# Weak measurement and its experimental realisation for non-zero mass particles

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# Emerging experiments

Emerging theories are only ever accepted if accompanied by repeatable experiments.

Experimentalists look for new experiments rather than new theories. Where do we look for new physics?

Just over one year ago Basil approached me about possible experiments to complement his and David Bohm's theoretical work.

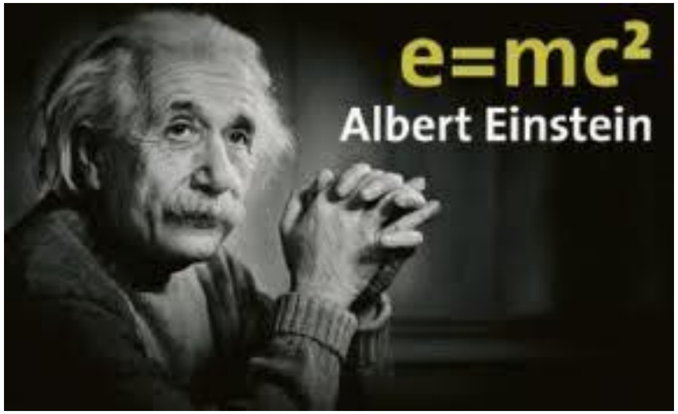
Specifically to reconstruct the tracks of Schrödinger particles in Young's 2-slit experiment using the weak measurement technique.

As a first step we decided to observe the weak measurement process with an extended version of the Stern-Gerlach experiment.

**These experiments are extremely challenging and I want report our progress and highlight the problems.**

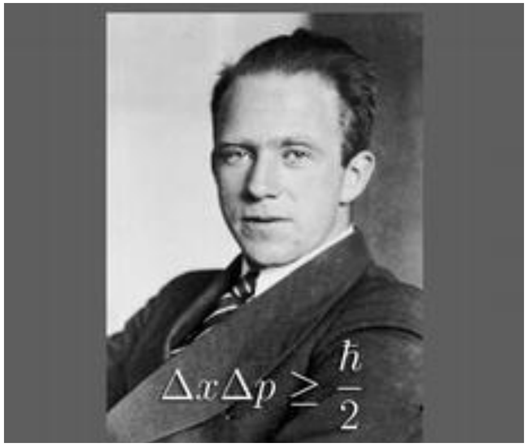
# Where do we look for new physics?

High energy LHC 14TeV




High background rates

Precision measurements, rare events, low energy



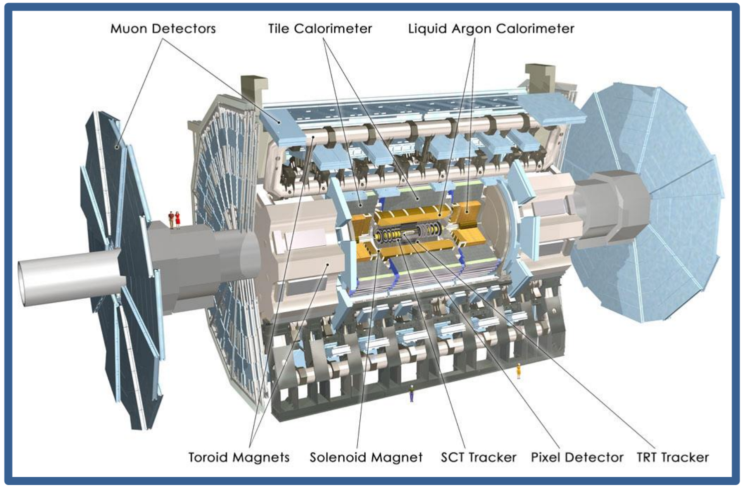

Low or zero background rates

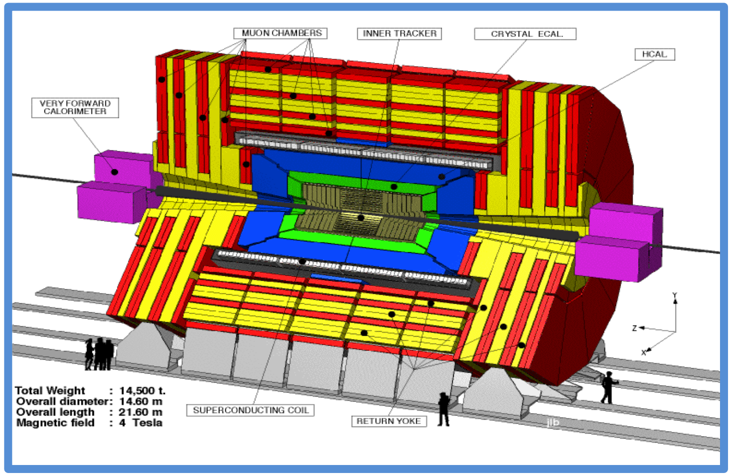
**Emerging QM:**

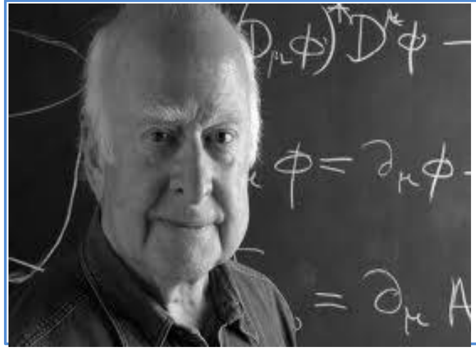
Are there new ways of looking at the problem?

Are there more fundamental issues to be addressed?

# High energy approach to physics beyond the SM



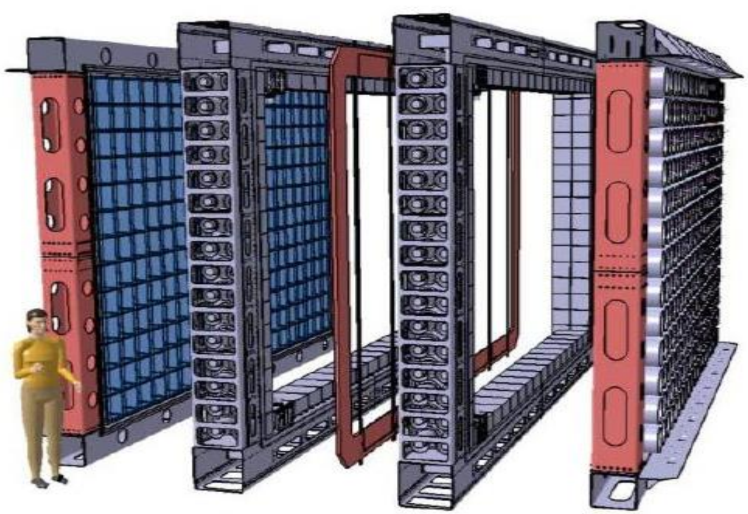
Discovered "a Higgs boson":
Confirmed mass 125 MeV and spin zero
Measuring couplings in expected channels.

No hint of SUSY
"The null results are not making people happy"
Phillip Schuster; Perimeter institute

# Precision measurements with neutrinos

Neutrino oscillations:
First observed by Ray Davis at Homestake (1970).
Atmospheric – SuperK (1998)
Solar – SNO – (2001)

## SuperNEMO



Neutrinolesss double beta decay: half life *>$10^{26}$ yrs*

- Neutrinos have mass and they mix.
- Precision measurements of mixing angles and $\Delta m^2$.
- **Nature of neutrinos** : Dirac ( $\nu \neq \bar{\nu}$ ) or Majorana ( $\nu = \bar{\nu}$ )
- **Absolute neutrino mass scale** : only limits so far :

  $$m_{\bar{\nu}_e} < 2.2 \text{ eV (Tritium end-point)}$$
  $$\Sigma m_{\nu_i} < 0.3 \text{ eV (Cosmology)}$$
- **Neutrino mass-hierarchy** :
  - Normal : $m_1 < m_2 < m_3$
  - Inverted : $m_3 < m_1 < m_2$
  - Quasi-degenerate : $m_1 \approx m_2 \approx m_3$
- **CP-violation in neutrino sector** :
  - Dirac phase : $\delta \neq 0, \pi$
  - **Majorana phases** : $\alpha_{21}, \alpha_{31} \neq 0, \pi$

# Precision measurements with muons: g-2

Brookhaven

Moving to FermiLab

$$a_\mu = \left(\frac{g-2}{2}\right)$$

$$a_\mu = \frac{\alpha}{2\pi} = 0.00116\ 140980$$
$$= 0.00116\ 591792\ (\text{SM all loops})$$

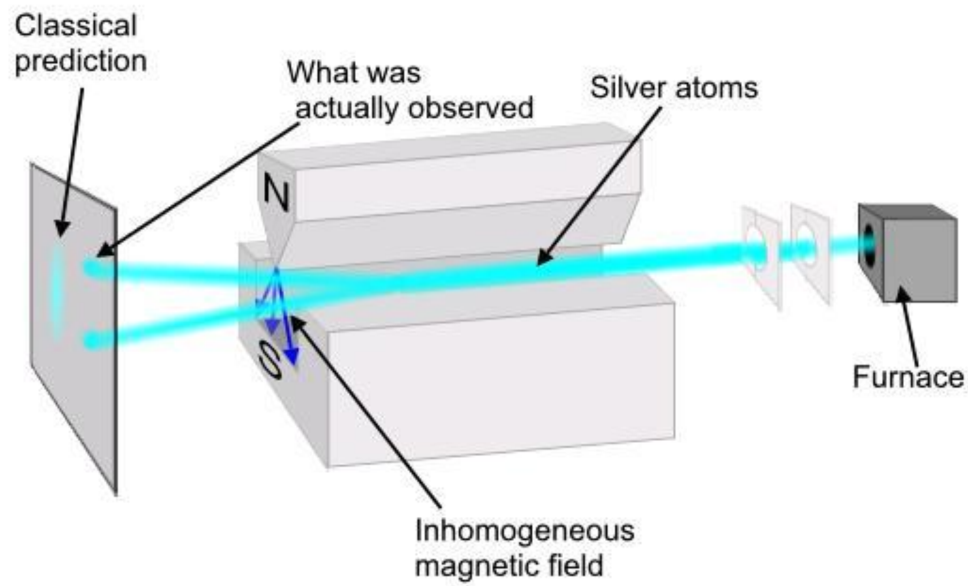
# Why weak (protective) measurement?

- Amplification of small signals:
  - Large amplification possible.
  - Unique in that the quantum system is effectively amplifying itself.
  - No need for external amplifier which always introduces noise.

- Tracks for Schrödinger particles:
  - Tracks for photons already demonstrated.
  - In principle can reconstruct the tracks for non-zero mass particles.
  - First evidence for the quantum potential.

# Original Stern-Gerlach experiment

Silver atoms were originally used. They have one valence electron around a filled core and it behaved like a spin-1/2 particle.




The classical prediction was the beam should spread out in a continuous manner.

The observation is a beam split into two parts: spin-up, spin-down.
**A STRONG MEASUREMENT**

Reduce the magnetic field the beams are not well separated but overlapping.
**A WEAK MEASUREMENT**

**Aharanov et al asked what would happen if you follow a weak measurement by a strong one (post-selection)?**

# Results from the original paper of Stern and Gerlach

**Der experimentelle Nachweisder Richtungs quantelung im Magnetfeld. Z. Phys 9 (1922) 349-352**

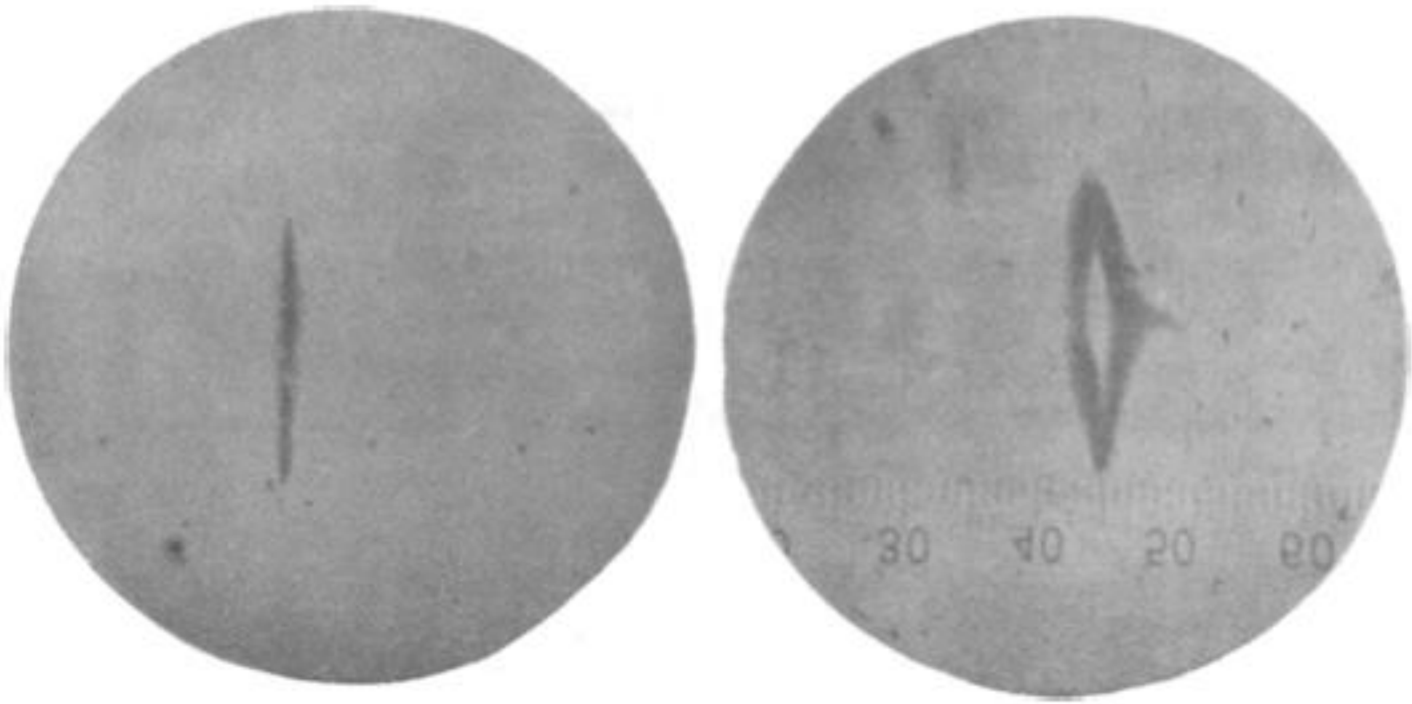



Fig. 2. Fig. 3.

