

# **Relational Causality and Classical Probability: Grounding Quantum Phenomenology in a Superclassical Theory**

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# THE CONCEPT OF PROBABILITY IN QUANTUM MECHANICS

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*From about the beginning of the twentieth century experimental physics  
presented us in a series of stages of strange phenomena which demonstrated the*

**R. P. Feynman, 1951:**

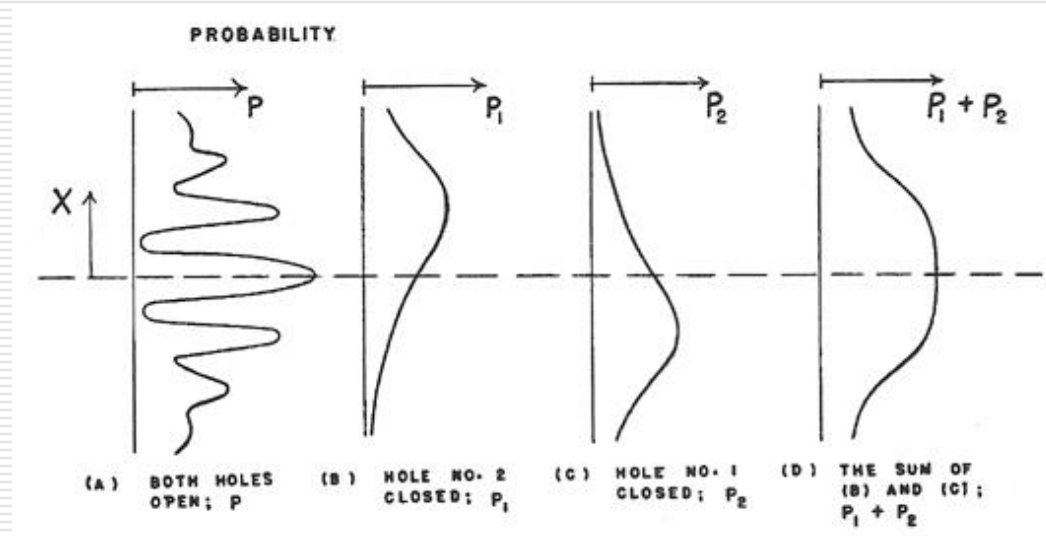
- Classical probability theory not applicable in QM
- separate “laws of probabilities of QM”

“We might at first suppose (since the electrons behave as particles) that

I. Each electron which passes from  $S$  to  $X$  must go either through hole 1 or hole 2. As a consequence of I we expect that:

II. The chance of arrival at  $X$  should be the sum of two parts,  $P_1$ , the chance of arrival coming through hole 1, plus  $P_2$ , the chance of arrival coming through hole 2."

*however:*



"Hence experiment tells us definitely that  $P \neq P_1 + P_2$  or that II is false. (...)

We must conclude that when both holes are open it is *not true* that the particle goes through one hole or the other."

**B. O. Koopman** 1955, **L. E. Ballentine** 1986:

- “Argument draws its radical conclusion from an incorrect application of probability theory” (Ballentine)
- use conditional probabilities (contexts!)  $P_1(X|C_1)$ ,  $P_2(X|C_2)$ ,  $P(X|C_3)$
- Experiment  $\rightarrow P(X|C_3) \neq P_1(X|C_1) + P_2(X|C_2)$
- “the QM superposition principle for amplitudes is in no way incompatible with the formalism of probability theory”

$\rightarrow$  starting in the wave picture (analogy to Couder!), we shall show that

$$P(X|C_3) = P_1(X|C_3) + P_2(X|C_3)$$

→ calculate wave intensity distributions  $\Leftrightarrow$  particle distributions

*classical double slit* ( $P_i = R_i^2$ ):

intensity after slit 1  $\rightarrow P(1) = P_1 + R_1 R_2 \cos \varphi$

*intensity from slit*

*from interference with other slit*

intensity after slit 2  $\rightarrow P(2) = P_2 + R_2 R_1 \cos \varphi$

→ total intensity:  $P = P(1) + P(2) = P_1 + P_2 + 2 R_1 R_2 \cos \varphi$

*analogously in quantum-like modelling & QM (phase differences  $\varphi_{ij}$ ):*

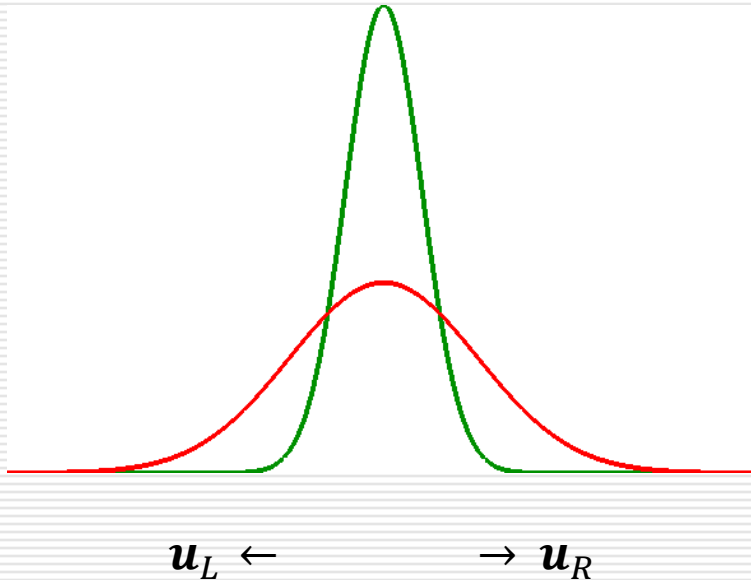
$$n \text{ slits: } P = \sum_{i=1}^n \left( P_i + \sum_{j=i+1}^n 2R_i R_j \cos \varphi_{ij} \right)$$

... however, with one additional ingredient:

## Dispersion of a wavepacket:

particle “bouncing” in fluctuating wave-like environment (Couder!)

→ diffusion to the right and left of mean path  
(i.e. of Ehrenfest trajectory in free particle case)



→ total velocity field:  $\boldsymbol{v} + \boldsymbol{u}_L + \boldsymbol{u}_R$ ,

$$\boldsymbol{u}_i = D \frac{\nabla_i P}{P} \propto \nabla_i Q \dots \text{diffusion velocity}$$

not necessarily falling off with distance !

→ issue of nonlocality /superluminal effects: see talk by Jan Walleczek !

→ 3 current channels per slit

## Decay of a Gaussian wavepacket: anomalous diffusion

$$u_0 = -\frac{\hbar}{2m} \frac{\overline{\nabla P}}{P} = \frac{D}{\sigma_0} : \quad \overline{x^2} = \overline{x^2}\Big|_{t=0} + u_0^2 t^2$$

$$\text{with } D(t) = u_0^2 t : \quad \overline{x^2} = \overline{x^2}\Big|_{t=0} + D(t)t$$

... Brownian-type displacement with time-dependent diffusivity  
("ballistic diffusion").

→ *average total velocity field of a Gaussian wave packet:*

$$v_{\text{tot}}(t) = v(t) + [x_{\text{tot}}(t) - vt] \frac{D(t)}{\sigma^2}$$

→ **Computer simulations with Coupled Map Lattices (CML):**

$$P[i, k + 1] = P[i, k] + \frac{D[k + 1]\Delta t}{\Delta x^2} \{P[i + 1, k] - 2P[i, k] + P[i - 1, k]\}$$

