma_y^{(S)}\sigma_x^{(k)}\sigma_y^{(E)}$	$(\pi/2;3\pi/2)$	$(0;\pi)$	$(\pi/2;3\pi/2)$	-0.9840(10)
$\sigma_y^{(S)}\sigma_y^{(k)}\sigma_x^{(E)}$	$(\pi/2;3\pi/2)$	$(\pi/2;3\pi/2)$	$(0;\pi)$	-0.9837(11)

$$M = 3.936(2) \nleq 2$$

Sponar *et al.*, New J. Phys, **14,** 053032 (2012).

M=4: for perfect circumstances

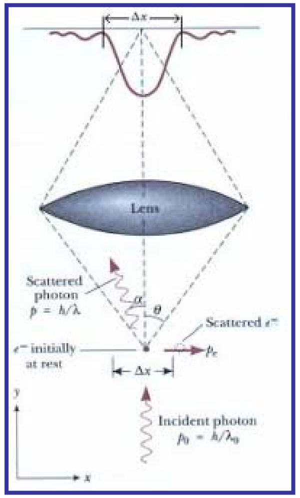
Uncertainty relation: historical 1

- In 1927 Heisenberg postulated an uncertainty principle:

  γ-ray thought experiment

  → $$p_1 q_1 \approx h$$

  with q_1 (mean error) & p_1 (discontinuous change)



- Sei q_1 die Genauigkeit, mit der der Wert q bekannt ist (q_1 ist etwa der mittlere Fehler von q), also hier die Wellenlänge des Lichtes, p_1 die Genauigkeit, mit der der Wert p bestimmbar ist, also hier die unstetige Änderung von p beim Compton-effekt, so stehen nach elementaren Formeln des Comptoneffekts p_1 und q_1 in der Beziehung

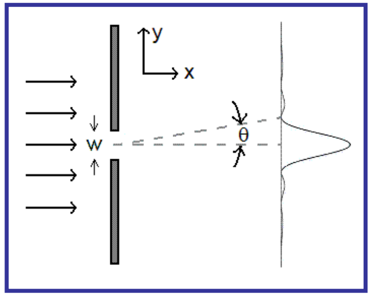
$$p_1 q_1 \sim h. \qquad (1)$$

Uncertainty relation: historical 2

- Kennard considered the spread of a wave function ψ

$$\sigma(Q)\sigma(P) \geq \frac{\hbar}{2}$$

σ: standard deviations



- Robertson generalized the relation to arbitrary pairs of observables in any states ψ

$$\sigma(A)\ \sigma(B) \geq \frac{1}{2}\left|\langle\psi|[A,B]|\psi\rangle\right|$$

→ dependent on the state but independent of the appartus

$$\textit{Is}\ \ \varepsilon(A)\eta(B) \geq \frac{1}{2}\left|\langle\psi|[A,B]|\psi\rangle\right|\ \ \textit{generally valid}\,?$$

Universally valid uncertainty relation by Ozawa

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2}\left|\langle\psi\left|[A,B]\right|\psi\rangle\right|$$

$$\begin{cases} \varepsilon: \text{error of the first measurmen} \ (A) \\ \eta: \text{disturbance on the second measurement} \ (B) \\ \sigma: \text{standard deviations} \end{cases}$$

First term: error of the first measuremt, disturbance on the second measurement

second and third terms: crosstalks between spreads of wavefunctions and error/disturbance

M. Ozawa, Phys. Rev. A **67**, 042105 (2003).

Error and disturbance for projective measurement

- **Error**:

$$\epsilon(A)^2 = ||\sum_\lambda O_\lambda(\lambda - A)|\psi||^2$$

If the O_λ are mutually orthogonal projection operators sum and norm can be exchanged

$$\epsilon(A)^2 = ||(O_A - A)|\psi\rangle||^2$$

output operator: $O_A = \sum_\lambda \lambda O_\lambda$

different expression for measurement (5 expectation values):

$$\epsilon(A)^2 = \langle\psi|A^2|\psi\rangle + \langle\psi|O_A^2|\psi\rangle + \langle\psi|O_A|\psi\rangle + \underbrace{\langle\psi|AO_AA|\psi\rangle}_{\langle\psi'|O_A|\psi'\rangle} - \underbrace{\langle\psi|(A+I)O_A(A+I)|\psi\rangle}_{\langle\psi''|O_A|\psi''\rangle}$$

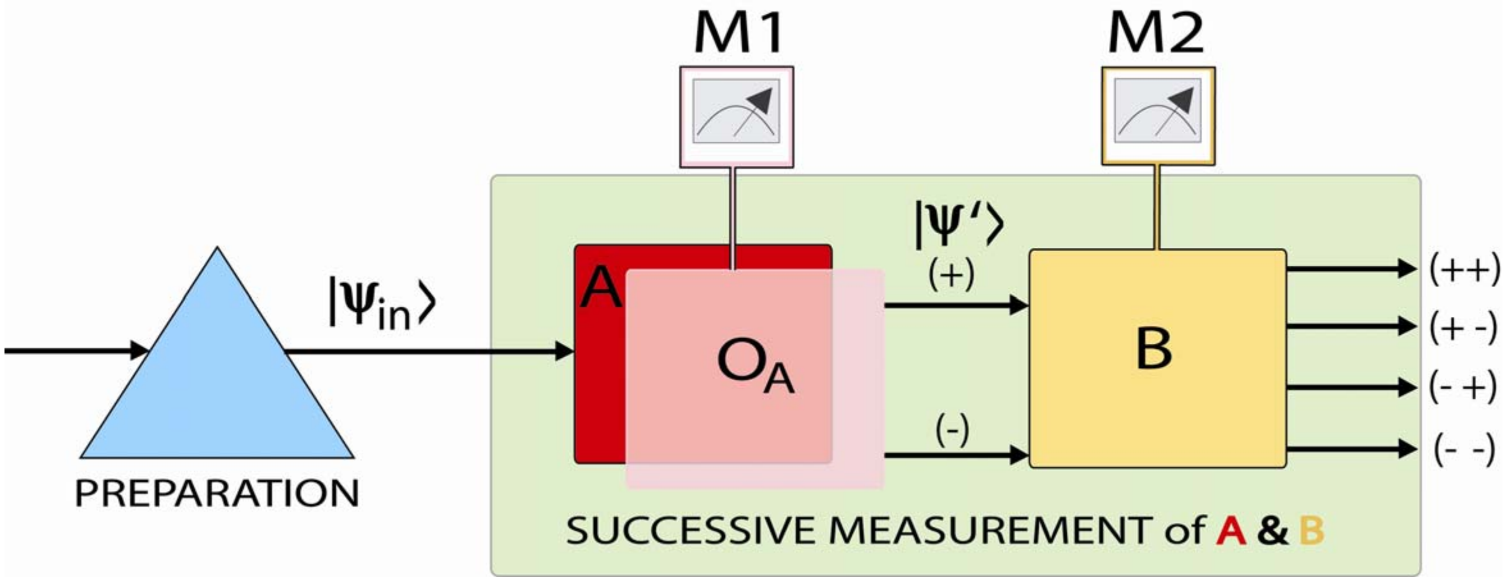
with $O_A^2 = \sum_\lambda \lambda^2 O_\lambda^\dagger O_\lambda$