**Magnet off** **Magnet on**

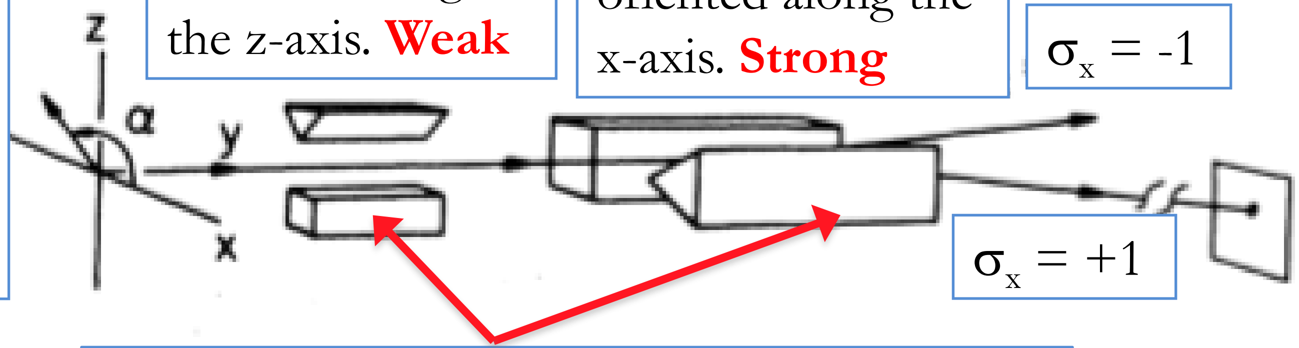
# Extended Stern-Gerlach apparatus

**I. M. Duck, P. M. Stevenson and Sudarshan, Phys. Rev. D, 1989**

The particles enter along the y-axis with spin oriented at angle $\alpha$ in the zx-plane.

The first S-G is oriented along the z-axis. **Weak**

The second S-G is oriented along the x-axis. **Strong**



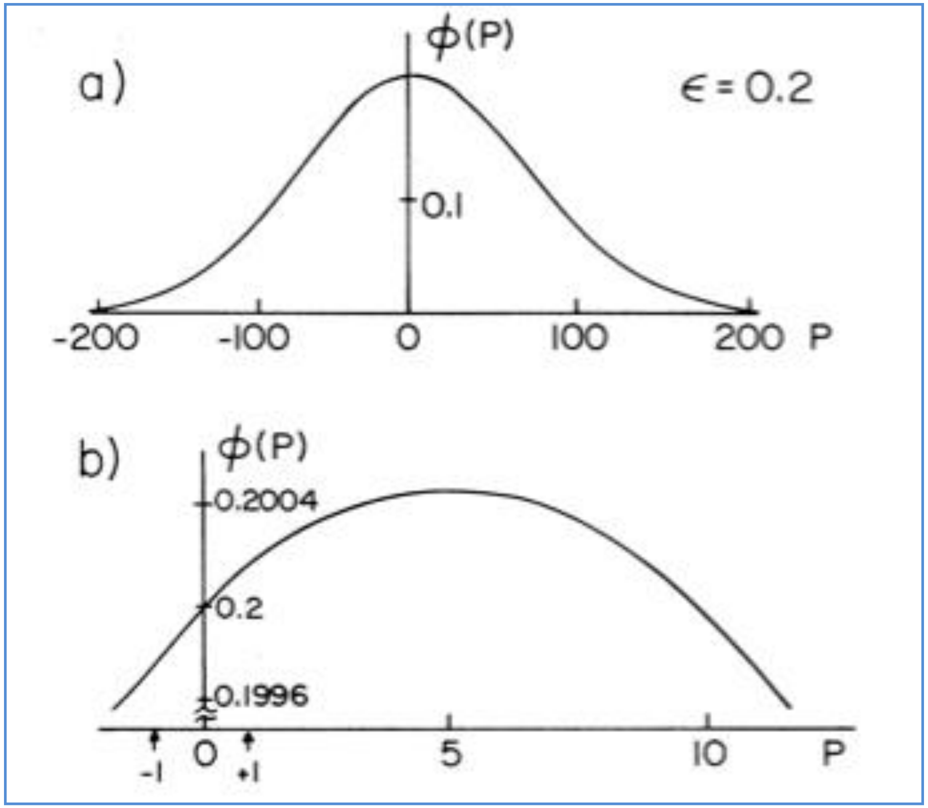

$\sigma_x = -1$

$\sigma_x = +1$

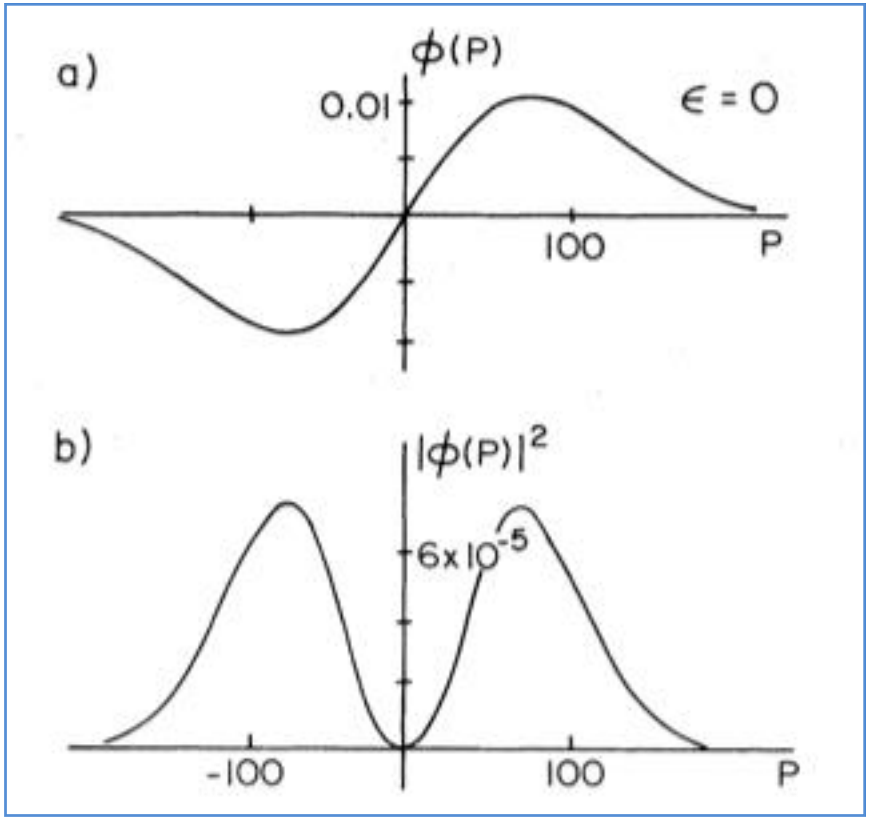
Non-commuting variables
spin z-axis, spin x-axis

- $A_w = \lambda \tan(\alpha/2)$, $\lambda \propto$ magnetic moment of the particle.
- Note what happens as $\alpha$ approaches $\pi$, $A_w$ gets very large.
- Need three magnets.

- What happens as $\alpha$ approaches $\pi$?
- Let $\varepsilon = \pi - \alpha$.
- $P = p_z/\lambda$,
- $p_z$ = transverse momentum $\lambda$ = wavelength of the particle




(a) $\phi(p)$ resembles a Gaussian.
(b) Close up shows the peak is at 5.



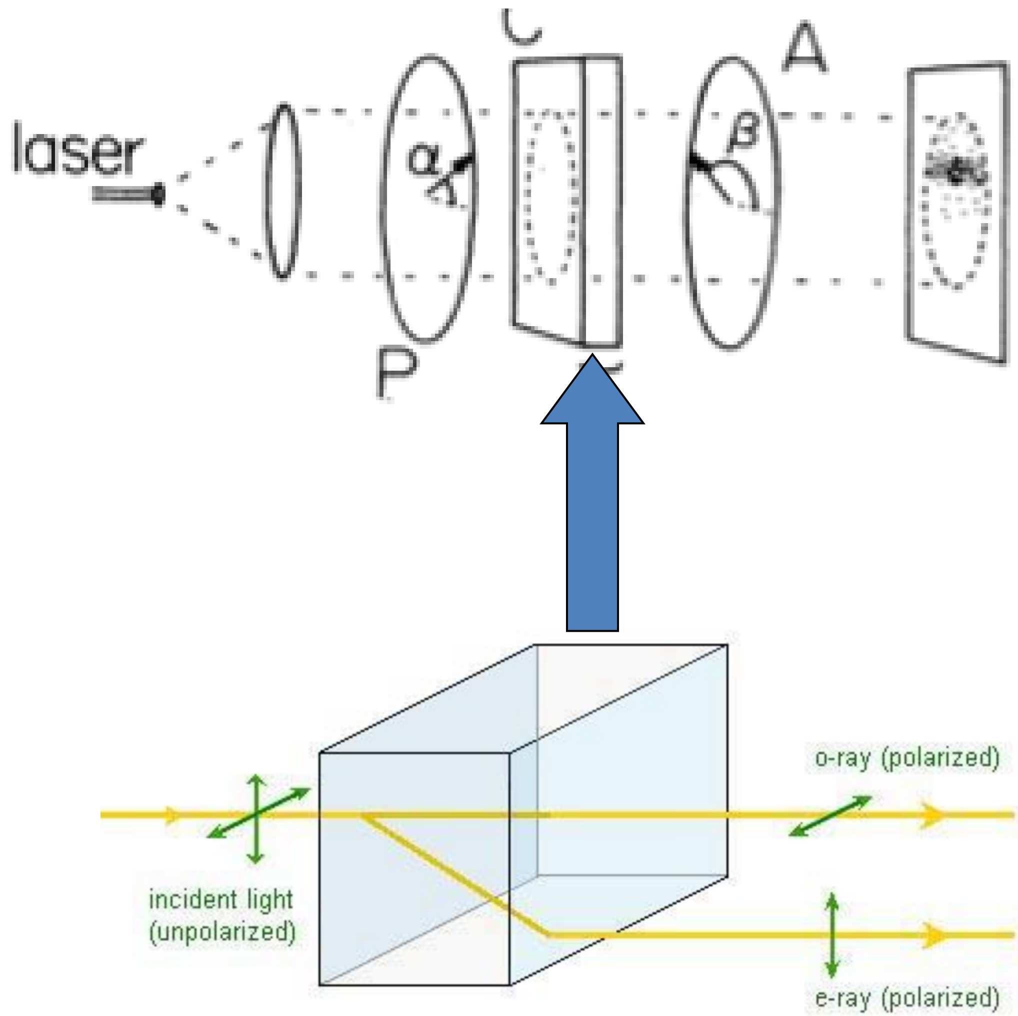

(a) $\phi(p)$ is now antisymmetric.
(b) The resulting probability distribution peaks at ±70.

# Optical analogue of extended Stern-Gerlach apparatus

Polarised light from a laser is used instead of spin-1/2 particles.

Polariser P and analyser A select the intial and final polarisations at angles α and βrespectively.