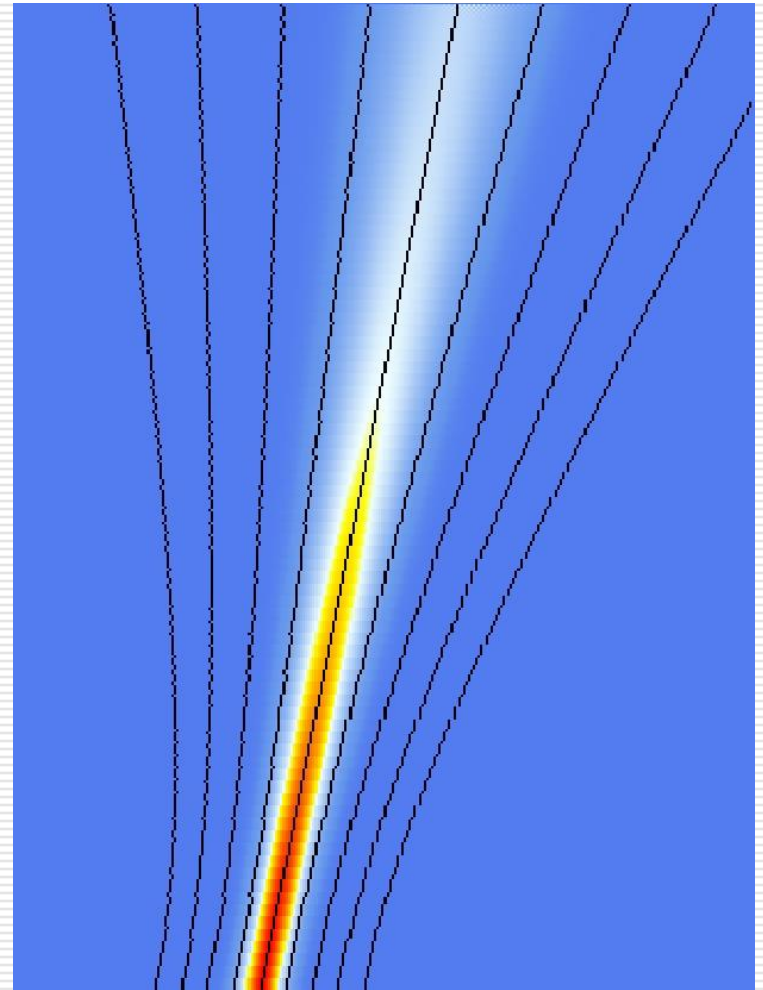
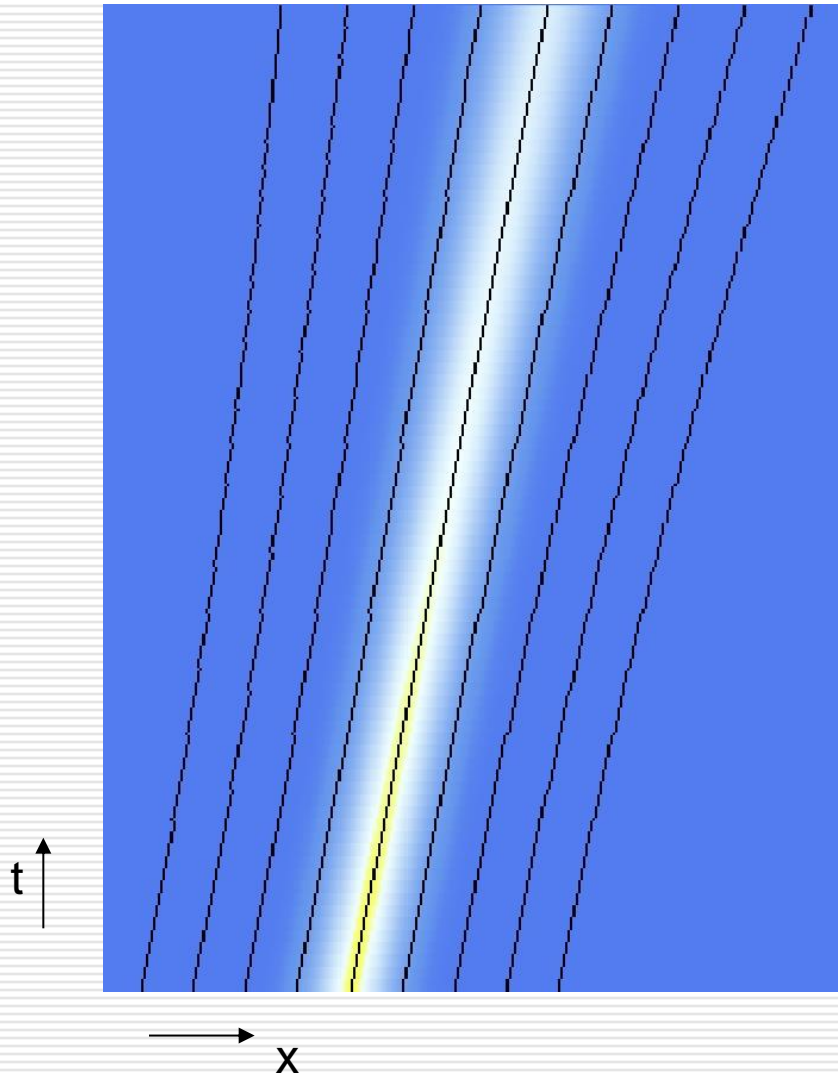
with  $D \rightarrow D(t) = u_0^2 t = \frac{D^2}{\sigma_0^2} t$

meaning of *time-dependent diffusivity*:

- reflects a **changing thermal environment**
- dissipation of “kinetic energy” from a constrained (i.e., by the slit potential) to an unconstrained vacuum



*Dispersion of a free Gaussian wave-packet  
in a classical (CML) simulation*



anomalous/ballistic diffusion ... signature of “superstatistics”

prototype:

*Brownian particle moving through (thermally) changing environment*



relatively fast dynamics  
(particle velocity)  
on a small scale



slow changes  
(temperature fluctuations)  
on a large scale



superposition of two statistics:

*Superstatistics*

...with unexpected emergent properties on intermediate scales!

→ *analogy for the modelling of quantum systems:*

quantum behavior as emerging from the interplay of classical processes at small ("sub-quantum") and large ("macroscopic") scales

→ "superclassical" physics

*Emergence* through the co-evolution of "microscopic", local processes and of "macroscopic" boundary conditions

- new type of behavior possible due to **emergent features** **unexpected** from very small or large scale physics

## "surreal trajectories" ?

Marlan O. Scully, *Physica Scripta* Vol. T76, 41-46, 1998

### Do Bohm Trajectories Always Provide a Trustworthy Physical Picture of Particle Motion?

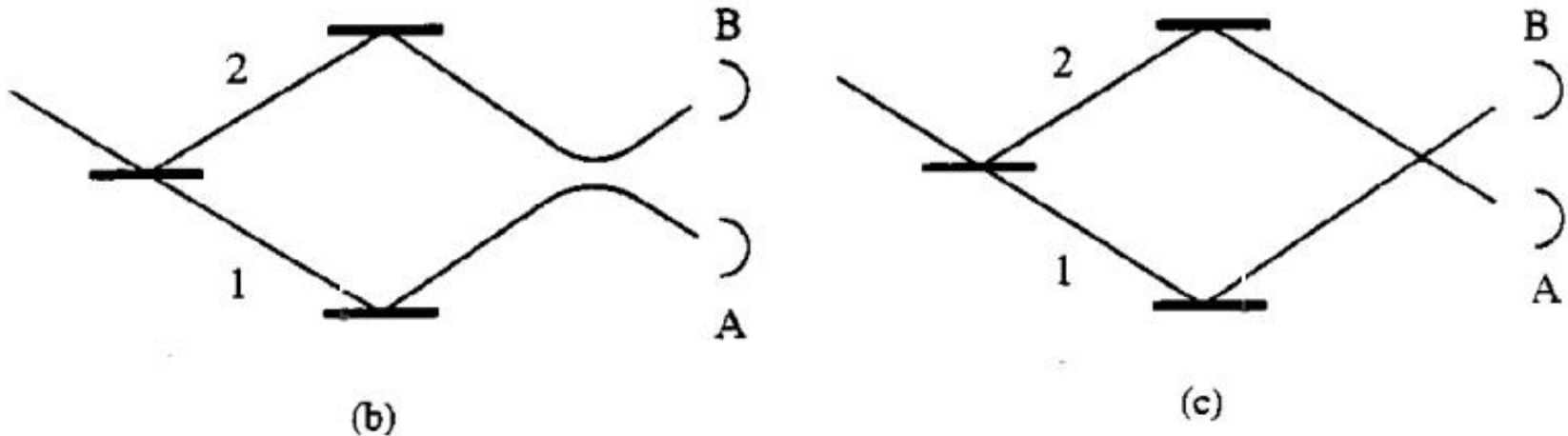


Fig. 1. Paths of quantum particles in incomplete inteferometers with the final beam splitter missing. b) Trajectories in an incomplete Mach-Zender Interferometer in which the final beam splitter is missing according to Bohmian Mechanics; notice non-crossing of trajectories c) Paths through an incomplete Mach-Zender Interferometer as described by quantum mechanics.

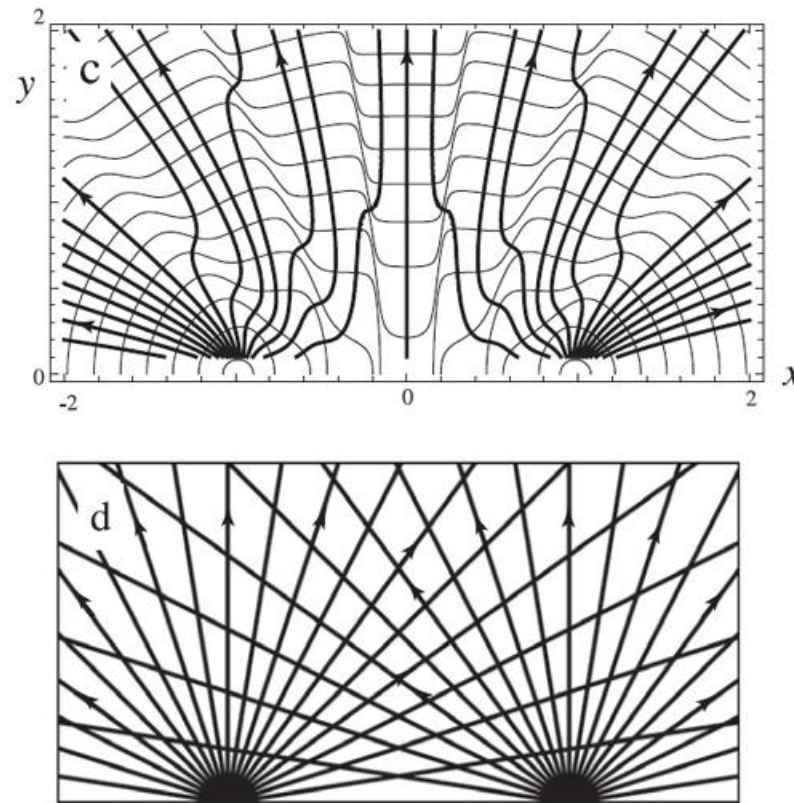
Example of Gaussian: With *total velocity field*, one obtains for the action  $S$

$$S = mvx + \frac{mu_0^2}{2} \left( \frac{\xi(t)}{\sigma(t)} \right)^2 t - Et$$

→  $\nabla S$  ... trajectories (= current streamlines)  
orthogonal to surfaces  $S = \text{const.}$

... *not* identical with rays of geometrical optics!  
(except in simple cases like wave from single point source)

→ What happens, e.g., in case of *two* point sources?



quantum case streamlines (top) vs. geometrical rays (bottom)

M. Berry (2009): *The streamlines of the superposition are different from the superposition of the streamlines*

→ velocity field is emergent! (compare: “surreal” trajectories)

Howard Wiseman (2007):

(real parts of) weak values of velocities  
*are* the Bohmian velocities

Kocsis *et al.* (2011) → we can now ask new  
questions w.r.t. “trajectories”

... and even ask questions “beyond Bohm”:  
where does the *guiding equation* (or the  
*quantum potential* ) come from?