- **Disturbance**: $\eta(B)^2 = \sum_\lambda ||[O_\lambda, B]|\psi\rangle||^2$

$$\eta(B)^2 = \langle\psi|B^2|\psi\rangle + \langle\psi|X_B^2|\psi\rangle + \langle\psi|X_B|\psi\rangle + \overbrace{\langle\psi|BX_BB|\psi\rangle}^{\langle\psi'''|X_B|\psi'''\rangle} - \overbrace{\langle\psi|(B+I)X_B(B+I)|\psi\rangle}^{\langle\psi''''|X_B|\psi''''\rangle}$$

with $X_B^2 = \sum_\lambda O_\lambda^\dagger B^2 O_\lambda$, and modified output operator: $X_B = \sum_\lambda O_\lambda^\dagger B O_\lambda$

Experimental scheme



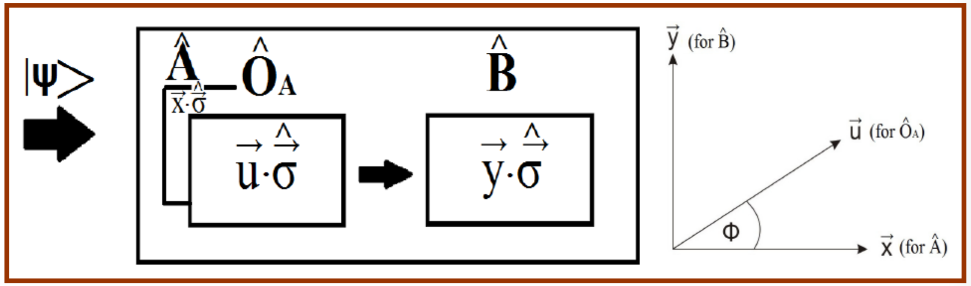
- Successively measurement of 2 noncommuting observables A and B
- Apparatus 1 measures O_A, Apparatus 2 measures B

Theoretical predictions 1

For error and disturbance:

$$\epsilon^2(A) = 2 - 2\,(\vec{x}\cdot\vec{u})$$

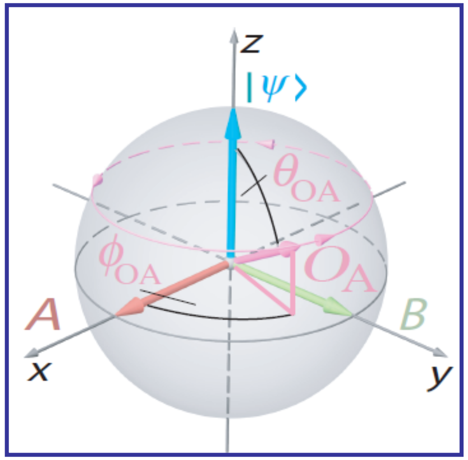
$$\eta^2(B) = 2 - 2\,(\vec{u}\cdot\vec{y})^2$$



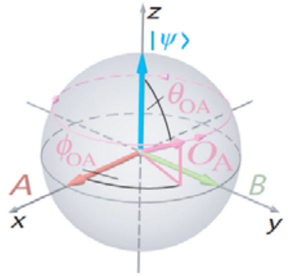
For the standard deviations:

$$\sigma^2(A) = \underbrace{\langle\psi|\,A^2\,|\psi\rangle}_{1} - (\langle\psi|\,A\,|\psi\rangle)^2$$

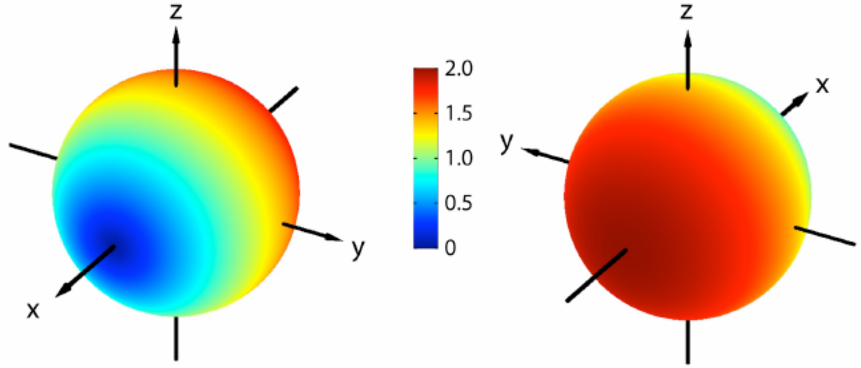
$$\sigma^2(B) = \underbrace{\langle\psi|\,B^2\,|\psi\rangle}_{1} - (\langle\psi|\,B\,|\psi\rangle)^2$$