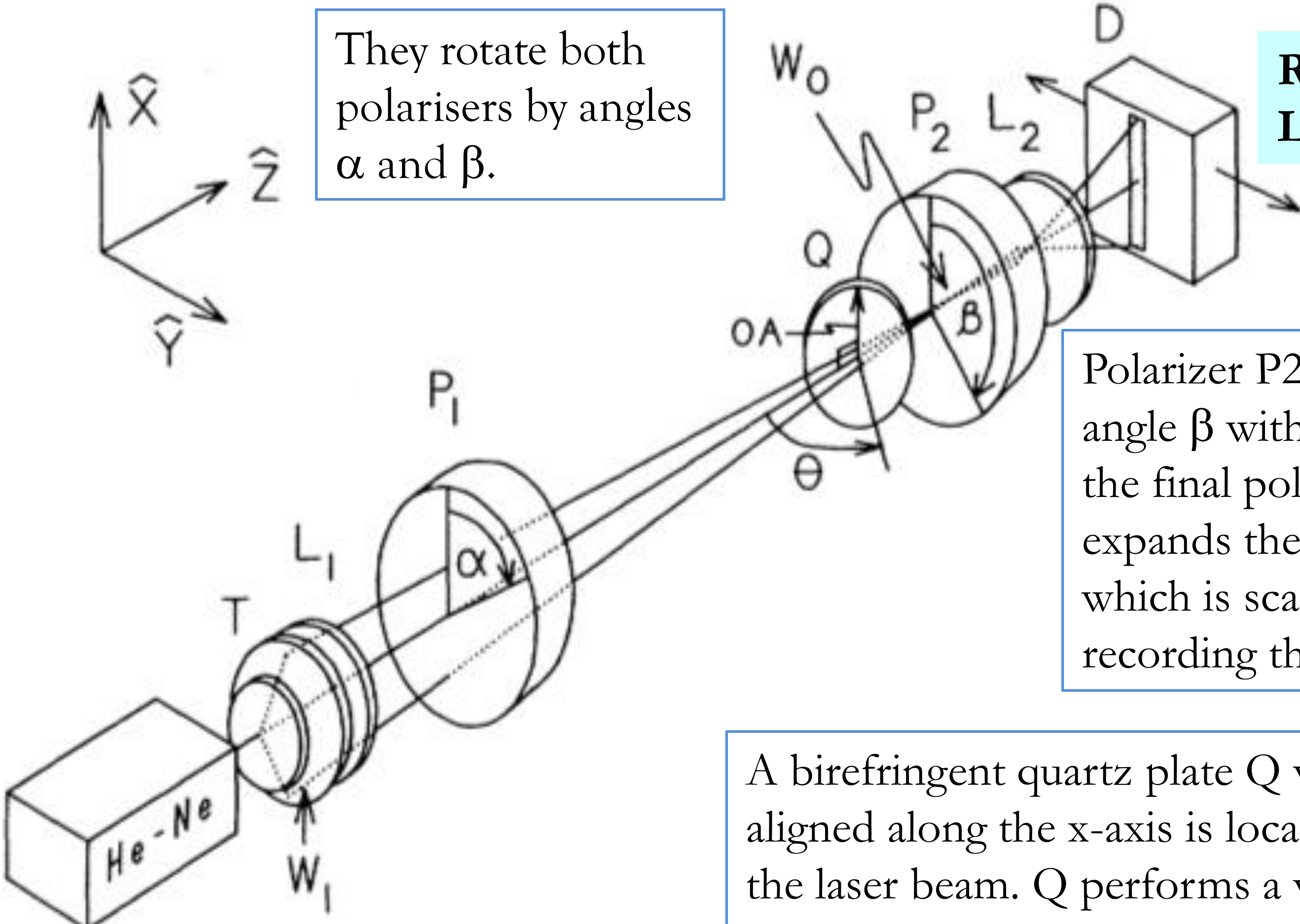
A birefringent crystal provides the weak measuring device. It introduces a small lateral displacement between the o-ray and the e-ray.



# Realisation of the optical analogue

They rotate both polarisers by angles $\alpha$ and $\beta$.

**Ritchie et al. Phys. Rev. Lett., 66, 1107**




Polarizer P2, whose axis makes an angle $\beta$ with the x-axis, post selects the final polarization state. Lens $L_2$ expands the image onto a detector D which is scanned along the y-axis, recording the intensity.

A birefringent quartz plate Q with optic axis (OA) aligned along the x-axis is located near the focus of the laser beam. Q performs a weak measurement by spatially separating the ordinary and extraordinary polarization components by a distance small compared to the waist of the beam.

Frequency-stabilized He-Ne laser is collimated, focused, and polarized at an angle $\alpha$ relative to the x-axis by telescope T, lens $L_1$, and polarizer $P_1$.

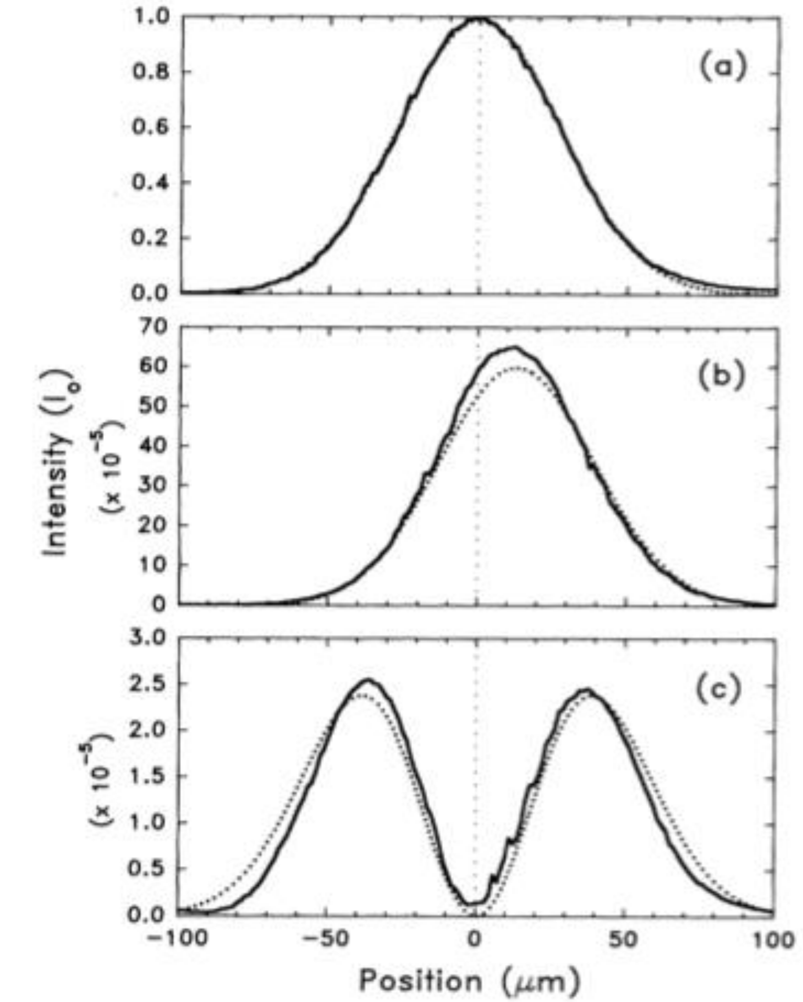
# Results of the optical analogue

The wavelength of the light a=0.64μm

(a) $\alpha=\beta=\pi/4$, corresponding to aligned polarizers. The measured intensity profile is the result of the constructive addition of two approximately Gaussian distributions.

(b) $\alpha= \pi/4$, $\beta =3 \pi/4+0.022$, corresponding to a measurement of the weak value. The centroid of the distribution is shifted by $A_w$=l2μm=20a.

(c) a= π/4, P=3π/4, corresponding to crossed polarizers, or orthogonal initial and final states. The separation of the two peaks is 120a.



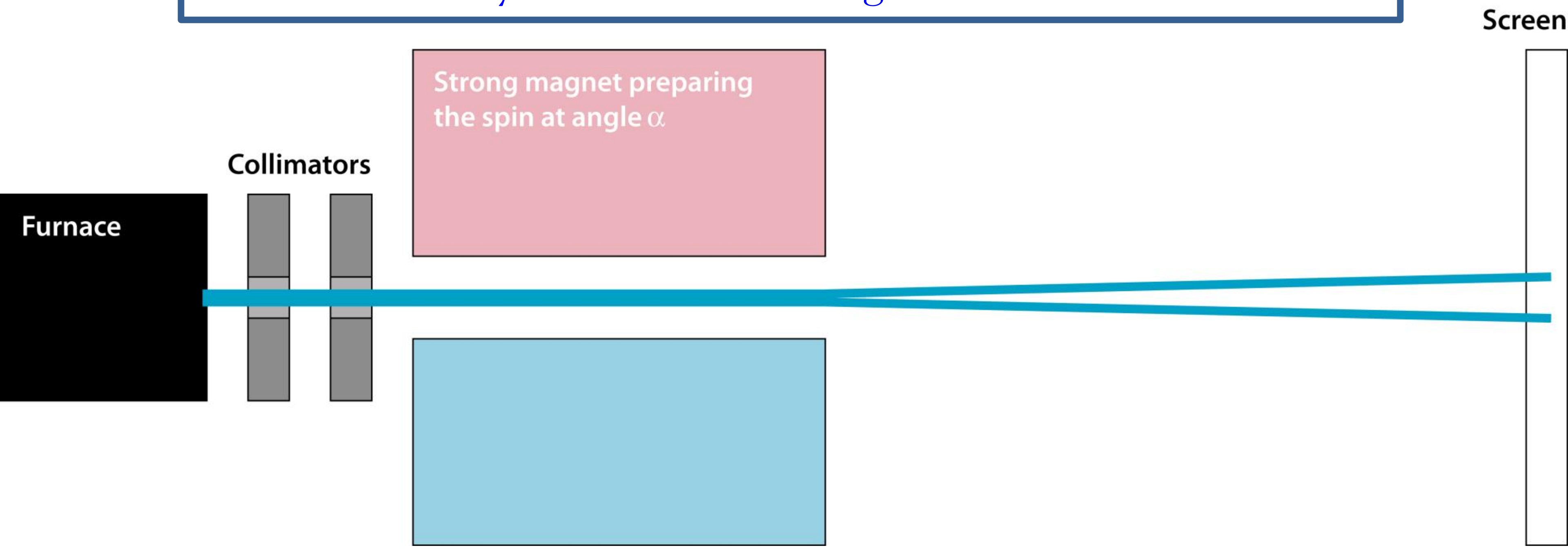
# Experimental proposal

- The first experiment is a modified Stern-Gerlach apparatus using neutral atoms specifically to see if we can observe the predicted weak measurement effect and measure its value within this context.

- The ultimate objective is to construct a Young's 2-slit experiment and reconstruct the trajectories of the non-zero mass particles using the weak measurement technique.

- **Compare them with the predicted Bohm trajectories assuming the quantum potential model.**

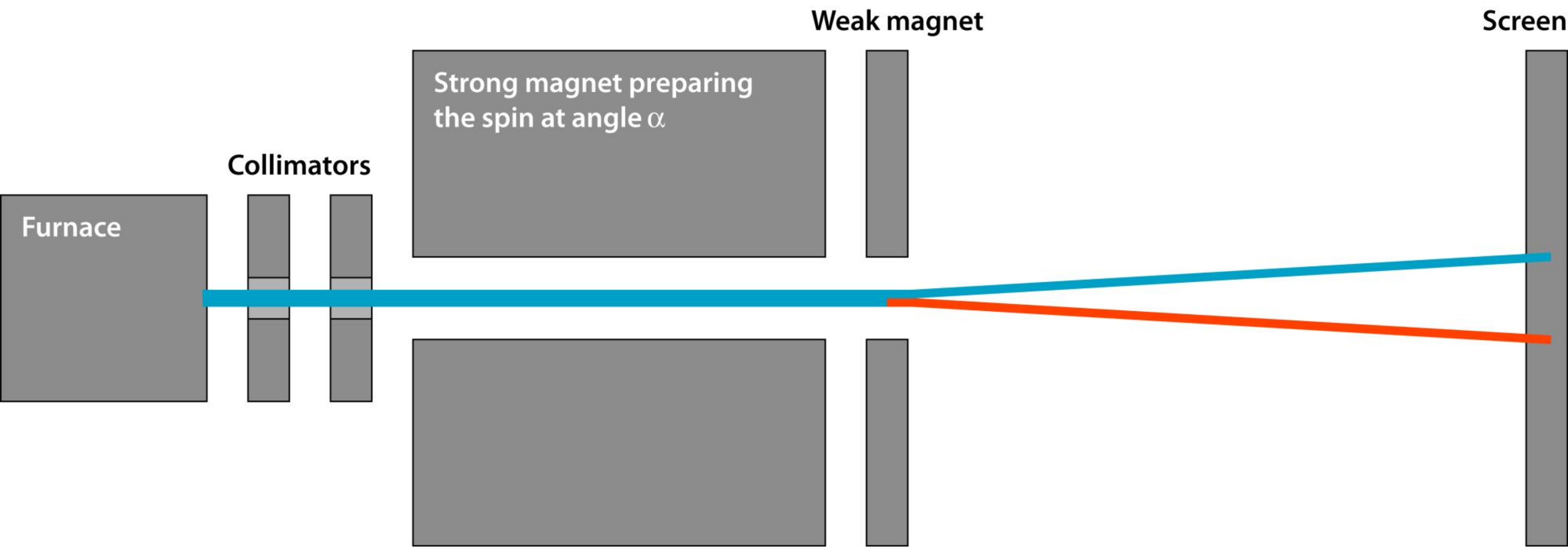
•Stage 1:
- •Produce a beam of atoms and perform standard S-G.
- •Rotate magnet and check rotation of beams.
- •Is it clearly observable with good statistics?




- •Polarisation of the beams are at angle $\alpha$, $\alpha+\pi$.
- •Plan to use both beams to double statistics.