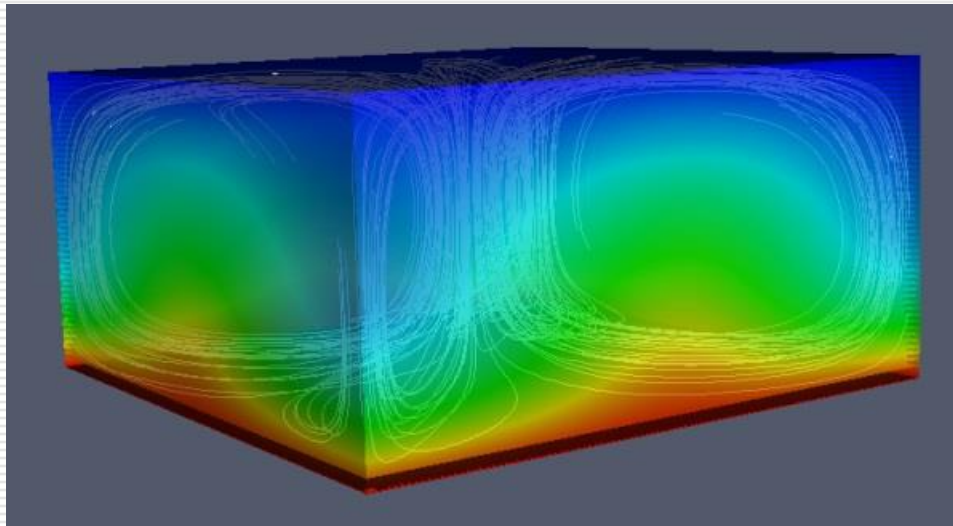


an *invisible hand* ? ... or:



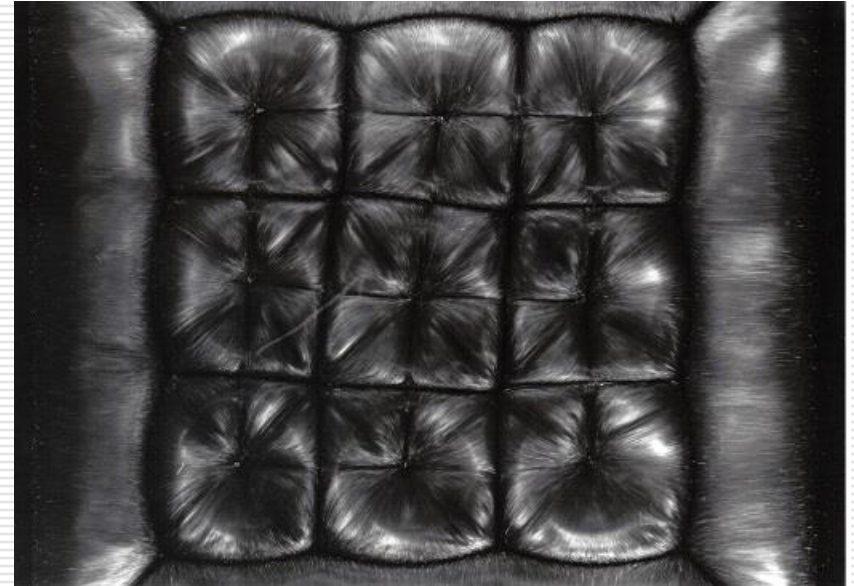
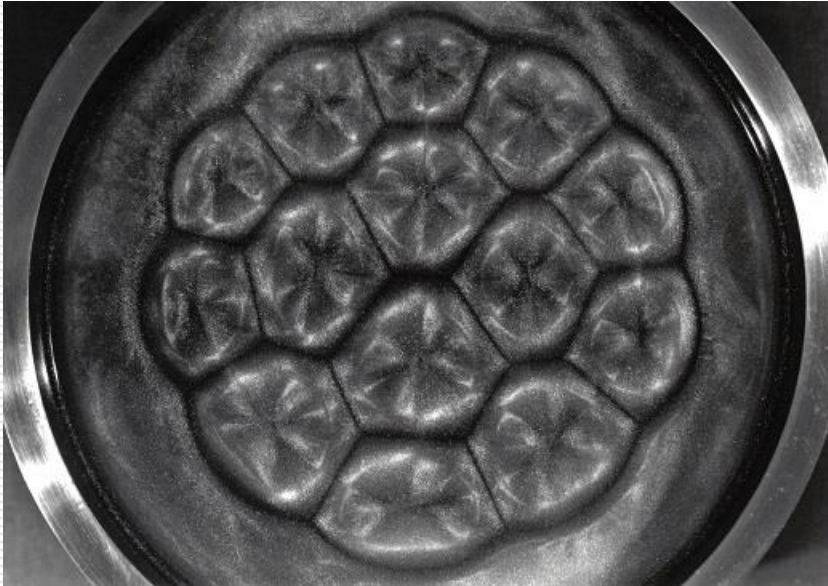
# Emergence:

## Example of Rayleigh-Bénard Cell



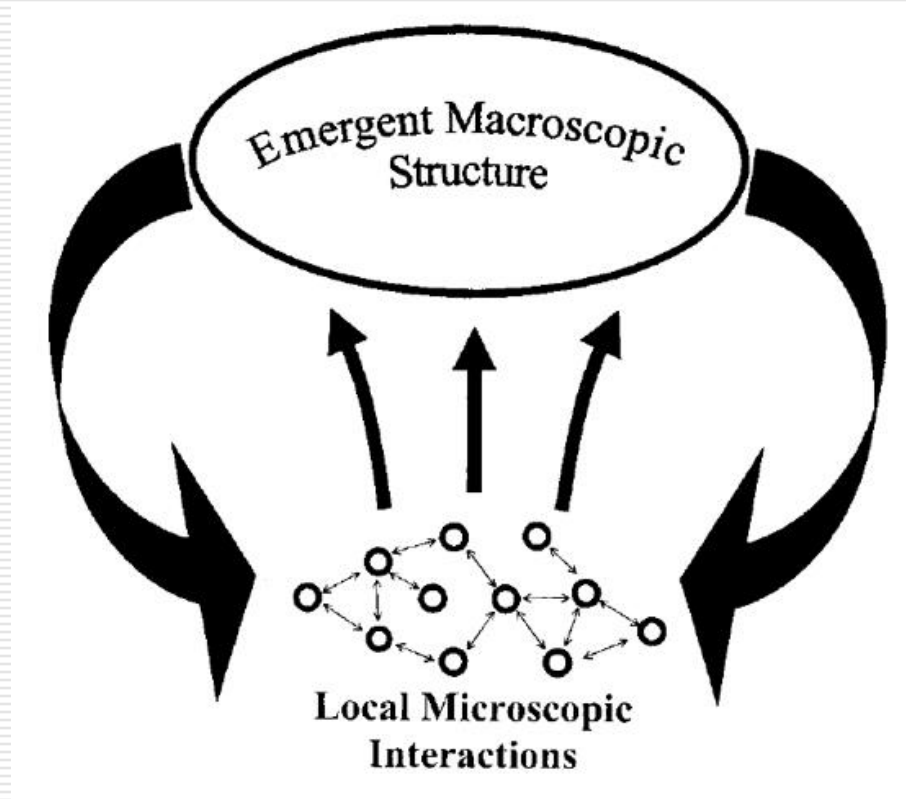
open system, temperature gradient, trajectories → convection rolls

# Emergence:



emergent *particle trajectories* depending on *boundary conditions*

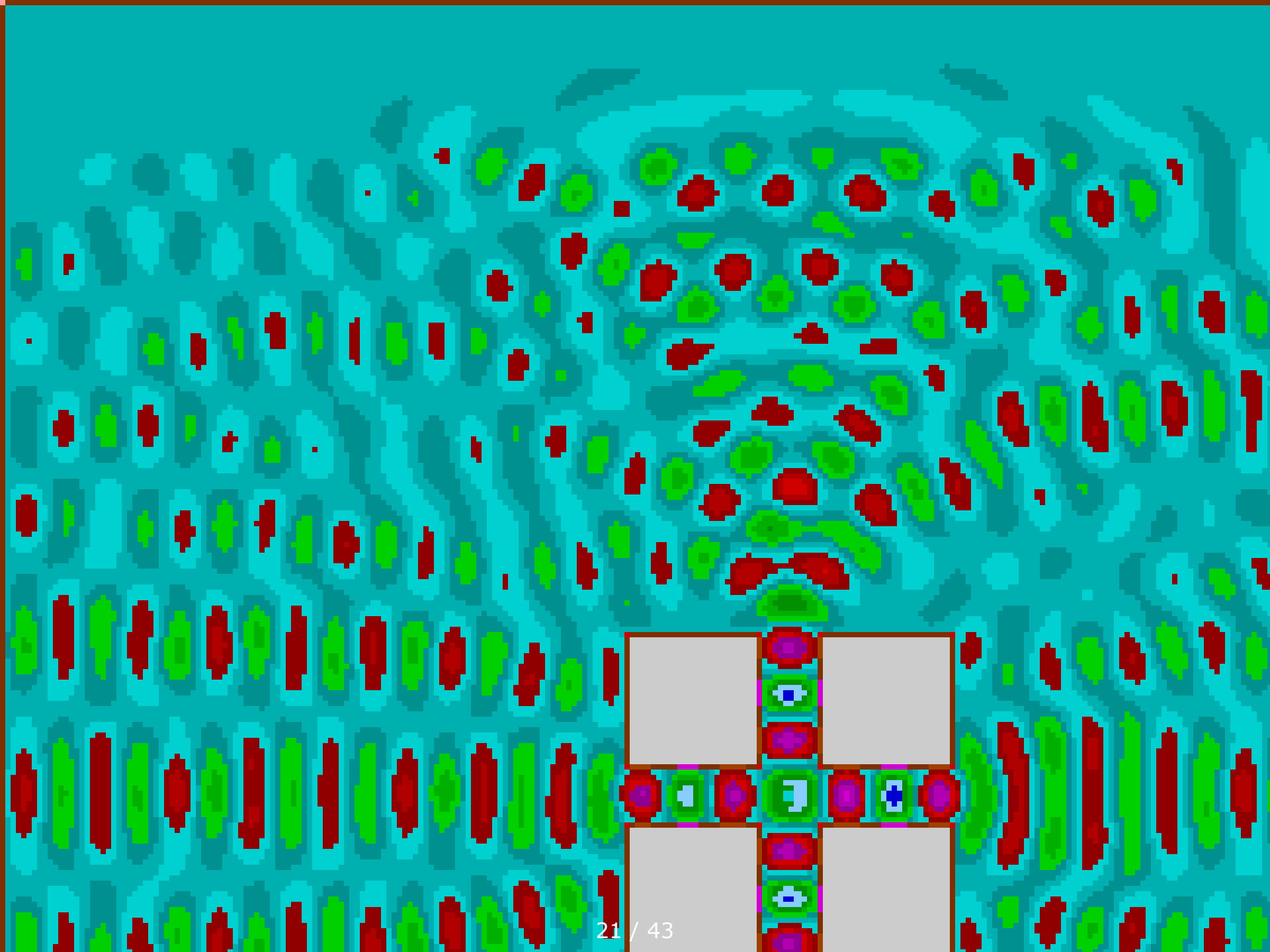
# Emergence:

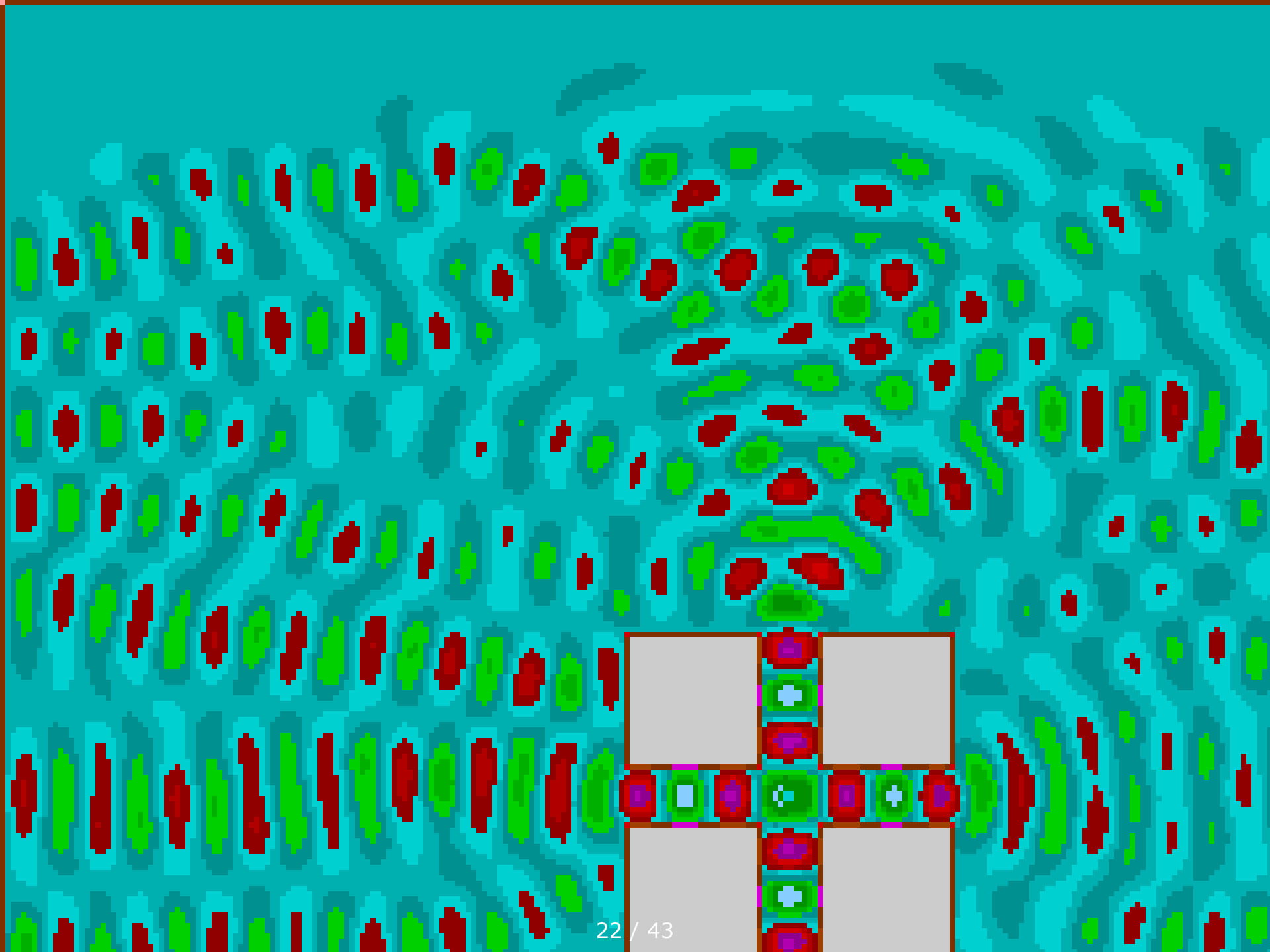


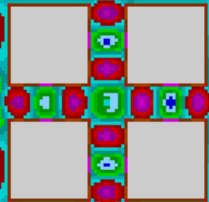
**Relational Causality:** bottom-up **and** top-down

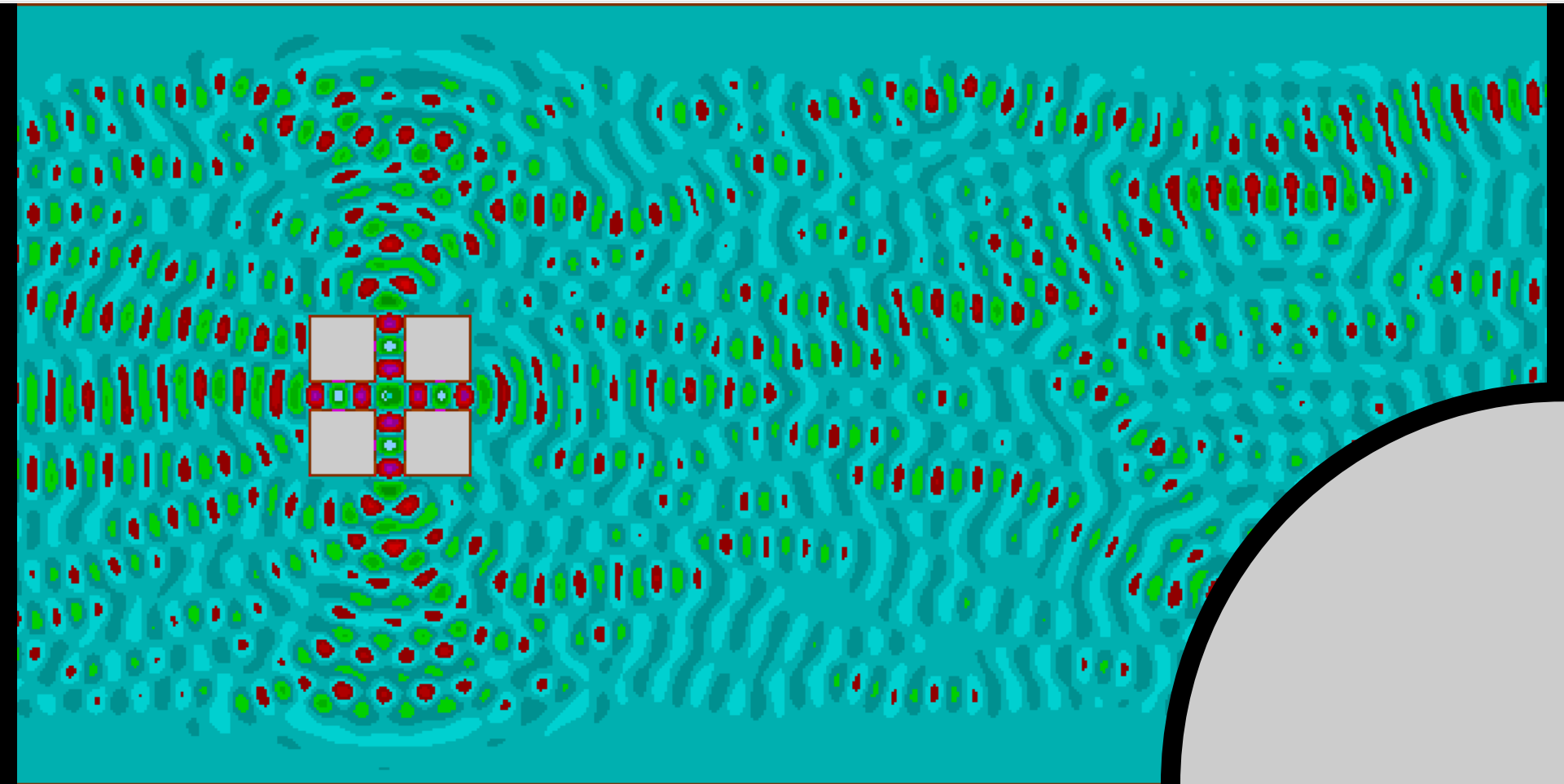
*Our claims:*

- usual formulation of de Broglie - Bohm **guidance equation** is of the “invisible hand” type i.e., “mysteriously” reaching out from configuration space into real space
- however, guidance equation is completely understandable in a “natural” way (Pearle) in real coordinate space, under the viewpoint of emergence
- a basic characteristic of emergent systems can be described via relational causality











### relational causality:

co-evolution of processes stemming from the "local" slits and those from the "global" environment, including the macroscopic boundary conditions ("confluence")