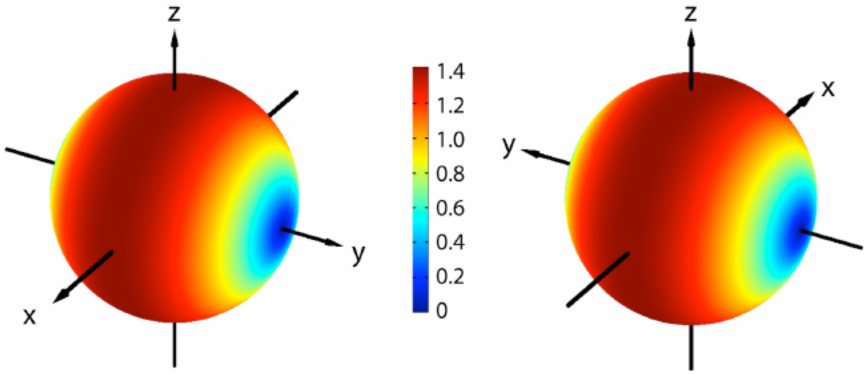
Theoretical predictions 2



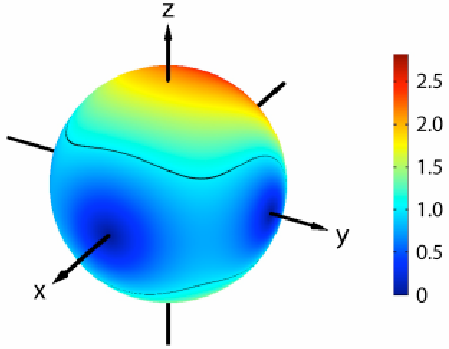
- **Error**: $\epsilon(A)$



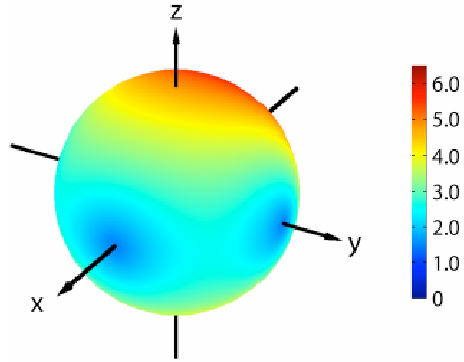
- **Disturbance**: $\eta(B)$



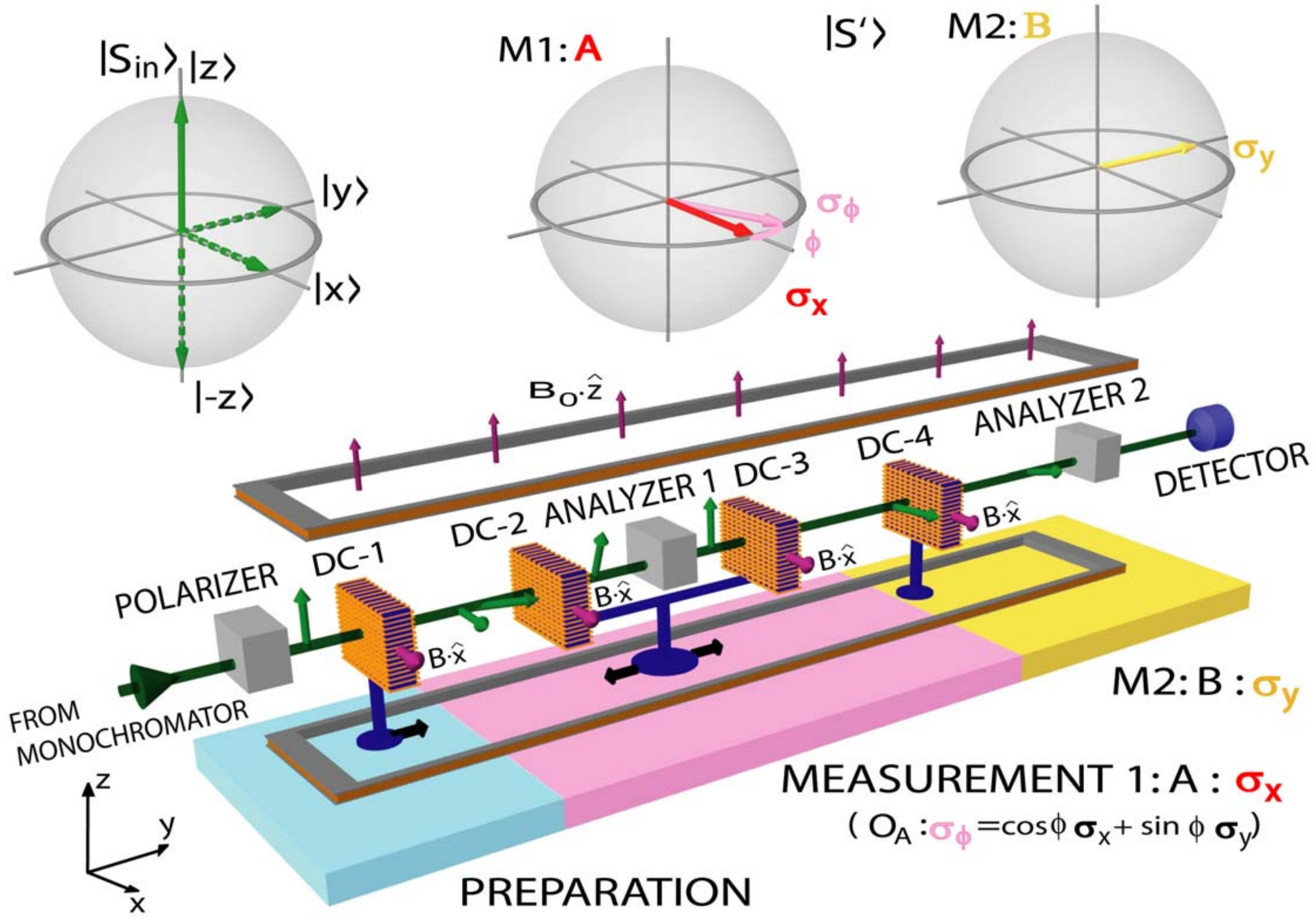
-Heisenberg's relation



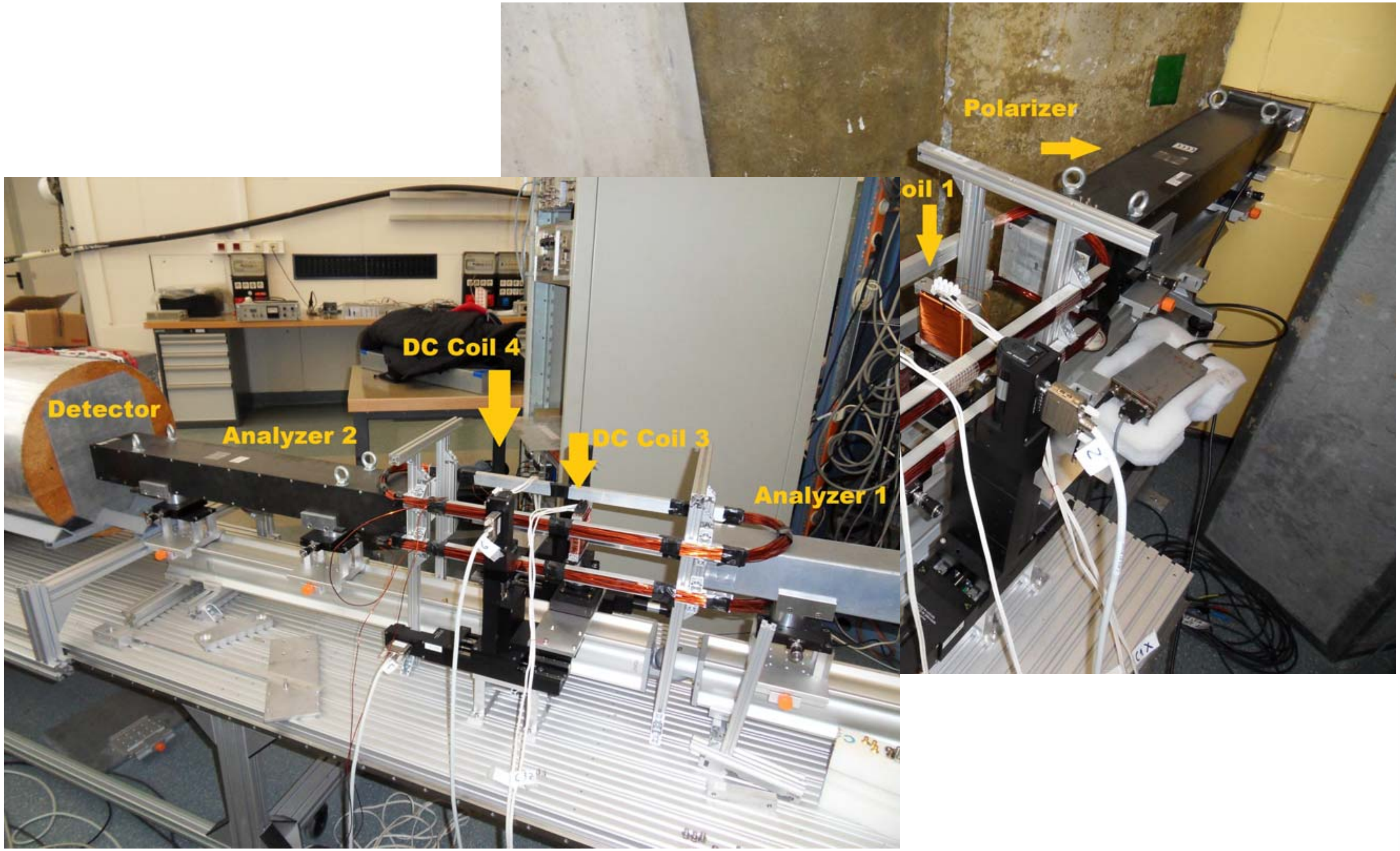
- new uncertainty relation



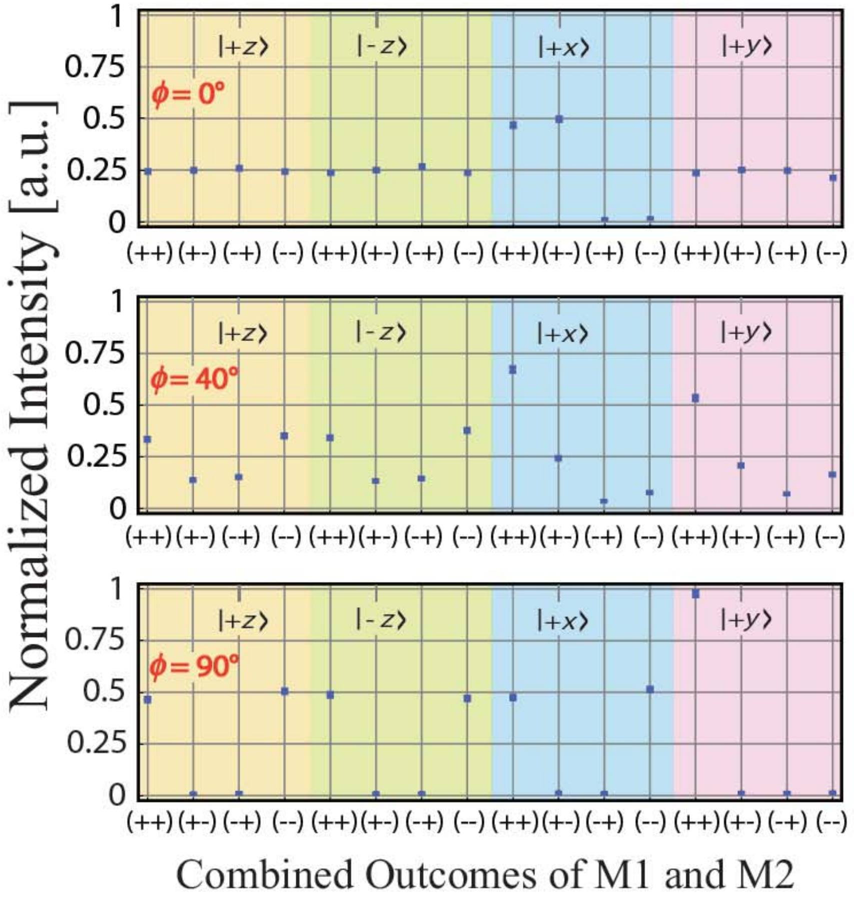
Experimental setup



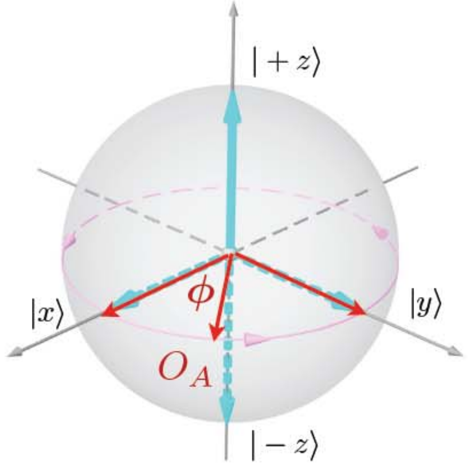
Experimental setup



Experimental data



$$A = \sigma_x$$
$$B = \sigma_y$$



$$O_A = \sigma_\phi = \sigma_x \sin\phi + \sigma_y \cos\phi$$

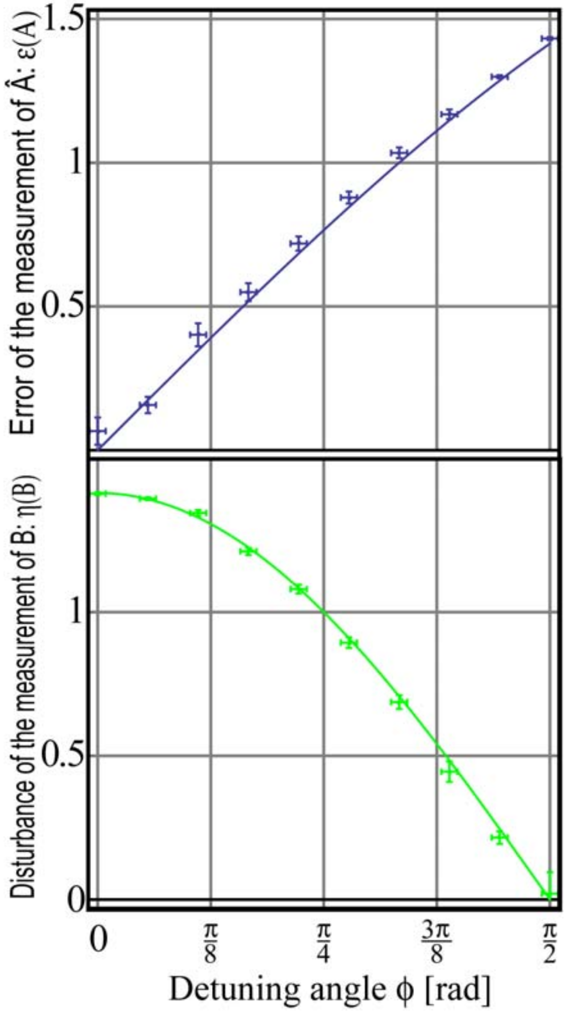
