Quantum emergence and role of the zero-point field

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INTRODUCTION

- Origin of quantum stochasticity
- The zero-point field
- Stochastic electrodynamics
  - Rise and fall
  - Revival
  - The emerging quantum.

Outline

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  - Equilibrium radiation field with nonthermal fluctuations

2 Quantum mechanics as a consequence of the zpf
  - 2a. The Schrödinger description:
    - Particle dynamics from phase space to configuration space
    - Energy balance in the radiationless approximation
  - 2b. The Heisenberg description:
    - Ergodic behavior of stationary solutions
    - Linear resonant response: the transition frequencies

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  - 3c. Quantum nonlocality and entanglement

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1. Planck's law as a consequence of the zpf

From Wien's law, $U(\omega, T) = \omega f(\omega/T)$, at zero temperature

$\mathcal{E}_0 \equiv U(\omega, 0) = A\omega \neq 0$ violates equipartition.

Take the equilibrium distribution ($\beta = 1/kT$),

$W_g(\mathcal{E})d\mathcal{E} = Z_g^{-1}(\beta)\, g(\mathcal{E})e^{-\beta\mathcal{E}}d\mathcal{E}$, with $Z_g(\beta) = \int g(\mathcal{E})e^{-\beta\mathcal{E}}d\mathcal{E}$.

For $\mathcal{E}_0 = A\omega \neq 0$, $g(\mathcal{E}) = ?$

Notice that $\sigma^2 \equiv \overline{\mathcal{E}^2} - U^2 = -dU/d\beta$, $\sigma^2 = kT^2 C_\omega$,

$\Longrightarrow$ W_g describes thermal fluctuations only!

1. Planck's law as a consequence of the zpf
Nonthermal fluctuations

To include the zero-point fluctuations we write $W_s(\mathcal{E}) = U^{-1}\, e^{-\mathcal{E}/U}$, which maximizes the statistical entropy. The (total) fluctuations are

$$\left(\sigma^2_{\mathcal{E}}\right)_s = U^2 = U_T^2 + 2\mathcal{E}_0 U_T + \mathcal{E}_0^2, \qquad (\sigma^2_{\mathcal{E}})_s\big|_0 = \mathcal{E}_0^2$$

where $U = U_T + \mathcal{E}_0$, and the thermal fluctuations are then

$$\sigma^2(U) = U^2 - \mathcal{E}_0^2 = -\frac{dU}{d\beta}\,.$$

Integrating,

$$\boxed{U = \mathcal{E}_0 \coth \mathcal{E}_0 \beta,} \qquad \text{with} \quad \mathcal{E}_0 = \frac{1}{2}\hbar\omega\,.$$

$$\langle f(E)\rangle = \underbrace{\int_0^\infty W_g(\mathcal{E}) f(\mathcal{E}) d\mathcal{E}}_{\text{continuous}} = \frac{1}{Z}\underbrace{\sum_{n=0}^{\infty} e^{-\beta\mathcal{E}_n} f(\mathcal{E}_n)}_{\text{discrete}}\,.$$

2. QM as a consequence of the zpf

Charged particle subject to an external binding force and the zpf; nonrelativistic, dipole approximation:

$$\dot{x} = p/m, \qquad \dot{p} = f(x) + m\tau \dddot{x} + eE(t)$$

with $\tau = 2e^2/3mc^3 (10^{-23} s)$.