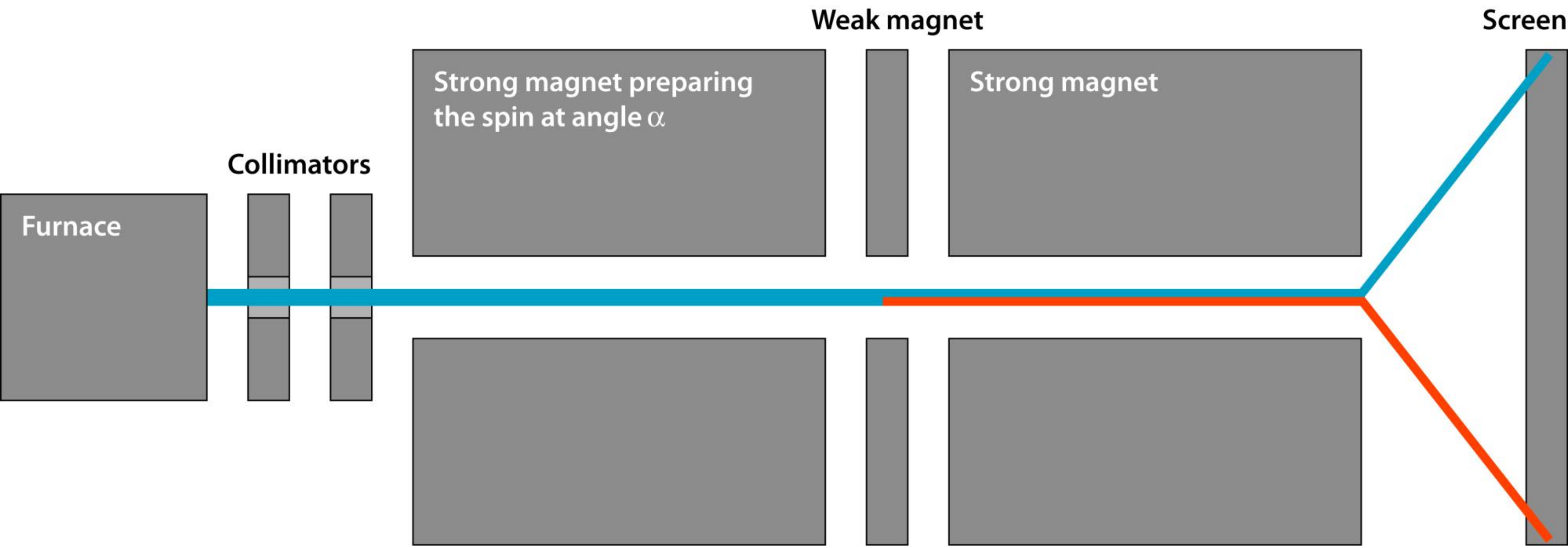
Stage 2:

- Put in weak magnet.
- How weak does it have to be (see latter discussion)?
- Is there a test I can carry out?



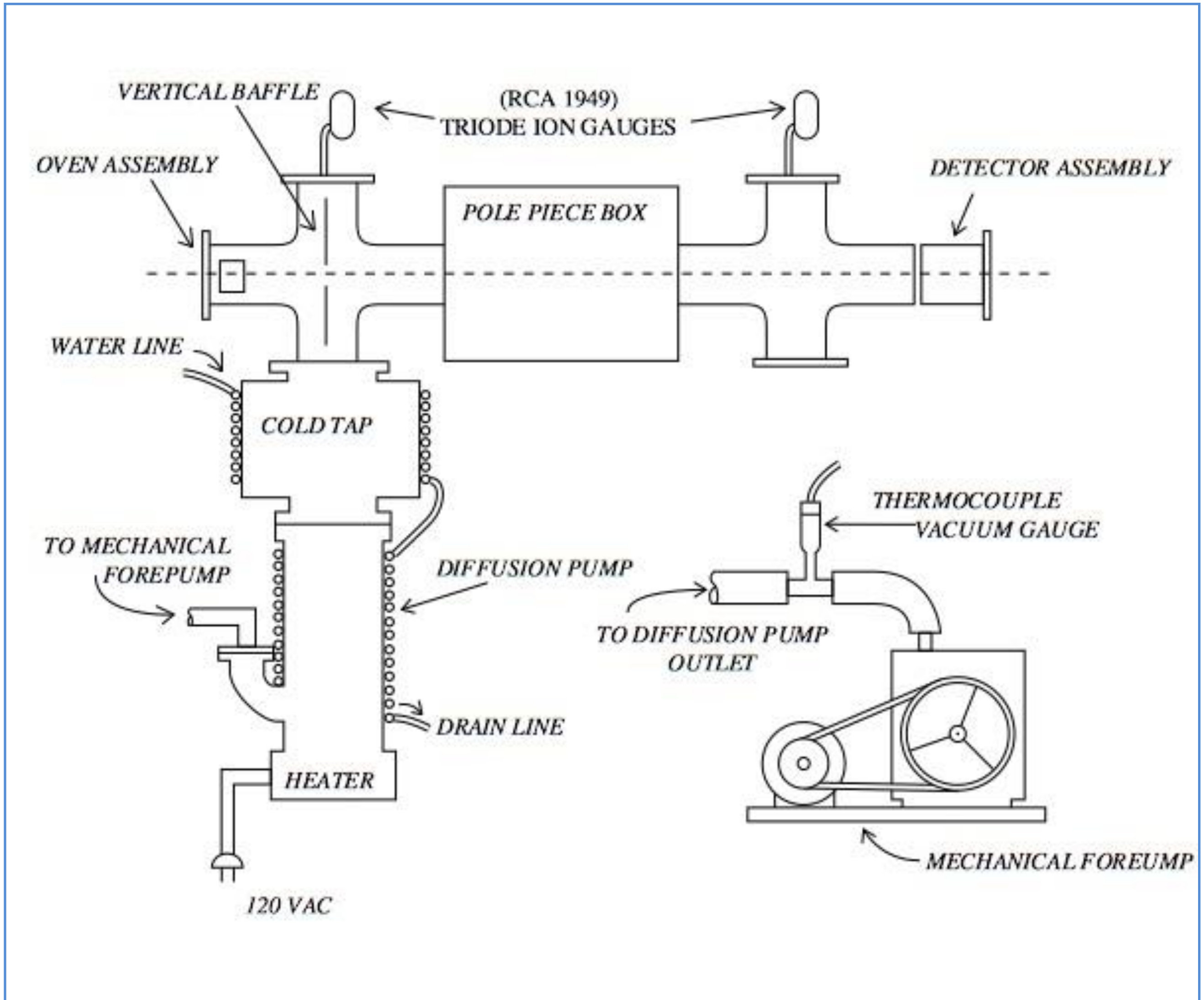

- The orientation of the weak magnet will define the *z*-axis

- Stage 3:
  - Put in third magnet.
  - Identical to the first.
  - Are the weak values observable?