$$\frac{(I_{++} + I_{+-}) - (I_{-+} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi_i | O_A | \psi_i \rangle$$

$$\frac{(I_{++} + I_{-+}) - (I_{+-} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi_i | X_B | \psi_i \rangle$$

$$|\psi_i\rangle = |+z\rangle,\ |-z\rangle,\ |x\rangle,\ |y\rangle$$

$$\epsilon(A)^2 = 2 + \langle +z|\sigma_\phi|+z\rangle + \langle -z|\sigma_\phi|-z\rangle - \langle x|\sigma_\phi|x\rangle$$
$$\eta(B)^2 = 2 + \langle +z|X_B|+z\rangle + \langle -z|X_B|-z\rangle - \langle y|X_B|y\rangle$$

Results: error-disturbance trade-off



$$|\psi_i\rangle = |+z\rangle$$

$$\hat{A} = \hat{\sigma_x} \quad \hat{O}_A = \hat{\sigma}_\phi = \cos(\phi)\hat{\sigma}_x + \sin(\phi)\hat{\sigma}_y$$

$$\hat{B} = \hat{\sigma_y}$$

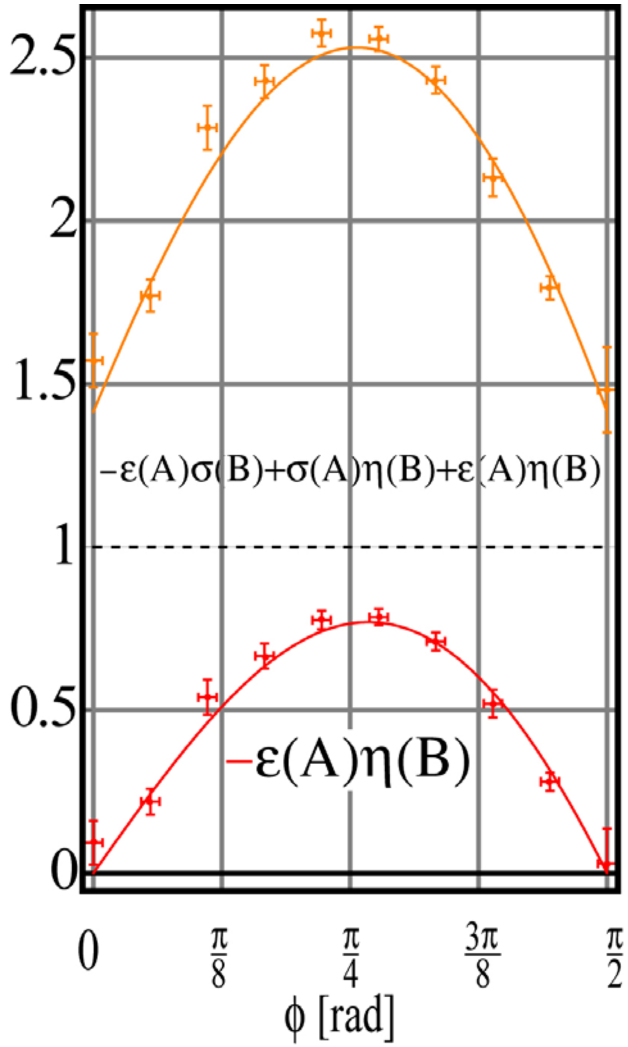
$$\varepsilon(A)^2 = \langle\psi|A^2|\psi\rangle + \langle\psi|O_A^2|\psi\rangle + \langle\psi|O_A|\psi\rangle + \langle A\psi|O_A|A\psi\rangle - \langle(A+I)\psi|O_A|(A+I)\psi\rangle$$

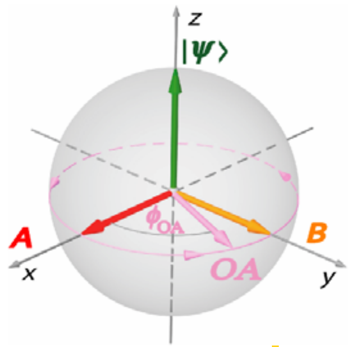
$$|\psi\rangle = |+z\rangle \qquad |\psi\rangle = |+z\rangle$$

$$|A\psi\rangle = |-z\rangle \qquad |B\psi\rangle = |-z\rangle$$

$$|(A+\mathbb{I})\psi\rangle = |+x\rangle \qquad |(B+\mathbb{I})\psi\rangle = |+y\rangle$$