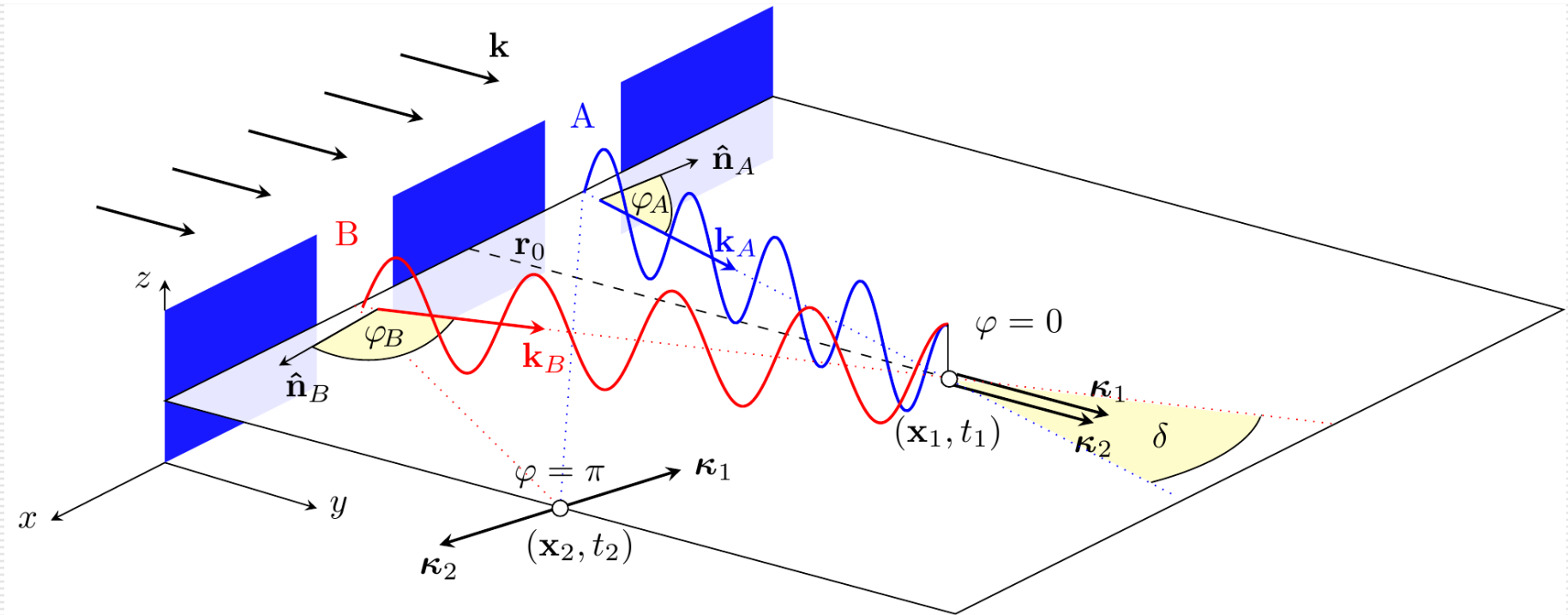
recall **classical** double slit formula (mean velocity  $v$ : 1 channel per slit)

→ total intensity: 
$$P = P(1) + P(2) = R_1^2 + R_2^2 + 2 R_1 R_2 \cos \varphi$$

now we look for **superclassical** interference formula:

3 channels per slit, due to additional sub-quantum diffusion velocities  $u_L$  and  $u_R$  next to  $v$ , *and all three co-evolving*

emergent velocities:  $v_i = \frac{1}{m} \hbar \kappa_i$



## 2 (or $n$ ) Gaussian slits: probability density field and emergent trajectories

$\mathbf{v}_i, \mathbf{u}_{iR}, \mathbf{u}_{iL}$  with average orthogonality  $\mathbf{v}_i \cdot \mathbf{u}_i = 0$   
i.e.,  $3n$  channels

$n = 2$  :

$$\mathbf{w}_1 := \mathbf{v}_1, \quad \mathbf{w}_2 := \mathbf{u}_{1R}, \quad \mathbf{w}_3 := \mathbf{u}_{1L}, \quad \mathbf{w}_4 := \mathbf{v}_2, \quad \mathbf{w}_5 := \mathbf{u}_{2R}, \quad \mathbf{w}_6 := \mathbf{u}_{2L}$$

→ conditional probability  $P(\mathbf{w}_i)$ :

all amplitude contributions  $R(\mathbf{w}_j)$  of total system's wave field  
projected on amplitude  $R(\mathbf{w}_i)$  at point  $(\mathbf{x}, t)$  of one channel  $(\mathbf{w}_i)$ :

$$P(\mathbf{w}_i) = R(\mathbf{w}_i) \hat{\mathbf{w}}_i \sum_{j=1}^6 \hat{\mathbf{w}}_j R(\mathbf{w}_j)$$

→ 'local' current  $\mathbf{J}(\mathbf{w}_i)$  per channel: 'local' intensity-weighted velocity

$$\mathbf{J}(\mathbf{w}_i) = \mathbf{w}_i P(\mathbf{w}_i)$$

Thus, e.g.

$$\mathbf{J}(\mathbf{v}_1) = \mathbf{v}_1 P(\mathbf{v}_1) = \mathbf{v}_1 R_1 \hat{\mathbf{v}}_1 \cdot (\hat{\mathbf{v}}_1 R_1 + \hat{\mathbf{u}}_{1R} R_1 + \hat{\mathbf{u}}_{1L} R_1 + \hat{\mathbf{v}}_2 R_2 + \hat{\mathbf{u}}_{2R} R_2 + \hat{\mathbf{u}}_{2L} R_2).$$

With

$$\hat{\mathbf{u}}_{iR} + \hat{\mathbf{u}}_{iL} = 0 \quad \rightarrow \quad \mathbf{J}(\mathbf{v}_1) = \mathbf{v}_1 (R_1^2 + R_1 R_2 \cos \varphi),$$

$$\mathbf{J}(\mathbf{v}_2) = \mathbf{v}_2 P(\mathbf{v}_2) = \mathbf{v}_2 (R_2^2 + R_1 R_2 \cos \varphi)$$

$$\mathbf{J}(\mathbf{u}_{1R}) = u_{1R} P(\mathbf{u}_{1R}) = \mathbf{u}_{1R} (R_1 \hat{\mathbf{u}}_{1R} \cdot \hat{\mathbf{v}}_2 R_2) = \mathbf{u}_{1R} R_1 R_2 \cos \left( \frac{\pi}{2} - \varphi \right) = \mathbf{u}_{1R} R_1 R_2 \sin \varphi$$

$$\mathbf{J}(\mathbf{u}_{1L}) = \mathbf{u}_{1L} P(\mathbf{u}_{1L}) = \mathbf{u}_{1L} (R_1 \hat{\mathbf{u}}_{1L} \cdot \hat{\mathbf{v}}_2 R_2) = \mathbf{u}_{1L} R_1 R_2 \cos \left( \frac{\pi}{2} + \varphi \right) = -\mathbf{u}_{1L} R_1 R_2 \sin \varphi$$

$$\mathbf{J}(\mathbf{u}_{2R}) = \mathbf{u}_{2R} P(\mathbf{u}_{2R}) = \mathbf{u}_{2R} (R_2 \hat{\mathbf{u}}_{2R} \cdot \hat{\mathbf{v}}_1 R_1) = \mathbf{u}_{2R} R_1 R_2 \cos \left( \frac{\pi}{2} + \varphi \right) = -\mathbf{u}_{2R} R_1 R_2 \sin \varphi$$

$$\mathbf{J}(\mathbf{u}_{2L}) = \mathbf{u}_{2L} P(\mathbf{u}_{2L}) = \mathbf{u}_{2L} (R_2 \hat{\mathbf{u}}_{2L} \cdot \hat{\mathbf{v}}_1 R_1) = \mathbf{u}_{2L} R_1 R_2 \cos \left( \frac{\pi}{2} - \varphi \right) = \mathbf{u}_{2L} R_1 R_2 \sin \varphi$$

total current for two-slit system:  $\mathbf{J}_{\text{tot}} = \sum_i \mathbf{J}(\mathbf{w}_i)$

$$\mathbf{J}_{\text{tot}} = R_1^2 \mathbf{v}_1 + R_2^2 \mathbf{v}_2 + R_1 R_2 (\mathbf{v}_1 + \mathbf{v}_2) \cos \varphi + \underbrace{R_1 R_2 (\mathbf{u}_1 - \mathbf{u}_2) \sin \varphi}_{\text{genuinely qm. term}}$$

... and analogously for  $n$  slits.

total probability density:  $P_{\text{tot}} = \sum_i P(\mathbf{w}_i)$

$$\begin{aligned} P_{\text{tot}} &= (R_1 \hat{\mathbf{v}}_1 + R_1 \hat{\mathbf{u}}_{1R} + R_1 \hat{\mathbf{u}}_{1L} + R_2 \hat{\mathbf{v}}_2 + R_2 \hat{\mathbf{u}}_{2R} + R_2 \hat{\mathbf{u}}_{2L})^2 \\ &= (R_1 \hat{\mathbf{v}}_1 + R_2 \hat{\mathbf{v}}_2)^2 = R_1^2 + R_2^2 + 2R_1 R_2 \cos \varphi \end{aligned}$$

$$\mathbf{J}_{\text{tot}} = P_{\text{tot}} \mathbf{v}_{\text{tot}}$$

$$\mathbf{v}_{\text{tot}} = \frac{R_1^2 \mathbf{v}_1 + R_2^2 \mathbf{v}_2 + R_1 R_2 (\mathbf{v}_1 + \mathbf{v}_2) \cos \varphi + R_1 R_2 (\mathbf{u}_1 - \mathbf{u}_2) \sin \varphi}{R_1^2 + R_2^2 + 2R_1 R_2 \cos \varphi}$$

... Bohmian velocity field!