Generalized Fokker-Planck equation:

$$\frac{\partial}{\partial t}Q + \frac{1}{m}\frac{\partial}{\partial x}pQ + \frac{\partial}{\partial p}\left[(f + \frac{\tau}{m}f'p)Q\right] = e^2\frac{\partial}{\partial p}\hat{D}(t)Q$$

In the time-asymptotic (Markovian) limit ($t >> \tau$),

$$e^2\frac{\partial}{\partial p}\hat{D}(t)Q = \frac{\partial}{\partial p_i}D_{ij}^{pp}\frac{\partial}{\partial p_j}Q + \frac{\partial}{\partial p_i}D_{ij}^{px}\frac{\partial}{\partial x_j}Q,$$

$$D_{ij}^{pp} = e^2\int_{-\infty}^{t} dt'\varphi(t-t')\frac{\partial p_j}{\partial p_i'}, \qquad D_{ij}^{px} = e^2\int_{-\infty}^{t} dt'\varphi(t-t')\frac{\partial x_j}{\partial p_i'}$$

$$\varphi(t-t') = \frac{4\pi}{3}\int_0^\infty \rho(\omega)\cos\omega(t-t')d\omega\,, \qquad \rho_{\text{ZPF}}(\omega) = \frac{\omega^2}{\pi^2 c^3}\cdot\frac{\hbar\omega}{2}.$$

2. Quantum mechanics as a consequence of the zpf

2a. The Schrödinger description

Multiplying the FPE by $p^n\,(n=1,2)$ and integrating, with

$\langle p^2\rangle_x - \langle p\rangle_x^2 = -\eta^2\left(\frac{\partial^2}{\partial x^2}\ln\rho\right)$, gives

$$\hat{M}_i\rho = \tau v_i \frac{\partial f_i}{\partial x_j}\rho - e^2\left\langle \hat{D}_i(t)\right\rangle_x .$$

Initially the terms with τ and e^2 have an important diffusive and dissipative effect;

in the time-asymptotic limit they become mere radiative corrections.

Radiationless approximation: $\hat{M}_i\rho = 0$, $\rho(x,t)=\psi^*(x)\psi(x)$ and

$$-\frac{2\eta^2}{m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = 2i\eta\frac{\partial\psi}{\partial t} \qquad \eta = ?$$

2. Quantum mechanics as a consequence of the zpf

2a. Energy balance

From the FPE multiplied by p^2, with $\langle H \rangle = \left\langle \frac{1}{2m} p^2 + V \right\rangle$,

$$\frac{d}{dt} \langle H \rangle = \frac{\tau}{m^2} \langle f' p^2 \rangle - \frac{e^2}{2m} \left\langle p \hat{D} \right\rangle .$$

Energy balance:

$$\frac{\tau}{m^2} \langle f' p^2 \rangle = \frac{e^2}{2m} \left\langle p \hat{D} \right\rangle$$

$$-m\tau \left| \omega_{0k} \right|^4 \left| x_{0k} \right|^2 = -\left(\hbar / 2\eta \right) m\tau \omega_{k0}^4 \left| x_{0k} \right|^2$$

whence $\eta = \hbar / 2$ and

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi .$$

2. Quantum mechanics as a consequence of the zpf

2b. The Heisenberg description

Stationary solutions α, characterized by $\mathcal{E}_\alpha$:

$$m\ddot{x} = f(x) + m\tau\dddot{x} + eE(t) \rightarrow m\ddot{x}_\alpha = f_\alpha + m\tau\dddot{x}_\alpha + eE_\alpha$$

where in general,

$$A_\alpha^{(i)}(t) = \sum_\beta \tilde{A}_{\alpha\beta}^{(i)} a_{\alpha\beta}^{(i)} e^{i\omega_{\alpha\beta} t}.$$

$\{\omega_{\alpha\beta}\}$: relevant frequencies (to be determined by the theory)

$a_{\alpha\beta}^{(i)}$: (partially averaged) stochastic field variables, $\left| a_{\alpha\beta}^{(i)} \right| = 1$.

Ergodic condition: for every dynamical variable g, $\overline{g}^t = \overline{g}^{(i)}$.

Applied to $\sigma_{A_\alpha}^2 \equiv \overline{\left| A_\alpha - \overline{A_\alpha(t)}^t \right|^2}^t \Rightarrow \tilde{A}_{\alpha\beta}^{(i)} = \tilde{A}_{\alpha\beta}$.