Results: new/old uncertainty relation





New uncertainty principle

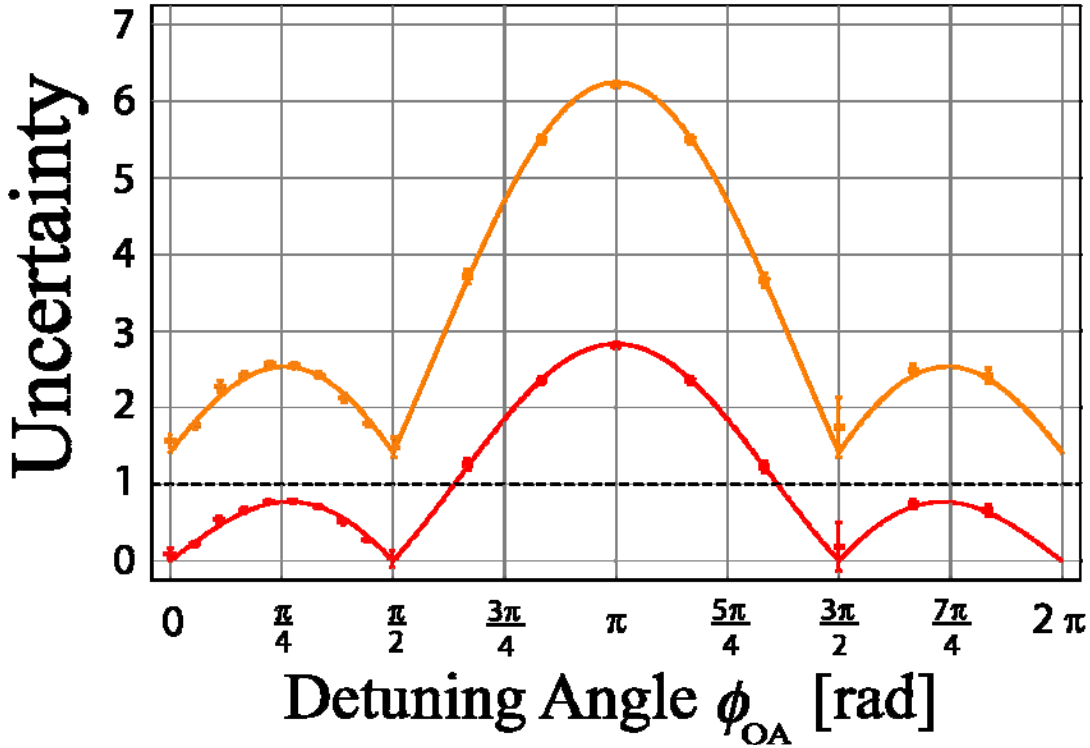
ε : error of the first measurmen (A)
η : disturbance on the second measurement (B)
σ : standard deviations

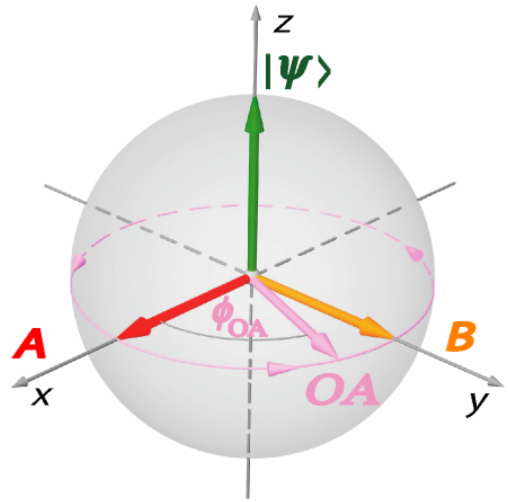
standard deviations:
σ(B)= 0.9999(1)
σ(A)= 0.9994(3)

Heisenberg product

J. Erhart et al., Nature Phys. **8**, 185-189 (2012)

Results 1: incident spin-state ($|s\rangle = |\theta,\phi\rangle$)