*Average Orthogonality Condition (fluctuations unbiased):*

$$\overline{\mathbf{p} \cdot \delta \mathbf{p}} = m^2 \overline{\mathbf{v} \cdot \mathbf{u}} = 0$$

$$\mathbf{p}_{tot} = \mathbf{p} + \delta \mathbf{p} = m(\mathbf{v} + \mathbf{u}) = \nabla S - \hbar \frac{\nabla R}{R}$$

With **Madelung transformation**  $\psi^{(*)} = R e^{(-)\frac{i}{\hbar} S}$ ,  $P = R^2 = |\psi|^2$

$$\overline{p_{tot}^2} = \overline{p^2} + \overline{(\delta p)^2} = \hbar^2 \left[ \overline{\left( \frac{\nabla S}{\hbar} \right)^2} + \overline{\left( \frac{\nabla R}{R} \right)^2} \right] = \hbar^2 \overline{\left| \frac{\nabla \psi}{\psi} \right|^2}$$

= *the "vocabulary"*, i.e., for translations between sub-quantum and quantum "languages"

## Translation into “quantum language”:

inserting “the vocabulary” into current

$$\begin{aligned}
 \mathbf{J}_{\text{tot}} &= R_{\text{tot}}^2 \mathbf{v}_{\text{tot}} \\
 &= (\psi_1 + \psi_2)^* (\psi_1 + \psi_2) \frac{1}{2} \left[ \frac{1}{m} \left( -i\hbar \frac{\nabla(\psi_1 + \psi_2)}{(\psi_1 + \psi_2)} \right) + \frac{1}{m} \left( i\hbar \frac{\nabla(\psi_1 + \psi_2)^*}{(\psi_1 + \psi_2)^*} \right) \right] \\
 &= -\frac{i\hbar}{2m} [\Psi^* \nabla \Psi - \Psi \nabla \Psi^*] = \frac{1}{m} \text{Re} [\Psi^* (-i\hbar \nabla_{\mathbf{n}}) \Psi], \\
 &\quad \text{with } \Psi = \psi_1 + \psi_2
 \end{aligned}$$

... exact qu. m. current

(and analogously for  $\mathbf{n}$ -particle systems !)

What looks like the necessity to superpose  
("invisible hand")  
wave functions in configuration space

can equally be obtained by

superpositions of all relational amplitude configurations,

i.e. of waves in real space.  
(emergence)



With  $\mathbf{w}_i = \frac{\mathbf{J}(\mathbf{w}_i)}{P(\mathbf{w}_i)}$  and the classical composition principles

$$\mathbf{J}_{\text{tot}} = \sum_i \mathbf{J}(\mathbf{w}_i) \quad \text{and} \quad P_{\text{tot}} = \sum_i P(\mathbf{w}_i)$$

$$\rightarrow \mathbf{v}_{\text{tot}} = \frac{\mathbf{J}_{\text{tot}}}{P_{\text{tot}}} = \frac{\sum_i \mathbf{J}(\mathbf{w}_i)}{\sum_i P(\mathbf{w}_i)} = \frac{\sum_i P(\mathbf{w}_i) \mathbf{w}_i}{\sum_i P(\mathbf{w}_i)}$$

the “guidance equation” *postulated* by de Broglie-Bohm is here derived and explained via relational causality with  $\mathbf{v}_{\text{tot}}$  being an *emergent* velocity

$$\mathbf{v}_{\text{tot}} = \mathbf{v}_{\text{tot}}(\text{Bohm}) = \mathbf{v}_{\text{tot}}(\text{QM})$$

$$\begin{aligned}
 &= \frac{\sum_i P(\mathbf{w}_i) \mathbf{w}_i}{\sum_i P(\mathbf{w}_i)} \\
 &= \frac{R_1^2 \mathbf{v}_1 + R_2^2 \mathbf{v}_2 + R_1 R_2 (\mathbf{v}_1 + \mathbf{v}_2) \cos \varphi + R_1 R_2 (\mathbf{u}_1 - \mathbf{u}_2) \sin \varphi}{R_1^2 + R_2^2 + 2R_1 R_2 \cos \varphi} \\
 &= \frac{\frac{1}{m} \text{Re} [\Psi^* (-i\hbar \nabla) \Psi]}{\Psi^* \Psi}, \quad \text{with} \quad \Psi = \sum_j \psi_j
 \end{aligned}$$

→ Sorkin's first and h.o. sum rules directly obtained:

$$I_A = P_A(\mathbf{v}_1) = R_1^2$$

$$I_{AB} = P_{AB} - P_A(\mathbf{v}_1) - P_B(\mathbf{v}_2) = 2R_1R_2 \cos \varphi$$

$$I_{ABC} = P_{ABC} - P_{AB} - P_{AC} - P_{BC} + P_A(\mathbf{v}_1) + P_B(\mathbf{v}_2) + P_C(\mathbf{v}_3) = 0$$

→ correspondence with standard quantum phenomenology,  
in particular with Born's rule

possible violations?

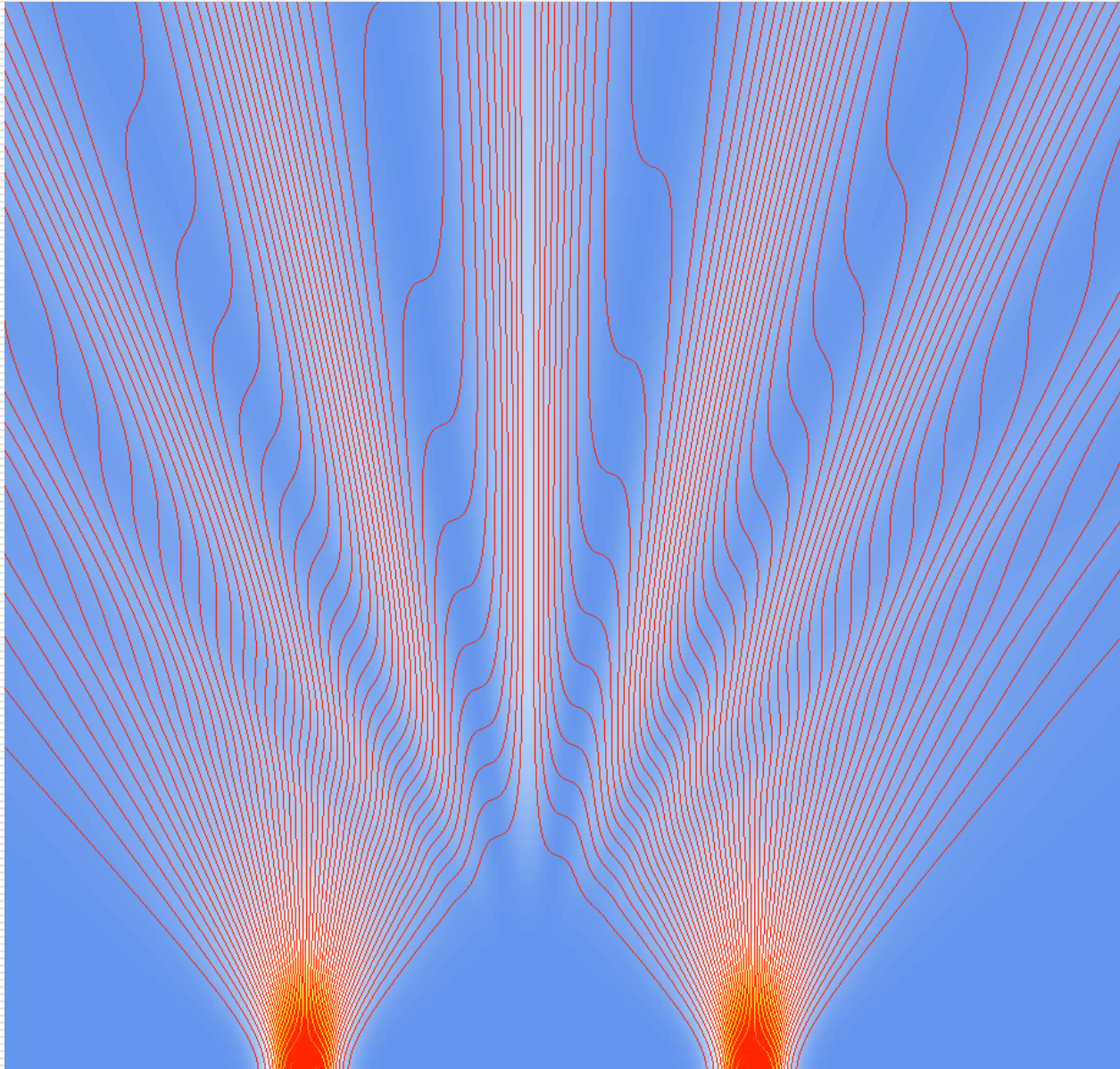
- 1) At very small time scales  $t \propto \frac{1}{\omega}$
- 2) Upon relaxation of "average orthogonality condition":  
is there a possible bias, e.g., between  $\mathbf{u}_L$  and  $\mathbf{u}_R$ ?