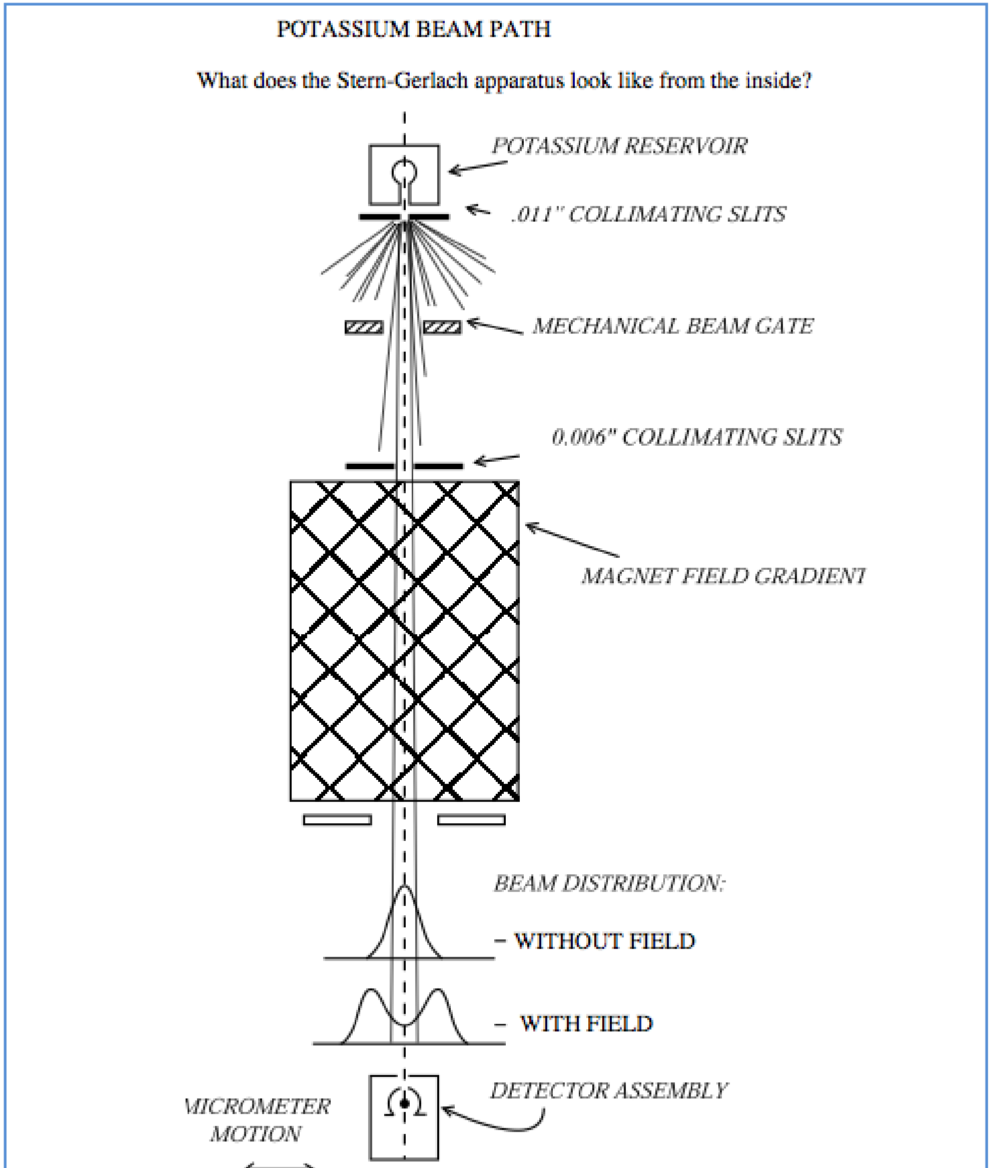



- **The orientation of the 3rd magnet will define the x-axis.**

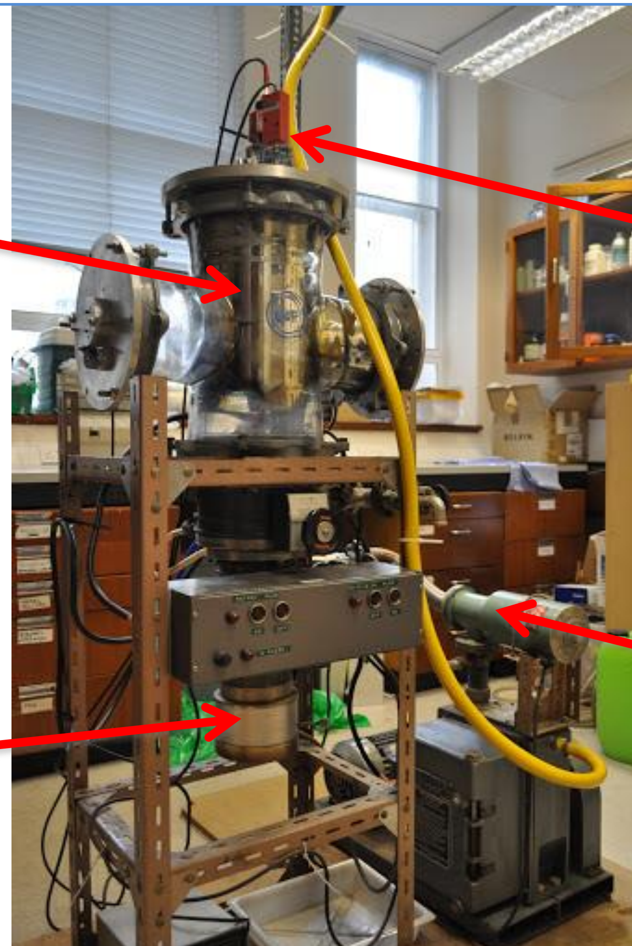
# Schematic of a Stern-Gerlach -1



# Schematic of a Stern-Gerlach -2

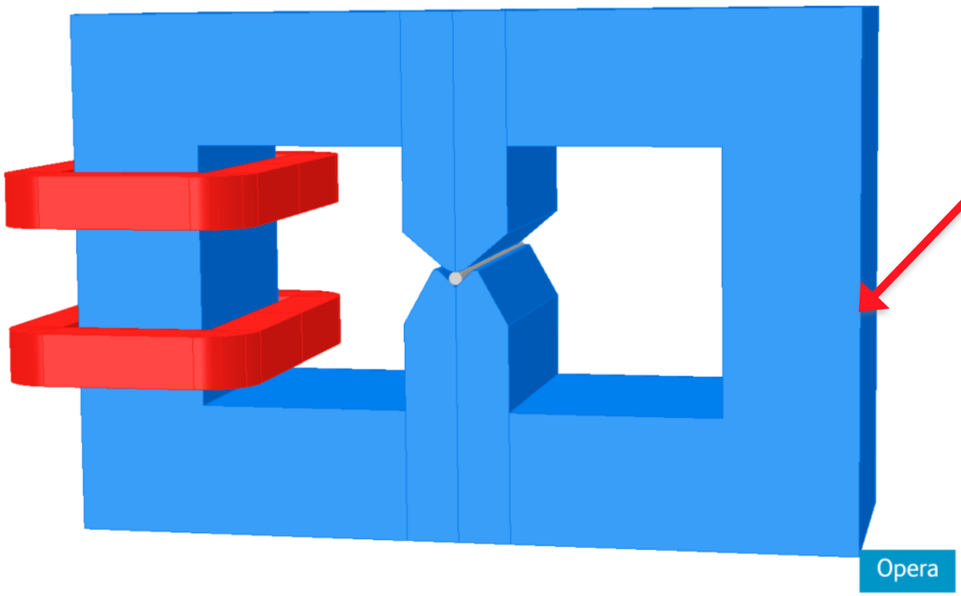


# Find a vacuum chamber

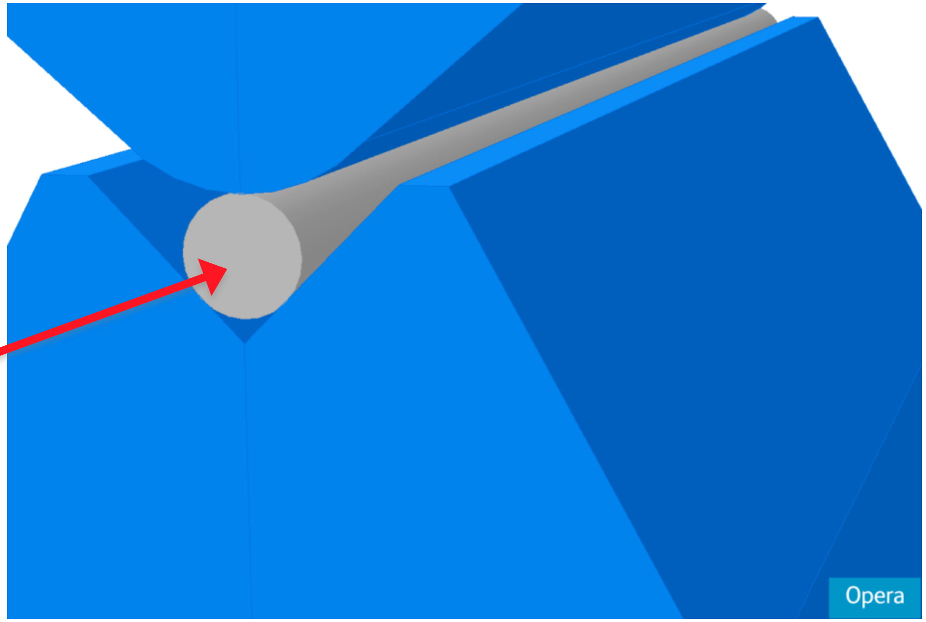


# Magnet design

Jim Clarke, Kiri Marinov Cockcroft Institute

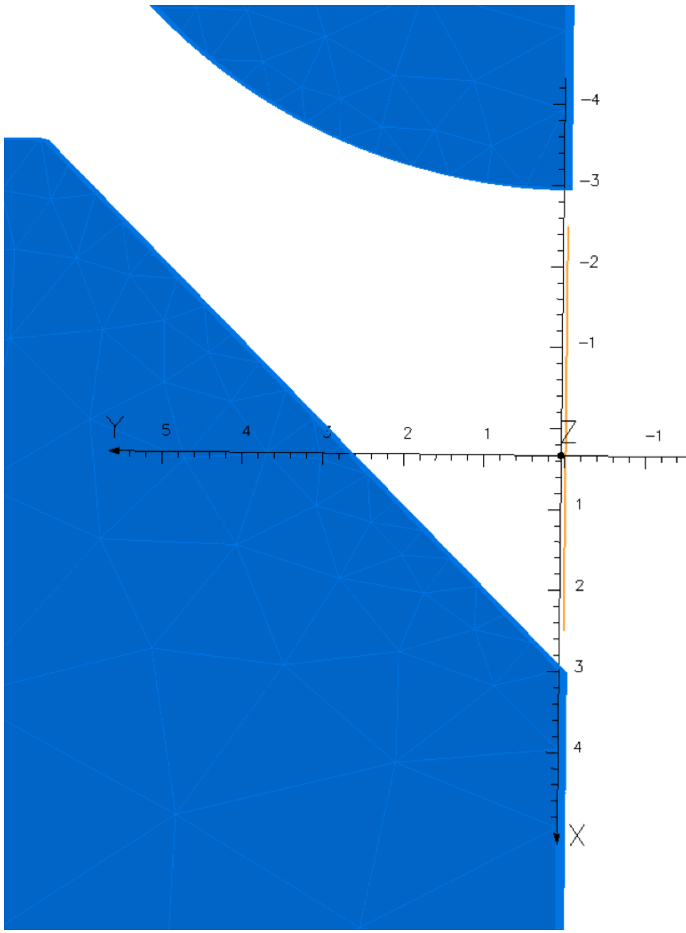



Coils are present on this side but not shown

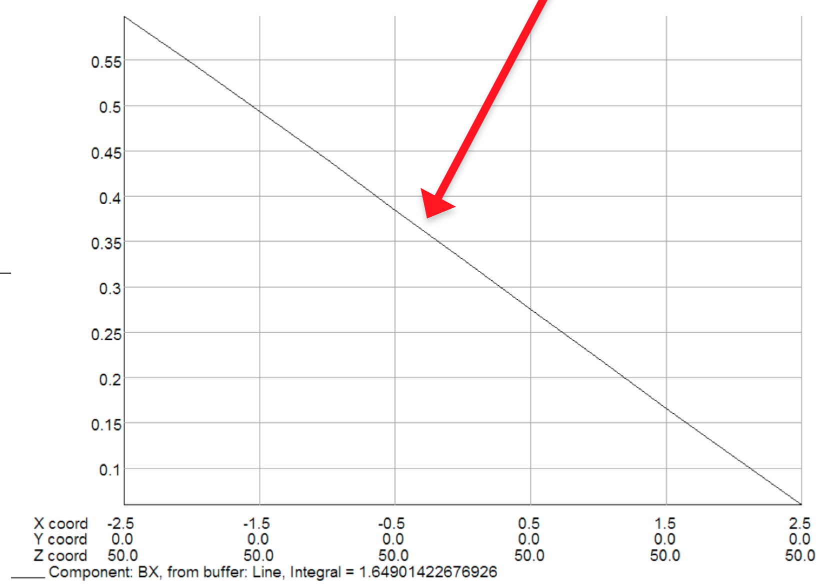


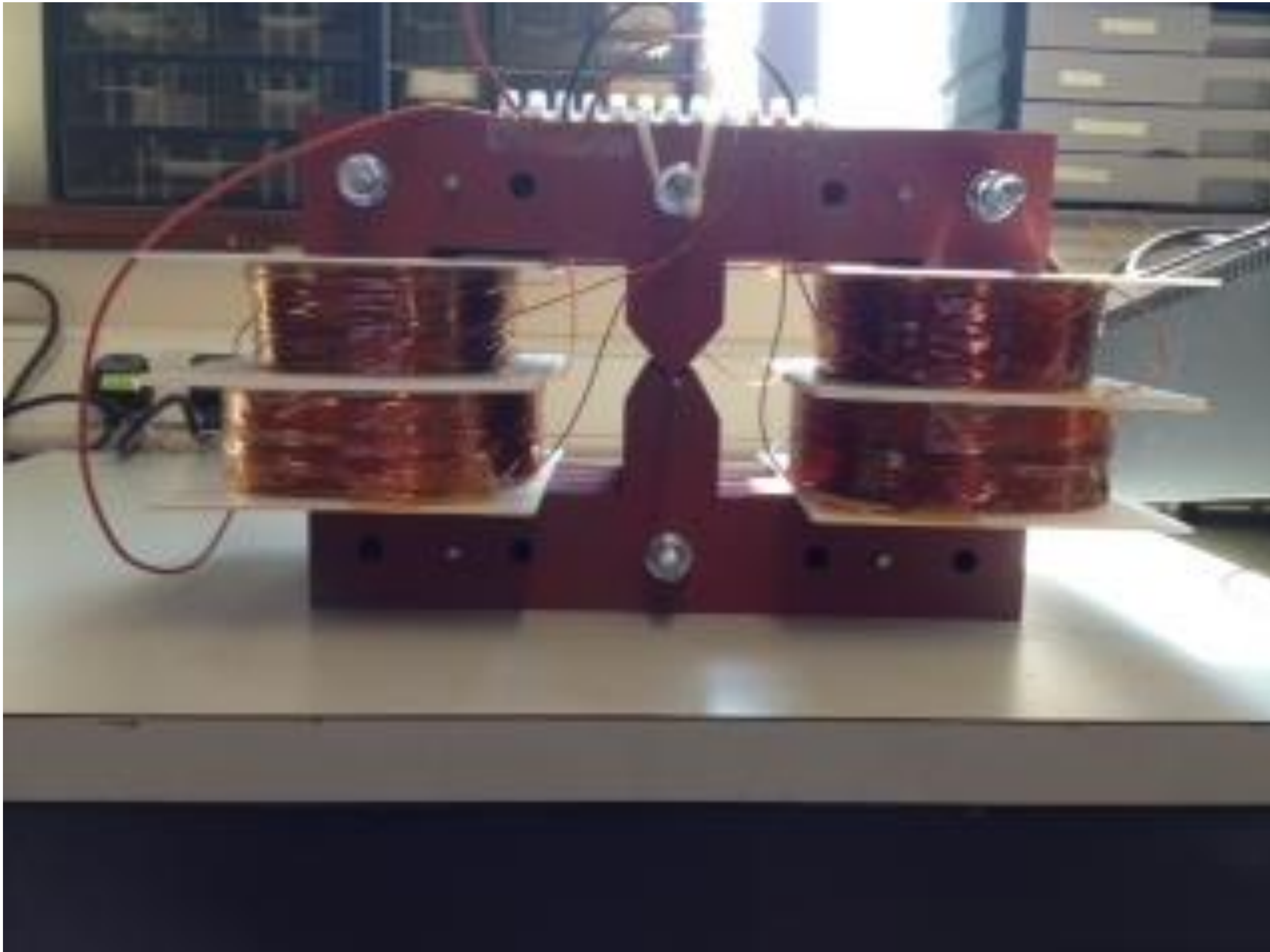

Quadrapole with space for a 4mm diameter pipe

# Magnet design




Predicted field gradient of 113 T/m

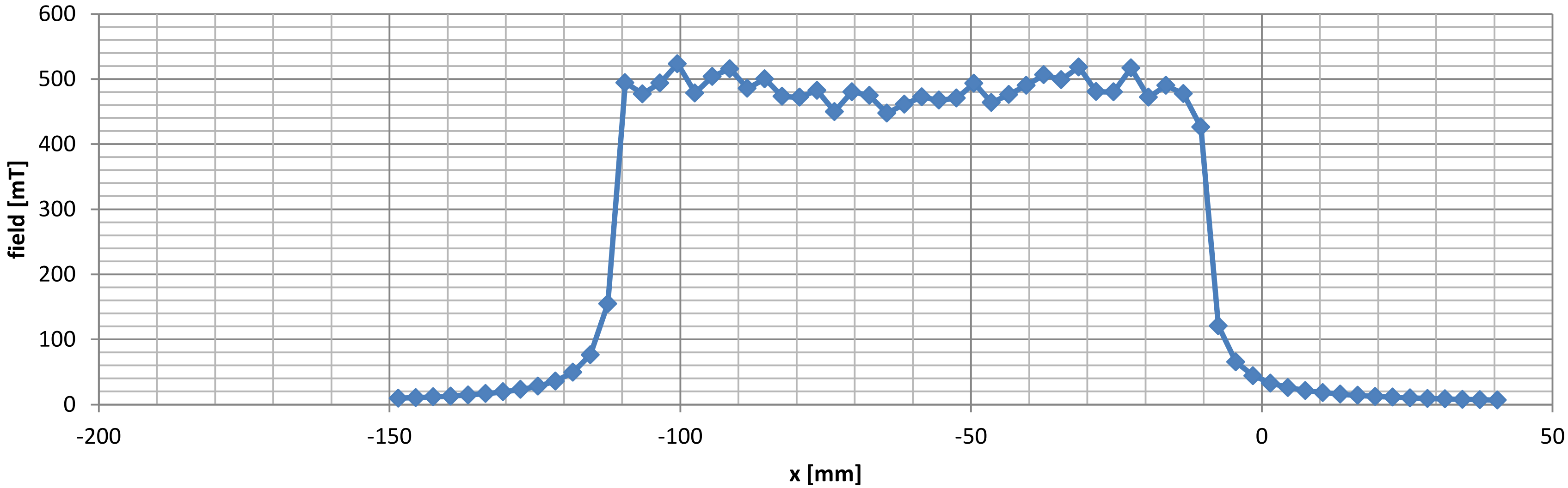




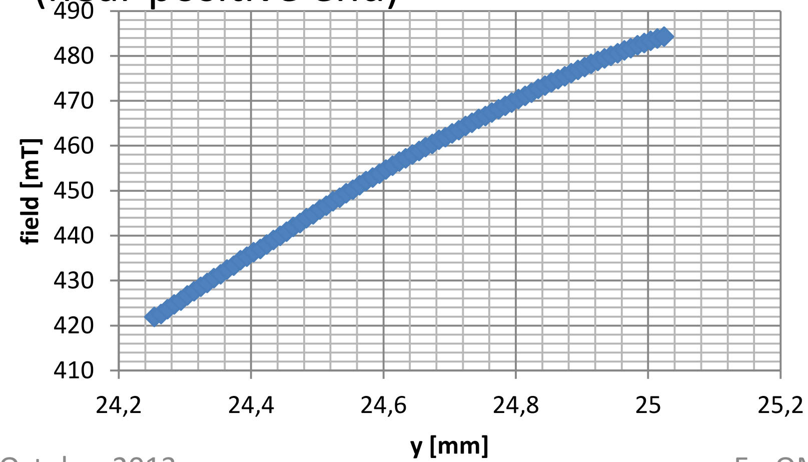
# Field plots

Longitudinal field plot

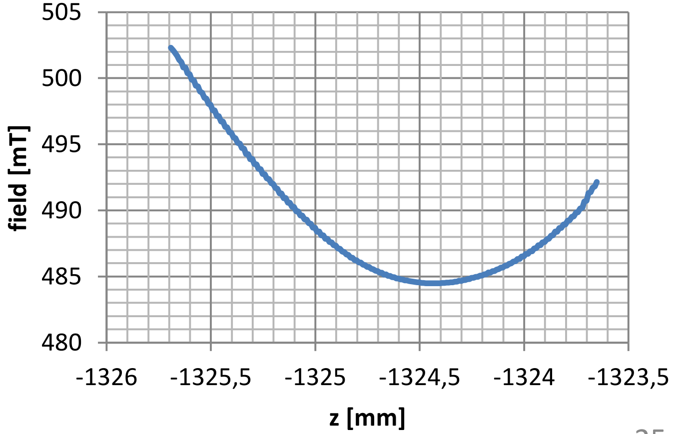
Ben Shepherd, Cockroft Institute




Field versus vertical position (near positive end)




Field versus horizontal position (near positive end)



# More field plots

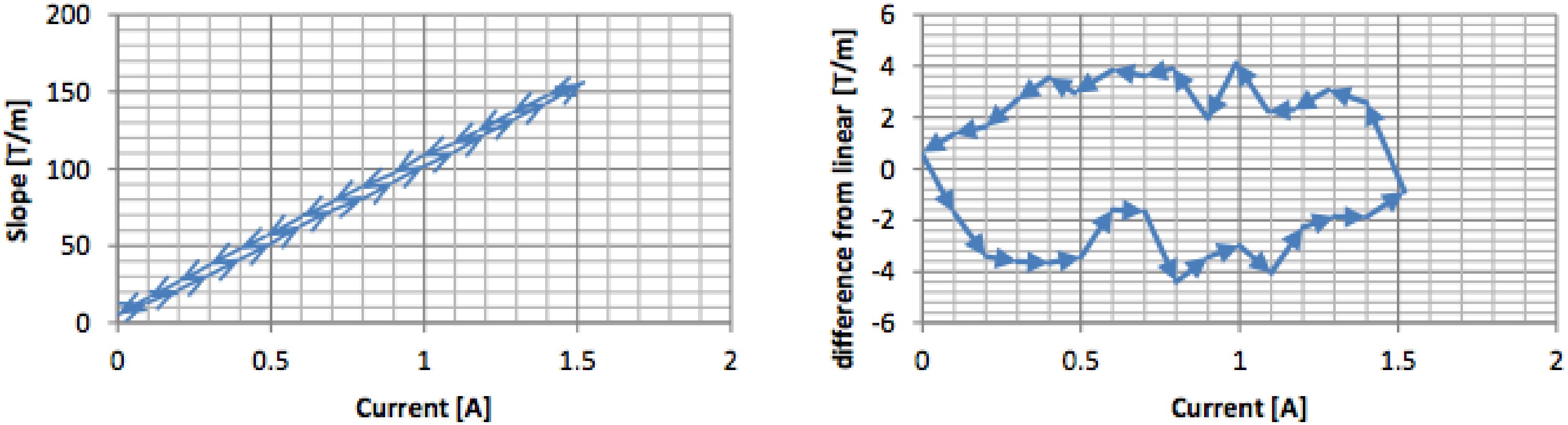



Figure 7. Gradient versus current. The right-hand plot shows the difference between the measured gradient and a linear fit to the data.

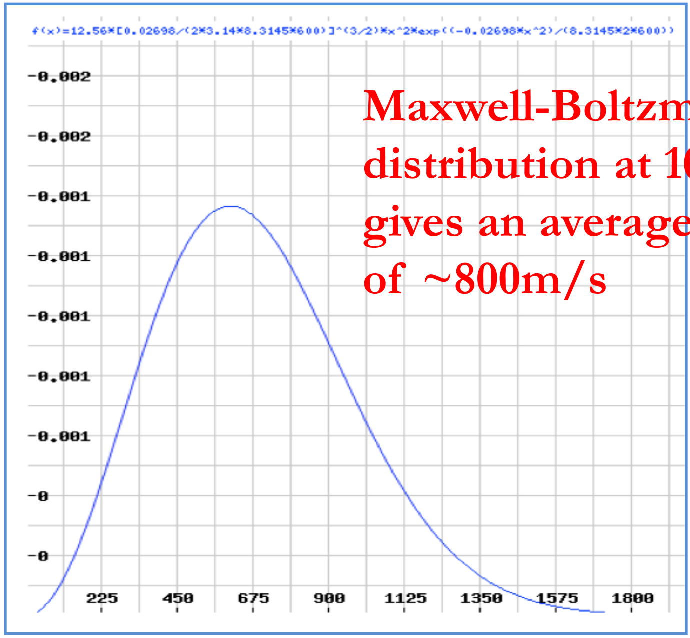
# Choose an atom

**Vapor Points for Common Materials Under Vacuum**

| Material | Symbol | Melt Point (°C) | $10^{-4}$ torr | $10^{-6}$ torr | $10^{-8}$ torr |
|---|---|---|---|---|---|
| | | | Temperature at Vapor Pressure (°C) | | |
| Aluminum | Al | 660 | 1010 | 821 | 677 |
| Copper | Cu | 1083 | 1017 | 857 | 727 |
| Gold | Au | 1062 | 1132 | 947 | 807 |
| Indium | In | 157 | 742 | 597 | 487 |
| Nickel | Ni | 1453 | 1262 | 1072 | 927 |
| Nichrome | NiCr | 1395 | 1217 | 987 | 847 |
| Silver | Ag | 961 | 1105 | 958 | 847 |
| Tin | Sn | 232 | 997 | 807 | 682 |
| Tungsten | W | 3410 | 2757 | 2407 | 2117 |

**Mean Free Path by Vacuum Level**

| Pressure (microns) | Pressure (torr) | Mean Free Path (inches) |
|---|---|---|
| 10 | $1.0 \times 10^{-2}$ | .191 |
| 5 | $5.0 \times 10^{-3}$ | .382 |