$$|s\rangle = |\theta = 0, \phi = 0\rangle$$

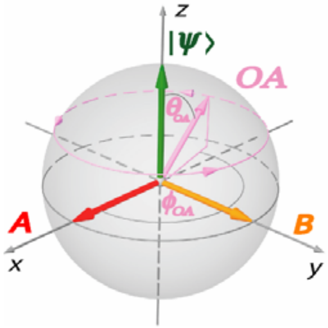
— $\varepsilon(A)\,\eta(B)$

— $\varepsilon(A)\,\eta(B) + \varepsilon(A)\sigma(B) + \sigma(A)\eta(B)$

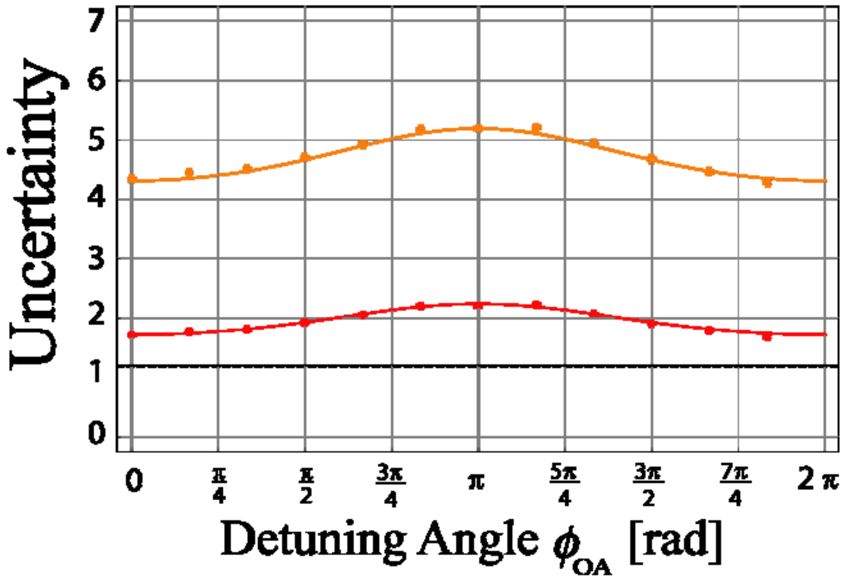
--- Heisenberg lower limit

G. Sulyok et al.,
PRA. **88**, 022110 (2013)

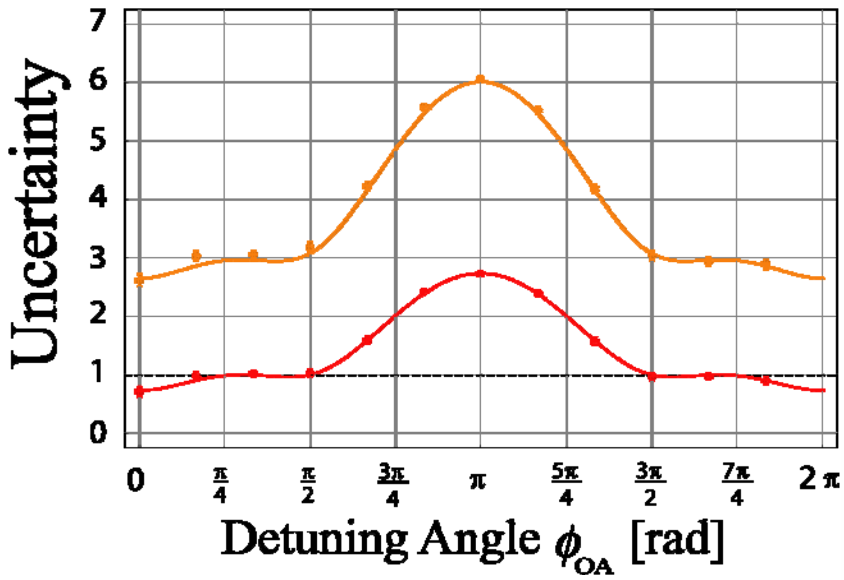
Results 2: polar angle of O_A [$\theta(O_A)$]



$\theta(O_A) = \pi/12$



$\theta(O_A) = \pi/3$

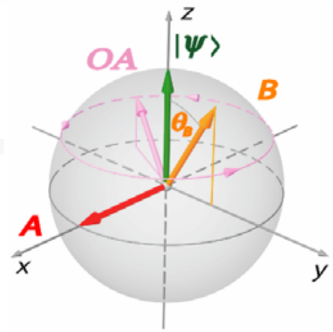


New sum is always above border!

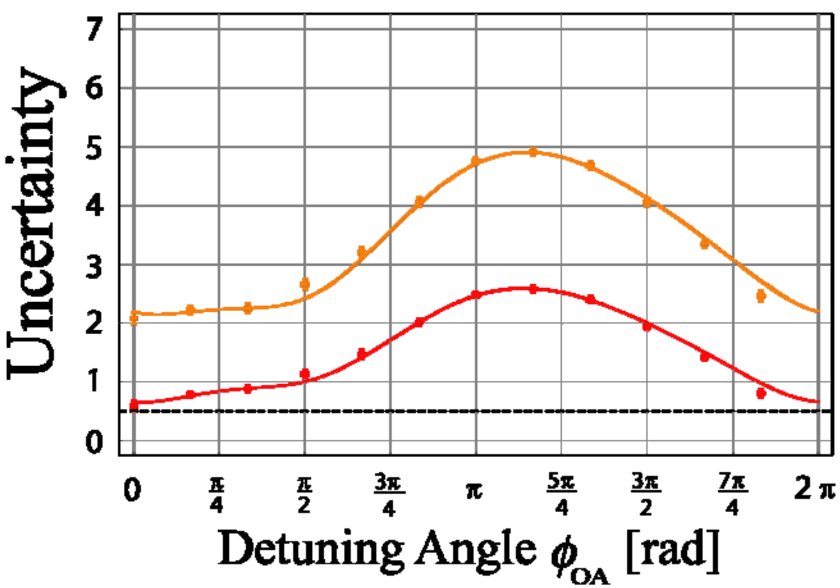
G. Sulyok et al.,
PRA. **88**, 022110 (2013)

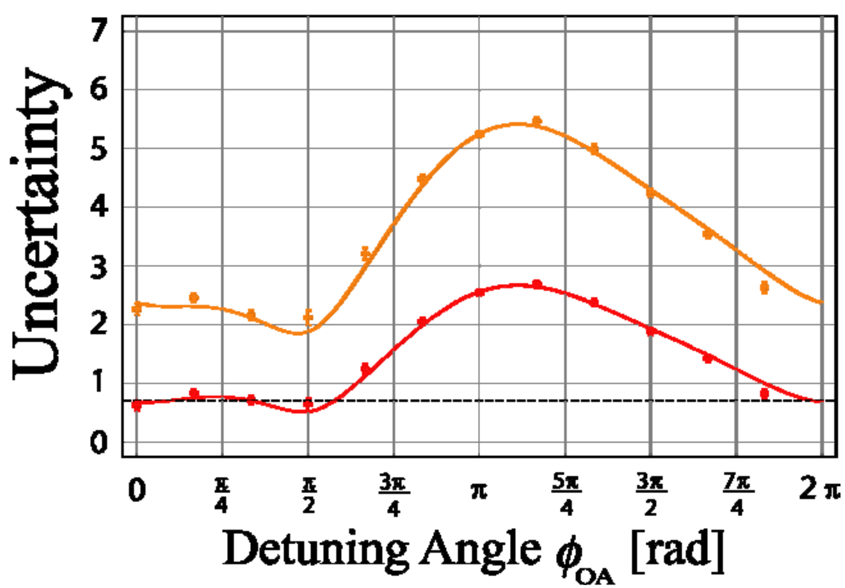
— $\varepsilon(A)\eta(B)$
— $\varepsilon(A)\eta(B)+\varepsilon(A)\sigma(B)+\sigma(A)\eta(B)$
--- Heisenberg lower limit

Results 3: polar angle of B [θ(B)]



θ(B) = π/6	θ(B) = π/4



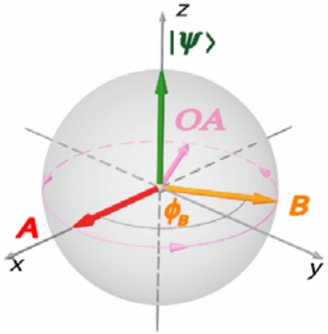


Asymmetry appears!