## Superluminal effects and non-signalling principle in EmQM approaches

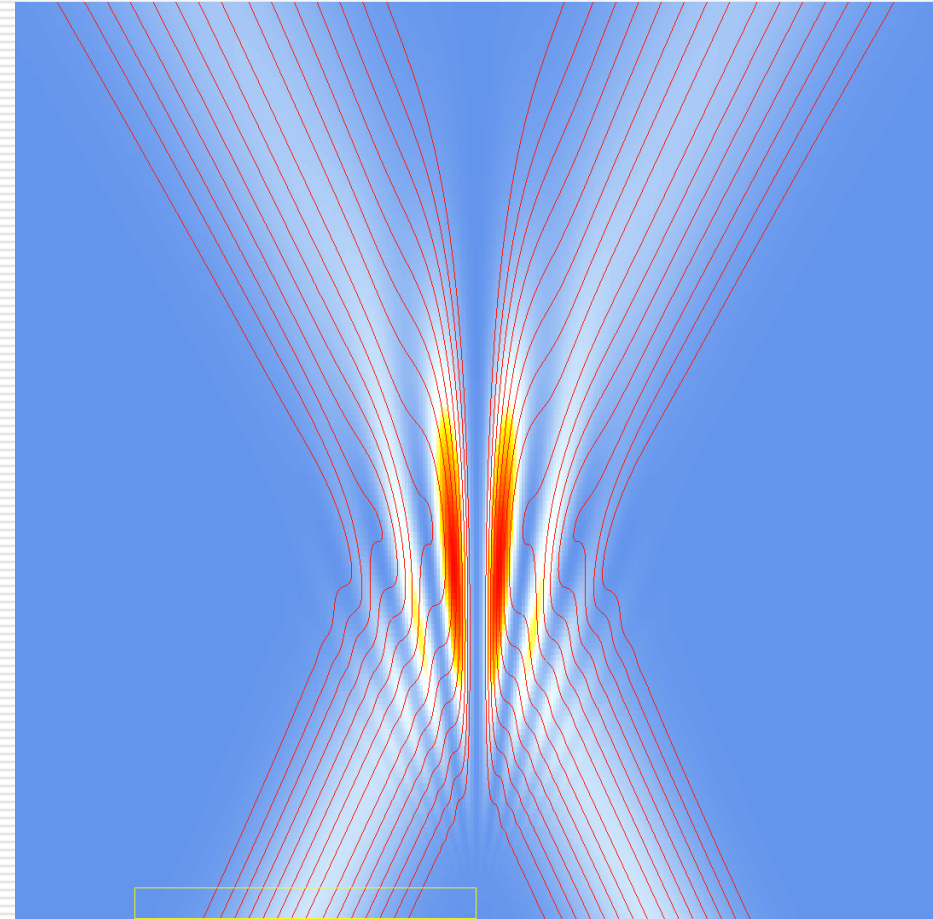
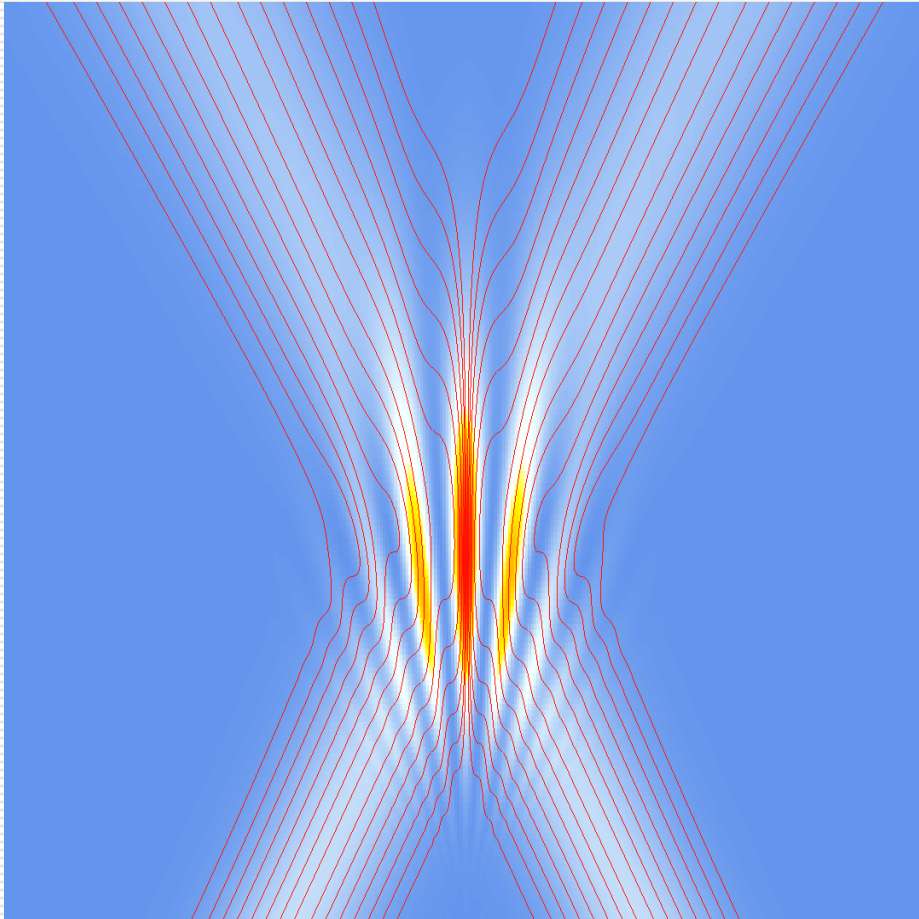
→ talk by Jan Walleczek (tomorrow)