| 2.5 | $2.5 \times 10^{-3}$ | .764 |
| 1 | $1.0 \times 10^{-3}$ | 1.91 |
| .5 | $5.0 \times 10^{-4}$ | 3.82 |
| .25 | $2.5 \times 10^{-4}$ | 7.64 |
| .1 | $1.0 \times 10^{-4}$ | 19.1 |
| .05 | $5.0 \times 10^{-5}$ | 38.2 |
| .025 | $2.5 \times 10^{-5}$ | 76.4 |
| .01 | $1.0 \times 10^{-5}$ | 191.0 |



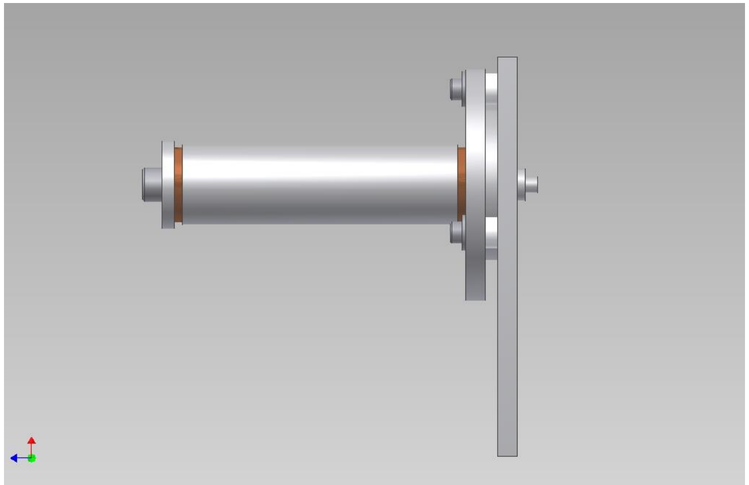

**Maxwell-Boltzman distribution at 1000 °C gives an average speed of ~800m/s**

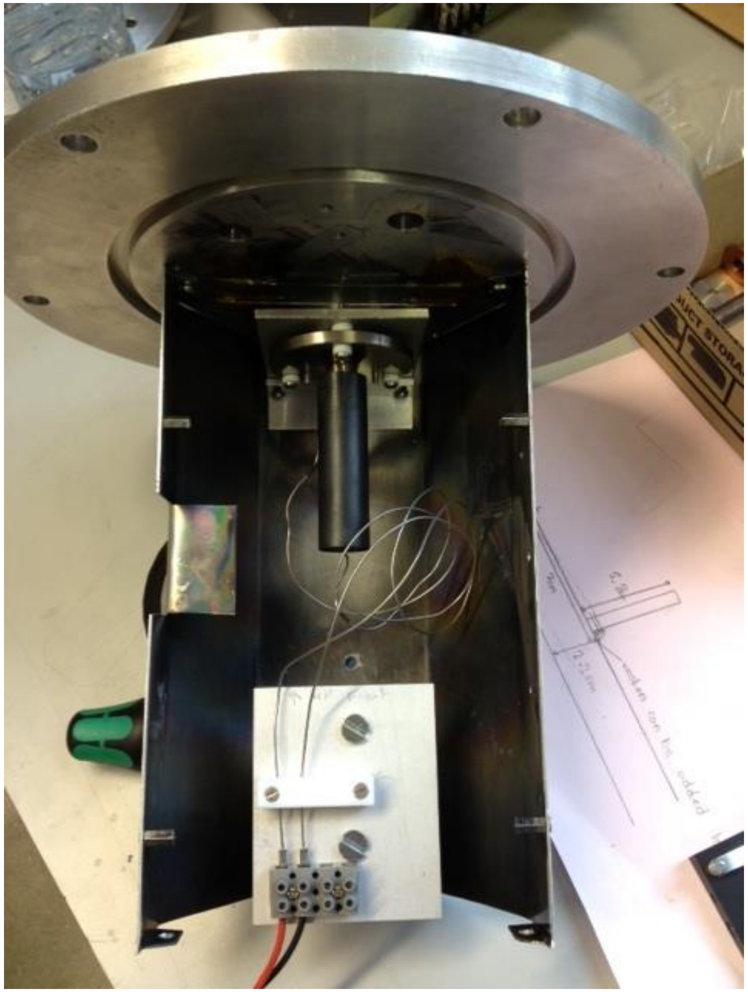
Choosing an atom:

- Correct electron structure – spin-1/2
- Temperature of vapourisation - < 1000 °C
- Required level of vacuum – $10^{-5}$ torr

# Furnace

Want to be able to use several types of atom. Designed to work up to temperature of 1000 $^{o}$C





# Weak magnet

•The two wave functions of the spin orientated along the *z*-axis have to be only partially separated but remain overlapping.

•This implies the wave function has extension in space. How do I calculate it?

•What strength does the weak magnet have to be?

•How do I know that I have achieved partial overlapping?

•Wavelength of the atoms is of the order of $10^{-11}$m. Is the wavelength the correct parameter to use?

•Using this wavelength gives a field gradient of the order of $10^{-3}$ T/m

# This autumn

- Build the experiment with one magnet and reproduce Stern-Gerlach.
- Introduce weak and second strong magnet.
- Carry out many runs and hopefully observe the weak measurement effect.
- Measure the weak value $\mathbf{A_w = \lambda tan(\alpha/2)}$.
- Could I explore the "width" of the wave function.

# Thanks

- **Basil Hiley for his advice and encouragement.**
- **Jim Clarke, Kiri Marinov and Ben Shepard from the Cockcroft for designing and testing the magnet.**
- **Mark Lancaster head of the HEP group at UCL for his support.**
- **Taher Gozel for financial support.**
- **Derek Atree for helping me build the apparatus.**

# Back up slides

# Remind ourselves about the measurement process

Von Neumann. Mathematical foundations of quantum mechanics. Springer Verlag,1932.