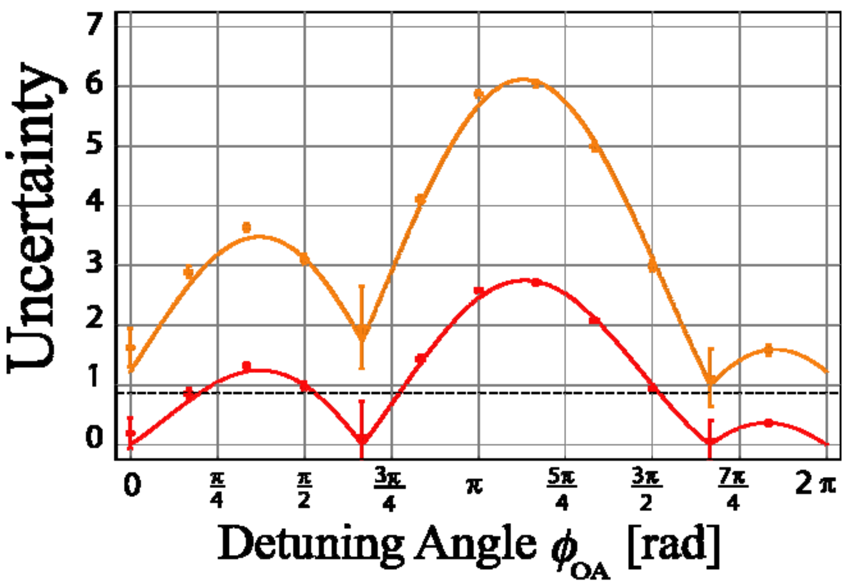
G. Sulyok et al.,
PRA. **88**, 022110 (2013)

— $\varepsilon(A)\,\eta(B)$
— $\varepsilon(A)\,\eta(B)+\varepsilon(A)\,\sigma(B)+\sigma(A)\,\eta(B)$
--- Heisenberg lower limit

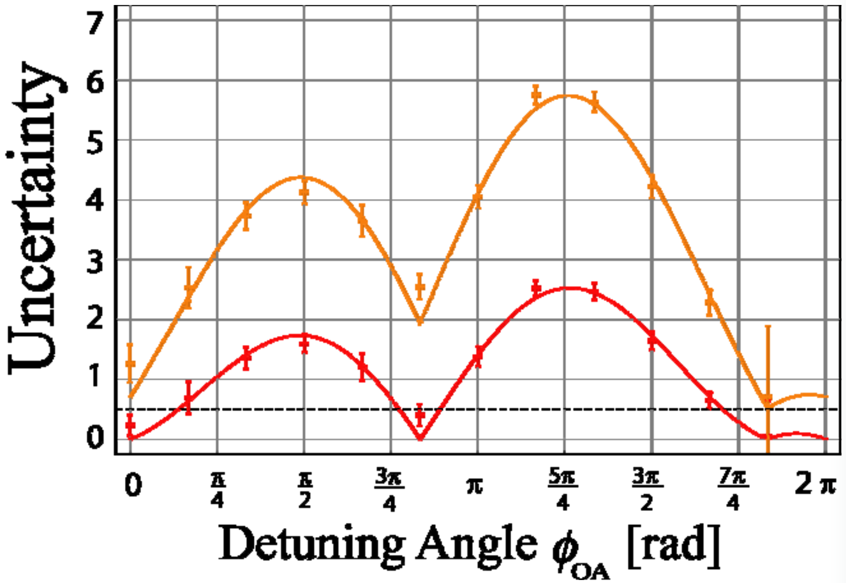
Results 4: azimuthal angle of B [ϕ(B)]



$\phi(B) = 2\pi/3$



$\phi(B) = 5\pi/6$



Sum touches the border!

G. Sulyok et al.,
PRA. **88**, 022110 (2013)

— $\varepsilon(A)\,\eta(B)$
— $\varepsilon(A)\,\eta(B)+\varepsilon(A)\,\sigma(B)+\sigma(A)\,\eta(B)$
--- Heisenberg lower limit

Publications by other groups

PRL **109,** 100404 (2012) PHYSICAL REVIEW LETTERS week ending 7 SEPTEMBER 2012

Violation of Heisenberg's Measurement-Disturbance Relationship by Weak Measurements

Lee A. Rozema, Ardavan Darabi, Dylan H. Mahler, Alex Hayat, Yasaman Soudagar, and Aephraim M. Steinberg

Ce

PRL **110,** 220402 (2013) PHYSICAL REVIEW LETTERS week ending 31 MAY 2013

Experimental Test of Universal Complementarity Relations

Morgan M. Weston, Michael J. W. Hall, Matthew S. Palsson, Howard M. Wiseman, and

Centre for Quantum

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DOI: 1

(a)

(b)

ArXiv; 1304.2071

How well can one jointly measure two incompatible observables on a given quantum state?

Cyril Branciard
Centre for Engineered Quantum Systems and Scho
The University of Queensland, St Lucia,
(Dated: April 9, 201

Heisenberg's uncertainty principle is one of the main tenets of
fundamental importance for our understanding of quantum foun
interpretation: although Heisenberg's first argument was that the
state necessarily disturbs another incompatible observable, stand
indeterminacy of the outcomes when either one or the other obse
precisely Heisenberg's intuition. Even if two incompatible obser
still approximate their joint measurement, at the price of intro
measurement of each of them. We present a new, tight relation
the error on one observable versus the error on the other. As
characterize the disturbance of an observable induced by the appr
derive a stronger error-disturbance relation for this scenario.

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QUANTUM OPTICS

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Experimental violation and reformulation of the Heisenberg's error-disturbance uncertainty relation

So-Young Baek[1]*, Fumihiro Kaneda[1], Masanao Ozawa[2] & Keiichi Edamatsu[1]

[1]Research Institute of Electrical Communication, Tohoku University, Sendai 980-8577, Japan, [2]Graduate School of Information Science, Nagoya University, Nagoya 464-8601, Japan.

The uncertainty principle formulated by Heisenberg in 1927 describes a trade-off between the error of a measurement of one observable and the disturbance caused on another complementary observable such that their product should be no less than the limit set by Planck's constant. However, Ozawa in 1988 showed a model of position measurement that breaks Heisenberg's relation and in 2003 revealed an alternative relation for error and disturbance to be proven universally valid. Here, we report an experimental test of Ozawa's relation for a single-photon polarization qubit, exploiting a more general class of quantum measurements than the class of projective measurements. The test is carried out by linear optical devices and realizes an indirect measurement model that breaks Heisenberg's relation throughout the range of our experimental parameter and yet validates Ozawa's relation.

FWF

Concluding remarks

Neutron interferometer and polarimeter are effective tools for investigations of quantum mechanics.

Universally valid uncertainty-relation by Ozawa is experimentally tested.

- **Neutron's spin measurement confirmed the new error-disturbance uncertainty relation.**
- **New sum is always above the limit!**
  Heisenberg product is often below the limit!

Neutron Quantum Optics generation