Classical computer simulations: examples

# *Quantum interference, strong dispersion*

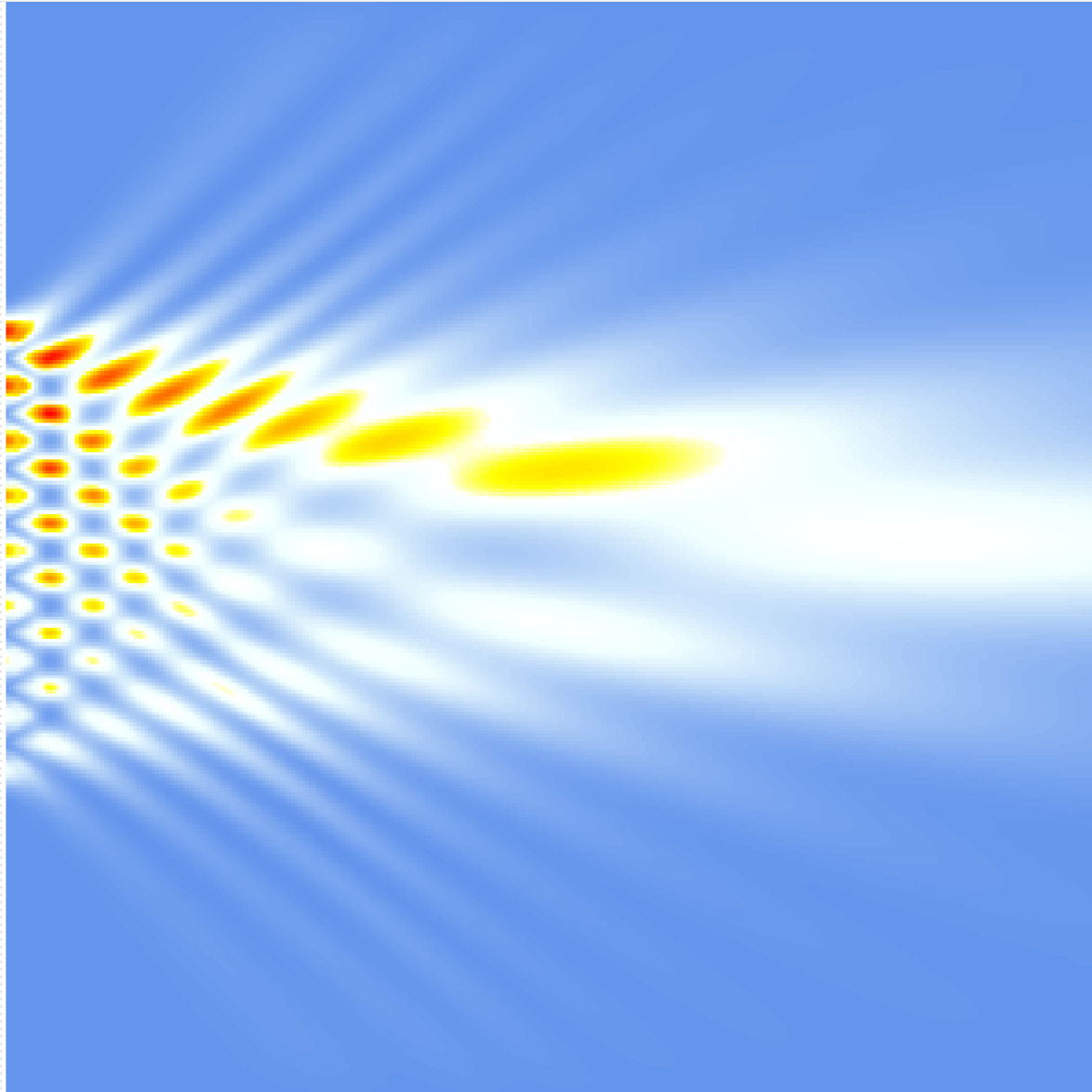


# *Quantum interference, weak dispersion*

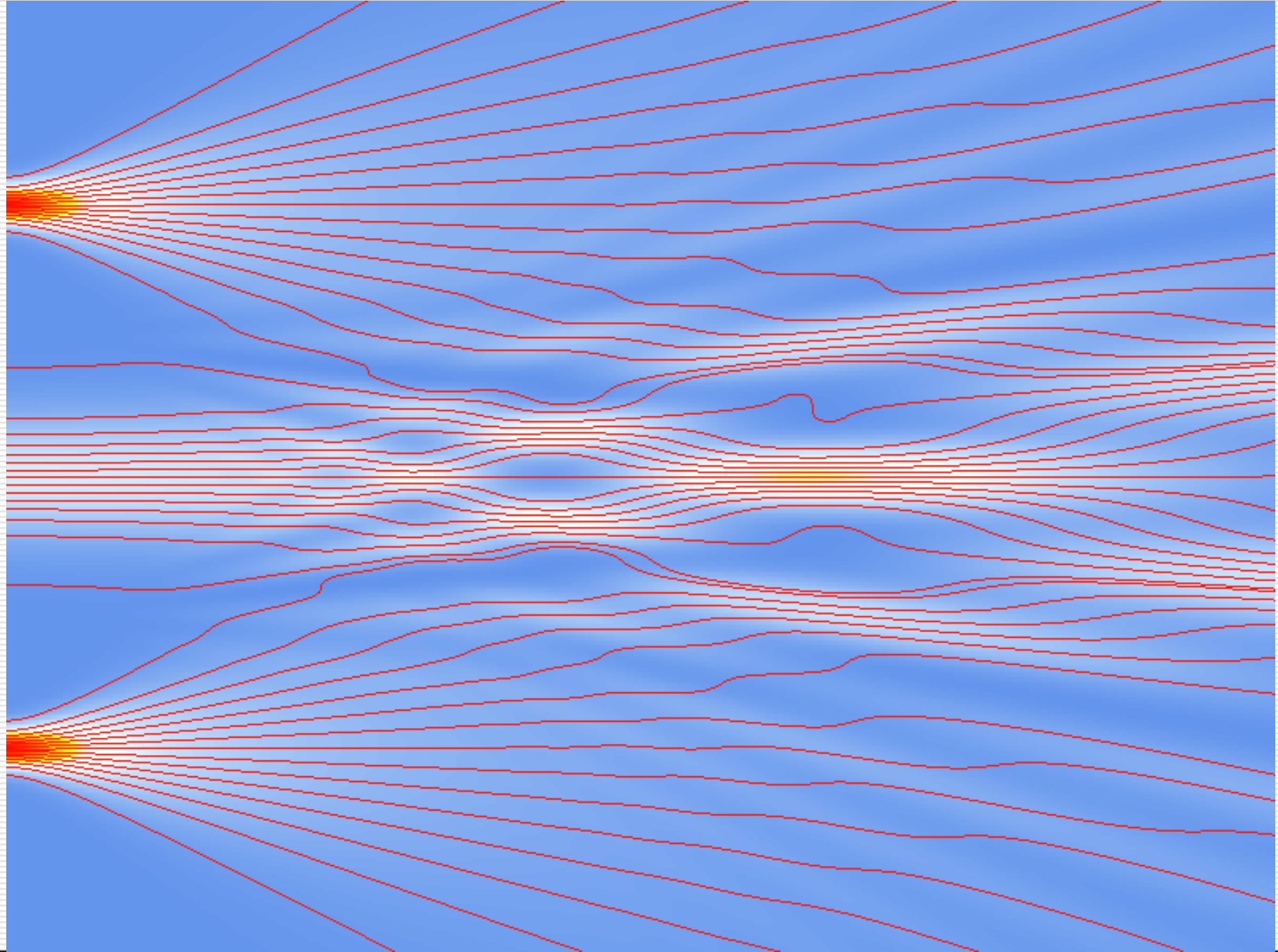


$$\Delta\varphi = \pi$$

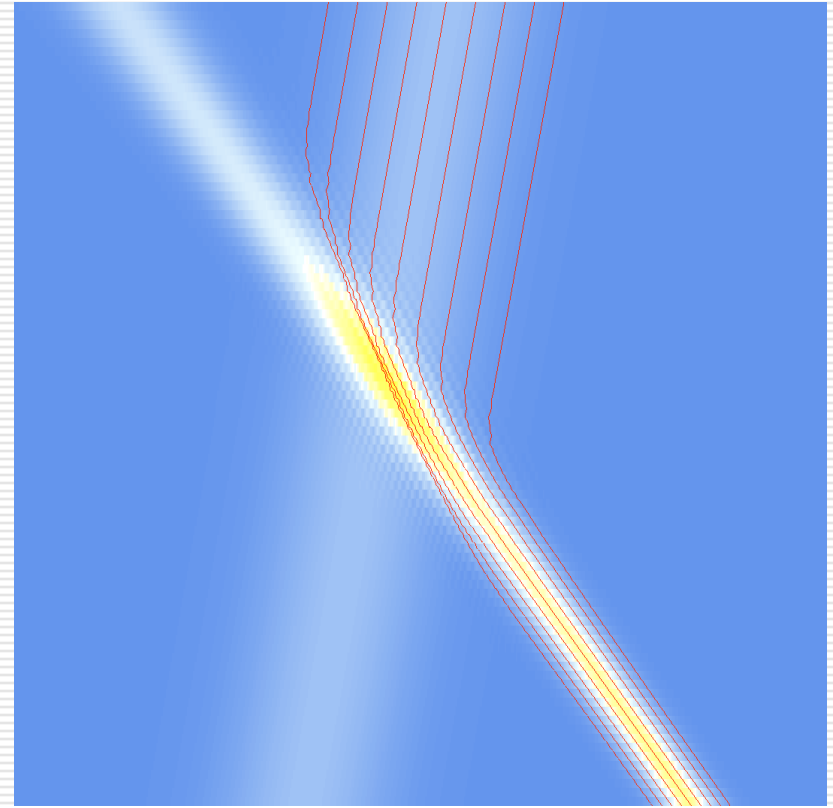
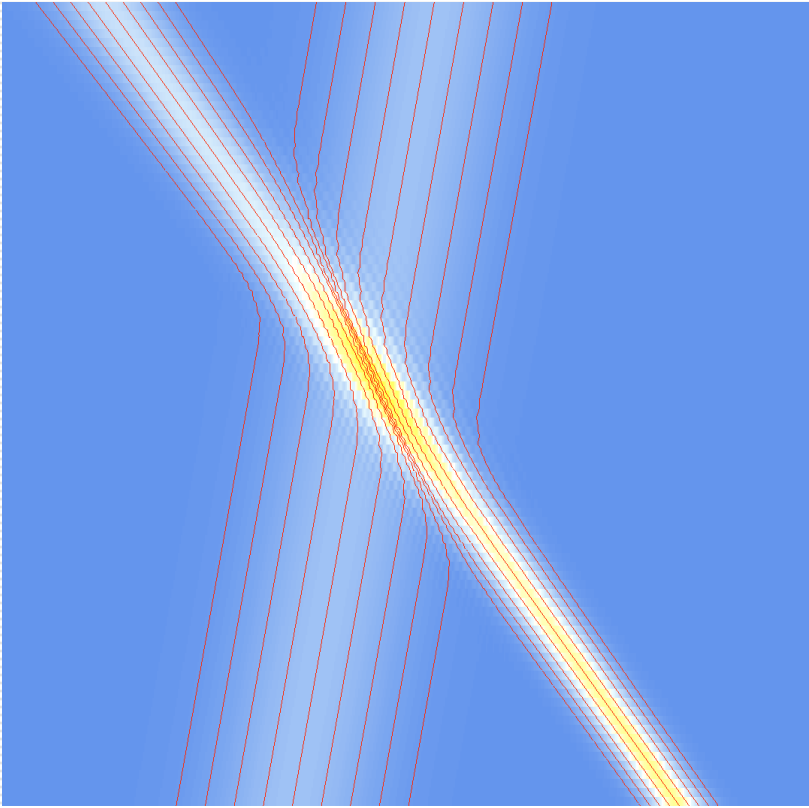
## Talbot carpet with gradually diminishing weights



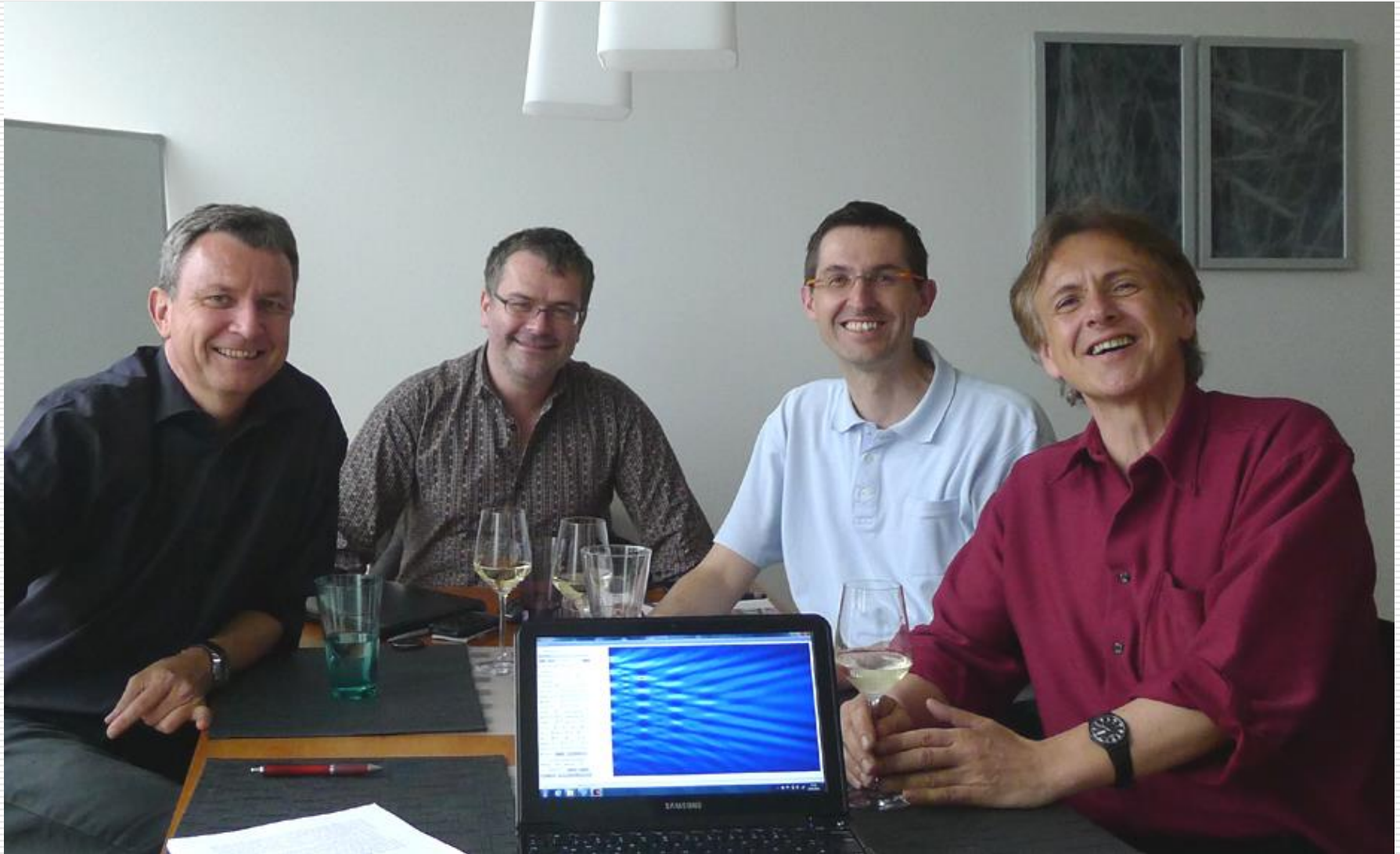




## A Sub-Quantum Cheshire Cat:



also: Scully's two alternatives in one scenario !



Gerhard Grössing

Herbert Schwabl

Johannes Mesa Pascasio

Siegfried Fussy