An experiment has a system, ***S***, and detector, ***D***, coupled in such a way as to make the measurement. The Hamiltonian, ***H***, is written as the sum of three parts,

$$H = H_s(x) + H_D(y) + H_I(x,y)$$

The third term, $H_I$, produces a correlation between the system and the detector and has the form: $H_I = -f\hat{q}\hat{A}$ , where ***A*** is the variable to be measured with values $a_n$ and ***q, p*** are generalised conjugate variables.

This term is switched on long enough to make the measurement and produce a strong correlation but not too long, **impulse measurement**. An example is the shutter of a camera. If it is left open for too long then the image is over-exposed.

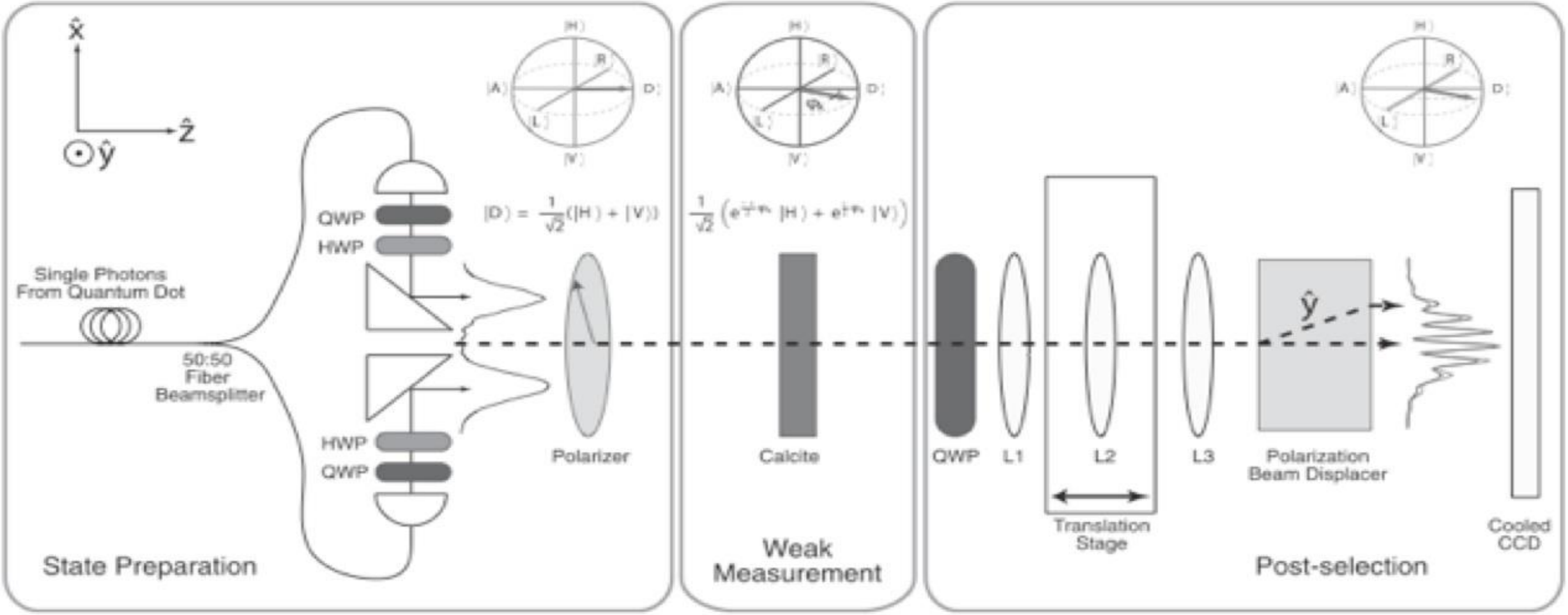
**An ideal measuring device has well defined initial and final values of *p* such that the pointer reading, $p_f - p_i$ gives the value of *A*.**

# Mapping trajectories of photons in 2-slit experiment

Kocsis et al, Science 332:1170, 2011

A quantum dot was used as a source of single photons. This made sure that only one photon was in the apparatus at a time.

The strong measurement of position is carried out by observing the fringe pattern at a range of horizontal positions

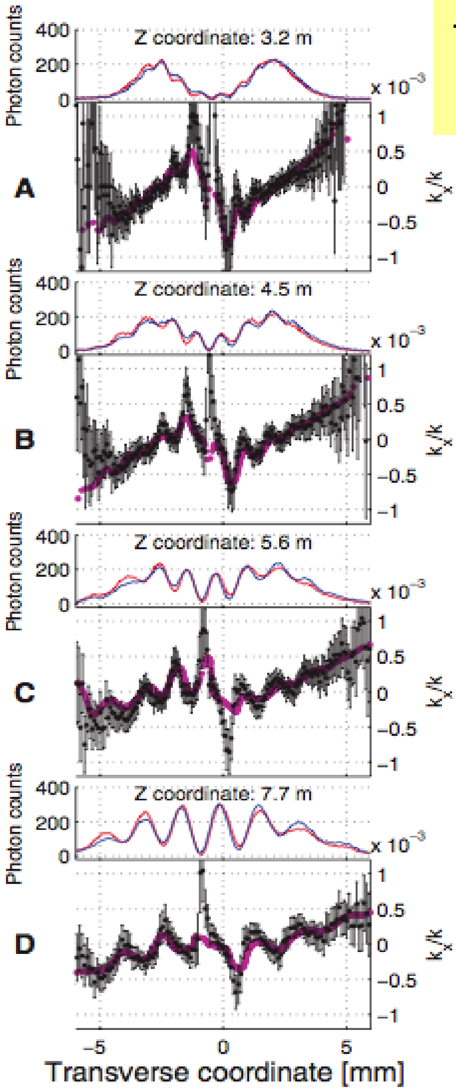


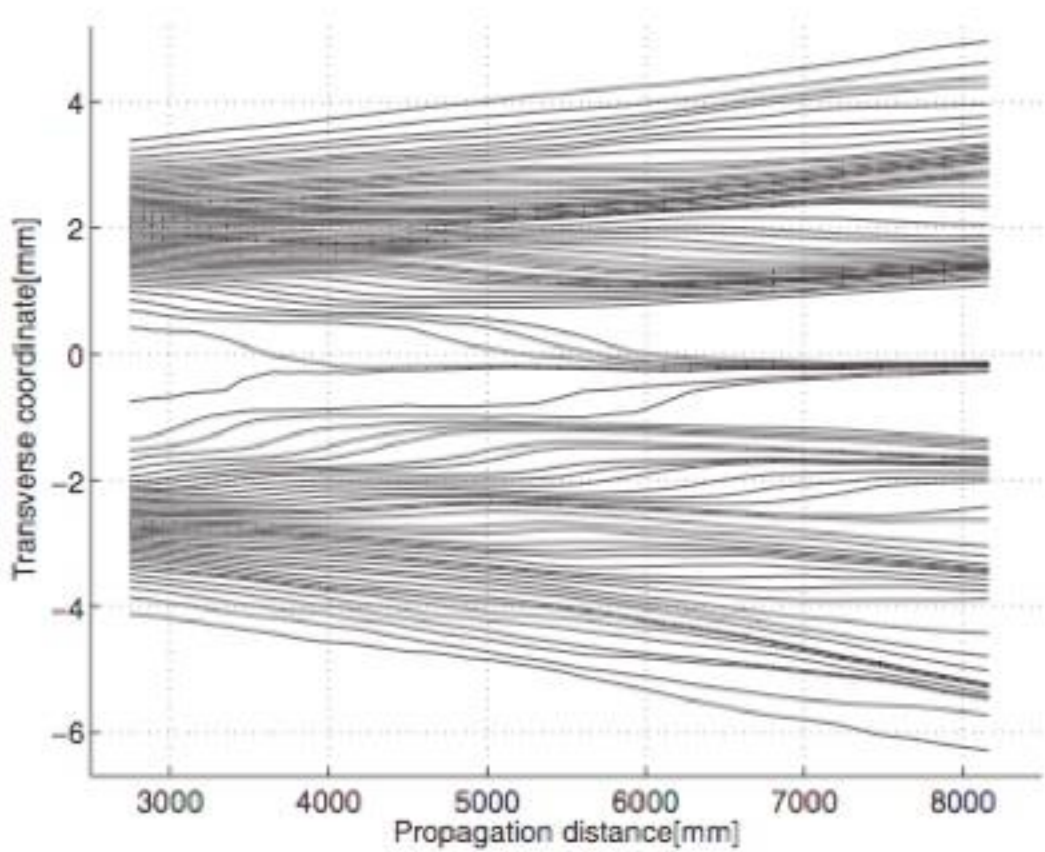

A 50:50 beam splitter.

Birefringent calcite is used to for the weak measurement of momentum. It imparts a small $k_x$-dependent phase shift (p=hk). Linear polarisation becomes slightly elliptical

# Reconstruction of the trajectories

The majenta line is after constant background has been subtracted



# Technical problems with 2-slit experiment

There are four main problems with carrying out this experiment with Schrödinger particles

- Producing a constant beam of Schrödinger particles i.e. atoms.
- Constructing the 2-slits of the correct dimension.
- A detector sensitive enough to count small numbers of atoms.
- The whole apparatus has to take place in a vacuum chamber.

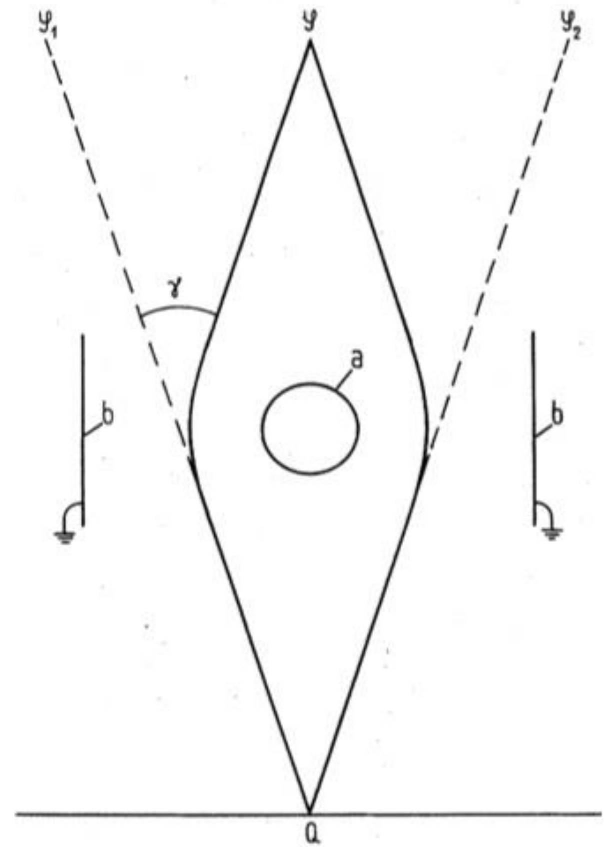
# Electrons in the 2-slit experiment

Mollenstedt and Duker. Zeitschrift fur Physik, 145 S:377-397, 1956.

Mollenstedt used an electrostatic biprism as the 2-slits used in Young's experiment.

The 10 - 20keV electron beam enters at the top at "y".

The biprism consists of a gold covered quartz fibre, "a" , diameter of 2.5 mm and 6 mm in length, set vertically and equidistant between two grounded conductors, "b", 4 mm apart. The fibre was held at a potential of 10 V.

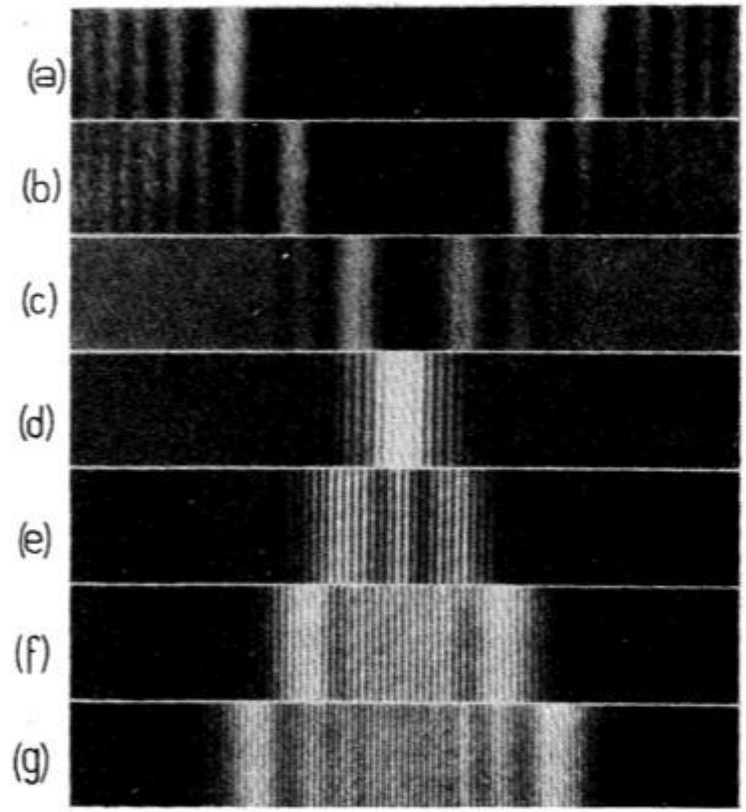



The working region estimated 15 μm from the fibre.