2. The Heisenberg description

2b. Linear resonant response

Under ergodicity, $m\ddot{x}_\alpha = f_\alpha + m\tau \dddot{x}_\alpha + eE_\alpha$ leads to the nonstochastic equations

$$m\widetilde{\ddot{x}}_{\alpha\beta}(t) = \tilde{f}_{\alpha\beta}(t) + m\tau \widetilde{\dddot{x}}_{\alpha\beta}(t) + e\tilde{E}_{\alpha\beta}(t)$$

from which

$$\tilde{x}_{\alpha\beta} = -\frac{e}{m}\frac{\tilde{E}_{\alpha\beta}}{\Delta_{\alpha\beta}}, \quad \Delta_{\alpha\beta} = \omega_{\alpha\beta}^2 - i\tau\omega_{\alpha\beta}^3 + \frac{\tilde{f}_{\alpha\beta}}{m\tilde{x}_{\alpha\beta}}.$$

- The system responds resonantly to the field modes with $\omega_{\alpha\beta}^2 \approx -\frac{\tilde{f}_{\alpha\beta}}{m\tilde{x}_{\alpha\beta}}$,
- $x_\alpha^{(i)}$ becomes a linear function of the field components,

$$x_\alpha^{(i)}(t) = -\frac{e}{m}\sum_\beta \frac{\tilde{E}_{\alpha\beta}}{\Delta_{\alpha\beta}} a_{\alpha\beta}^{(i)} e^{i\omega_{\alpha\beta}t}$$

whence the name Linear Stochastic Electrodynamics.

2. The Heisenberg description

2b. Matrix mechanics

The $\tilde{A}_{\alpha\beta}$ in $A_\alpha(t) = \sum_\beta \tilde{A}_{\alpha\beta} a_{\alpha\beta} e^{i\omega_{\alpha\beta} t}$ satisfy a matrix algebra,

and the relevant frequencies satisfy $\omega_{\alpha\beta} = \Omega_\alpha - \Omega_\beta$. From the equation of motion for x_α,

$$m \frac{d^2 \hat{x}(t)}{dt^2} = \hat{f}(t) + m\tau \frac{d^3 \hat{x}(t)}{dt^3} + e\hat{E}(t).$$

from the zpf

Moreover, $[\hat{x}, \hat{p}] = i\hbar \mathbb{I}$ and $\hbar\omega_{\alpha\beta} = \mathcal{E}_\alpha - \mathcal{E}_\beta$,

the resonance frequencies are the frequencies of transition between stationary states.

In the radiationless approximation, $i\hbar \frac{d\hat{A}(t)}{dt} = \left[\hat{A}(t), \hat{H}\right]$.

3. Further consequences of the zpf

3a. Radiative corrections

For a field with $\rho_{\text{ZPF}}(\omega) g(\omega) = \rho_{\text{ZPF}}(1 + g_a(\omega))$ and an atom in state n, energy balance is broken. Then,

$$\frac{dH_n}{dt} = m\tau \left|\omega_{nk}\right|^4 \left|x_{nk}\right|^2 \left[\underbrace{(g_a)_{\omega_{kn}>0}}_{\text{ind.abs.}} - \underbrace{(2 + g_a)_{\omega_{kn}<0}}_{\text{spont \& ind.em.}} \right]$$

'Spontaneous' emission rate: $dH_n = -\hbar\omega_{nk} A_{nk} dt$

Induced transition rates: $dH_n = \pm\hbar\omega_{nk} B_{nk} \rho_a dt$

$$\Rightarrow \quad A_{nk} = \frac{4e^2 \left|\omega_{nk}\right|^3}{3\hbar c^3} \left|x_{nk}\right|^2, \qquad B_{nk} = \frac{4\pi^2 e^2}{3\hbar^2} \left|x_{nk}\right|^2.$$