# Observation of the interference fringes for electrons

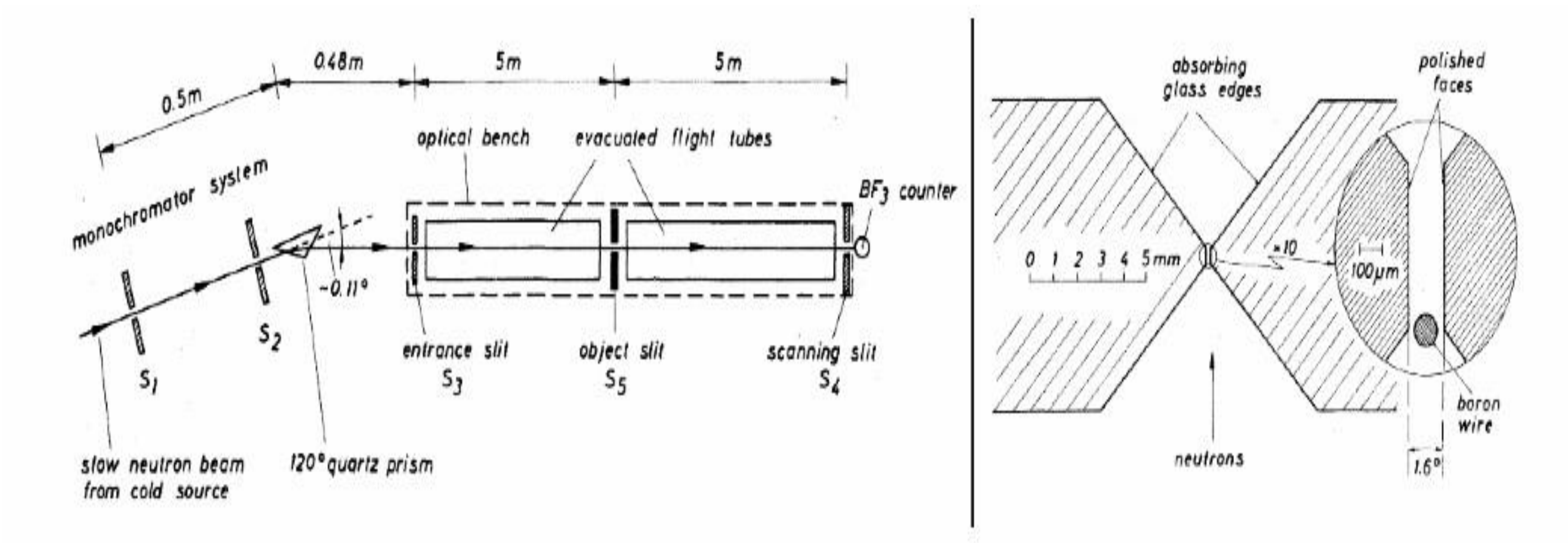
Effectively the biprism produces two virtual images of the filament emitting the electrons.The interference pattern is obtained as the superposition of the electron waves arriving in the observing plane.

By adjusting the voltage on the on the fibre the interference pattern gradually comes into sharp relief.

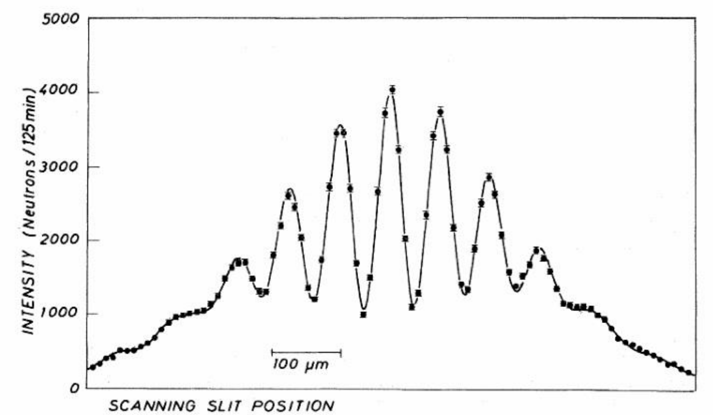


# Neutron 2-slit experiment

Zeilinger et al. Rev.Mod.Phys., 60:1067-1073, 1988.




The neutrons are selected and focused onto a slit similar to that used by Mollenstadt.



# Weak measurement

Sudarshan et al Phys. Rev. D, 1989

Aharonov et al. Phys. Rev. Lett., 60:1351, 1988.

In a non-ideal (real) measurement ***p*** has a width ***Δp*** the wave function of the detector will have the form:

$$|\Phi_{in}\rangle = \int exp(-\frac{p^2}{2(\Delta p)^2}dp]$$

The intial state of the system is prepared thus:

$$|\Psi_{in}\rangle = \Sigma_n \alpha_n |\mathrm{A_n}\rangle$$

After the measurement has taken place the final wave function has the form:

$$|\Phi_f\rangle = exp[-i\int H_I dt]|\Phi_{in}\rangle|\Psi_{in}\rangle$$

Putting these all together gives a sum of Gaussians one for each $a_n$.

$$|\Phi_f\rangle = \Sigma_n \alpha_n \int exp[-\frac{(p-a_n)^2}{2(\Delta p)^2}]dp|A = a_n\rangle|p\rangle$$

**A strong measurement depends on *Δp* being smaller than the spacing of the $a_n$ making each Gaussian narrow and non-overlapping.**

Consider the situation where $\Delta p$ is much larger than the spacing between the $a_n$. Instead of a set of widely separated Gaussians with narrow $\Delta p$ we have a set of overlapping Gaussians with wide $\Delta p$.

This will approximate a single large Gaussian-like function peaked at a mean value of $A$ i.e. $\langle A \rangle$.

This corresponds to a weak measurement and on its own gives little or no information.

If the weak measurement is carried out many times then it is possible to map out the distribution and obtain an estimate of the centroid. The accuracy depending only on the number of measurements.

**Is this what we mean as an effective measurement?**

# Particle physicists use weak measurement

G. J. Feldman et al., Phys. Rev. Lett. 48, 66 (1982)

In the early measurement of the τ-lifetime the position resolution of the detectors used was not good enough to resolve the decay length of the τ-lepton in a single event.

τ-lifetime: 460 x $10^{-15}$ ± 190. Feldman et al

The "apparent decay length" has a very broad distribution, dominated by the position-measurement errors. With a large sample of events the life-time can be deduced from the mean (centroid) of the distribution.

The lifetime has since been measured more accurately 290 x $10^{-15}$ ± 1.0 Abdallah et al.

# Foundations

It is a surprise to me that the two most basic parts of the quantum theory, the wave function and the acts of observation and measurement, are still subjects of debate and remain misunderstood.

It is my belief that the foundations of quantum mechanics needs more experimental input. Specifically concerning the nature of the wave function.

# Observation and measurement

- Core to any physical theory.
- Corner stone of quantum mechanics.
- Do we truly understand the process?
- As an experimentalist I am beginning to view measurement differently.

To move forward in our understanding we have to find ways of investigating matter, space and time at smaller and smaller dimensions.

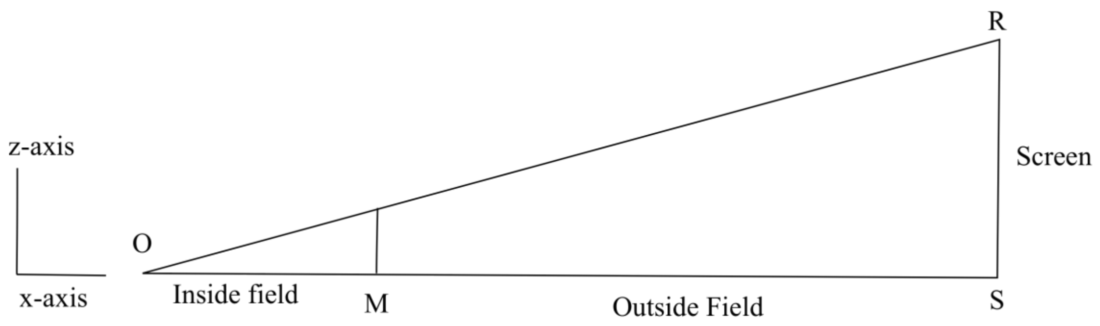
At the moment we use high energy colliders: The tank shell approach.

**Weak measurement may be a more subtle, less destructive method.**

# Summary of calculations

$$\therefore \mu_J(\text{magnetic moment for aluminium}) = \frac{g_{\text{Landé}}}{\sqrt{J(J+1)}}\mu_B = 0.77\mu_B$$

$$F_z = 0.77\mu_B \frac{dB}{dz}$$




Beam of atoms moves along the x-axis.

**For a field gradient of 70 T/m a separation of ~4mm should be observed**

# Crucial points

1. The measurement process has to clearly separate the possible values i.e. the separation has to be larger than the error on the measurement.
2. The source of the error is the quality of the measurement not the uncertainty principle.
3. It is an assumption that all errors in measurement are Gaussian (Roger Barlow, Statistics, Wiley 1989).

**In this talk I will refer to this as a strong measurement and I will now give examples of what I think of as measurement.**

# Conclusion

I have explained the difference between a weak and strong measurement from a practical standpoint and claim that particle physics uses weak measurement without realising it.

Claim: principle of weak measurement has been observed using photons in:

- An analogue of double Stern-Gerlach experiment.
- A Young's 2-slit experiment.

Weak measurement could be used to directly observe the quantum potential.

**I want to further explore experimentally the weak values using non-zero mass particles such as neutrons.**

# A request from Basil

**Weak values when $\hat{P}$ is involved.**

Form: $$\langle \boldsymbol{x}|\hat{\boldsymbol{P}}|\psi(t)\rangle = \int \langle \boldsymbol{x}|\hat{\boldsymbol{P}}|\boldsymbol{x}'\rangle\langle \boldsymbol{x}'|\psi(t)\rangle d\boldsymbol{x}' = -i\boldsymbol{\nabla}\psi(\boldsymbol{x},t)$$

Weak value $$\langle \boldsymbol{P}\rangle_W = \frac{\langle \boldsymbol{x}|\boldsymbol{P}|\psi(t)\rangle}{\langle \boldsymbol{x}|\psi(t)\rangle}$$

Write $\psi(\boldsymbol{x},t) = R(\boldsymbol{x},t)e^{iS(\boldsymbol{x},t)}$ then

$$\langle \boldsymbol{P}\rangle_W = \boldsymbol{\nabla}S(\boldsymbol{x},t) - i\boldsymbol{\nabla}\rho(\boldsymbol{x},t)/2\rho(\boldsymbol{x},t)$$ with $\rho(\boldsymbol{x},t) = |\psi(\boldsymbol{x},t)|^2$

Bohm momentum. osmotic momentum.

**The Bohm kinetic energy.**

$$\Re[\langle \boldsymbol{P}^2\rangle_W] = (\boldsymbol{\nabla}S(\boldsymbol{x}))^2 - \frac{\boldsymbol{\nabla}^2 R(\boldsymbol{x})}{R(\boldsymbol{x})} = P_B^2 + Q.$$

# Experiments using weak measurement

The weak and strong measurements are carried out on non-commuting variables: Spin in the z and x axes; momentum and position.

There is a debate as to whether this is really a meausrement process. I will illustrate that by looking at an experiment using a modified Stern-Gerlach where the usual spin measurement is ampilified.

Then I will look at how the particle trajectories in a Young's 2-slit experiment can be mapped out.

# A couple of quotes

"I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through I cannot understand; and this is why I cannot get the electromagnetic theory." **Sir William Thompson, Lord Kelvin 1884.**

"All the fifty years of conscious brooding have brought me no closer to the answer to the question, "what are light quanta?" Of course today every rascal thinks he knows the answer, but he is deluding himself." **A. Einstein 1951**

**I think it is human nature to try and build mechanical models of the universe and fundamentally we still don't understand light quanta i.e. the wave function.**

# Another quotation

All the world's a stage, and all the men and women merely players: they have their exits and their entrances; and one man in his time plays many parts, his acts being seven ages.
**William Shakespeare – As you like it**

All the universe is a stage, and all the particles merely players: they have their exits and their entrances; and one particle in his time plays many parts, his acts being the universe of ages.
**Rob Flack – Dreadful misquote**

In a stage play the actors (particles; quarks and leptons) are few and are what we observe. We also know that behind the scenes there is a production team many dressed in black (dark matter) who we don't see plus the mechanical parts of the staging.

**I think this is a good metaphor for where we are in physics. We know the principal players and how they interact but we still don't understand stage direction, the support staff, the scenery, or the stage lighting etc.**

# Weak value

Aharonov et al proposed a weak measurement followed by a strong measurement of a **conjugate variable** and a single value is selected.

Duck et al talk about the wave functions being partially overlapping after the weak measurement stage.

This implies the wave function have extension in space. How do I calculate it?

In fact how do I know as an experimentalist that I have achieved partial overlapping?

# Problems with the modified Stern-Gerlach

Problem with producing enough atoms to be able to observe a signal. Is a detector for neutral atoms sensitive enough?

What would constitute a signal for the weak measurement process?

What are overlapping wave functions? How do we specify the amount of overlapping?

Can we test if two wave functions are overlapping?

If we observe the process of weak measurement for different strengths of the weak magnetic field could we get information about the width of a wave function and what overlapping means?