3a. Radiative corrections
The Lamb shift

Again take the FPE, multiply by xp and integrate:

$$\underbrace{\frac{d}{dt}\langle xp\rangle = \frac{1}{m}\langle p^2\rangle + \langle xf\rangle}_{\text{virial theorem}} + \underbrace{m\tau\langle x\dddot{x}\rangle - e^2\left\langle x\hat{\mathcal{D}}\right\rangle}_{\text{radiative corrections}}$$

Hence $\delta\langle T\rangle_n = \cancel{-\frac{m\tau}{2}\langle x\dddot{x}\rangle_n} + \frac{e^2}{2}\left\langle x\hat{\mathcal{D}}\right\rangle_n$, and the Lamb shift is correctly predicted:

$$\delta\mathcal{E}_n = \frac{e^2}{2}\left\langle \boldsymbol{x}\cdot\hat{\boldsymbol{\mathcal{D}}}\right\rangle_n = -\frac{2e^2}{3\pi c^3}\sum_k |\boldsymbol{x}_{nk}|^2\,\omega_{kn}\int_0^\infty d\omega\,\frac{\omega^3}{\omega_{kn}^2-\omega^2}\,.$$

3. Further consequences of the zpf

3b. The spin of the electron

Again take the FPE, multiply by $x_i p_j$ and integrate. For central forces $\boldsymbol{f}(r) = g(r)\boldsymbol{r}$,

$$\tau \langle g(r) L_{ij} \rangle = -m \langle D^{px}_{ji} - D^{px}_{ij} \rangle .$$

Recall the intrinsic angular momentum of the field, made of circularly polarized modes, with $\boldsymbol{\epsilon}_{\boldsymbol{k}\pm} = \frac{1}{\sqrt{2}} \left(\boldsymbol{\epsilon}_{\boldsymbol{k}1} \pm i \boldsymbol{\epsilon}_{\boldsymbol{k}2} \right)$,

$$\hat{\boldsymbol{J}}_S = \sum_{\boldsymbol{k}, \sigma = \pm 1} \hbar \hat{\boldsymbol{k}} \sigma \left(\hat{n}_{\boldsymbol{k}\sigma} + \tfrac{1}{2} \right) .$$

✧ The electron interacts with circularly polarized modes of the field, hence we use

$$x^{+}_{mn} = \frac{1}{\sqrt{2}} \left(x_{mn} - i y_{mn} \right), x^{-}_{mn} = \frac{1}{\sqrt{2}} \left(x_{mn} + i y_{mn} \right)$$

3. Further consequences of the zpf

3b. The spin of the electron

Thus we get

$$\langle L_z \rangle_n = m_0 \sum_m \omega_{mn} \left(|x^+_{nm}|^2 - |x^-_{nm}|^2 \right) \equiv \left\langle \hat{O}^+_z \right\rangle_n + \left\langle \hat{O}^-_z \right\rangle_n .$$

And since

$$\hbar = m_0 \sum_m \omega_{mn} \left(|x^+_{nm}|^2 + |x^-_{nm}|^2 \right) = \left\langle \hat{O}^+_z \right\rangle_n - \left\langle \hat{O}^-_z \right\rangle_n ,$$

☞ $\left\langle \hat{O}^\pm_z \right\rangle_n = \frac{1}{2} \langle L_z \rangle_n + \bar{S}^\pm_z$; $\left\langle \hat{O}^\vartheta_z \right\rangle_n = \langle n\vartheta | \hat{O}_z | n\vartheta \rangle = \frac{1}{2} \langle L_z \rangle_n + \bar{S}^\vartheta_z .$

$\Rightarrow$ $O^\vartheta_{z,n} = \langle n\vartheta | \left(\frac{1}{2} \boldsymbol{L} + \boldsymbol{S} \right) \cdot \hat{\boldsymbol{z}} | n\vartheta \rangle , \ \hat{\boldsymbol{S}} = \frac{1}{2} \hbar \hat{\boldsymbol{\sigma}}, \ \vartheta = \pm .$

In presence of a field B, $\mathcal{H}^+_L = \frac{\mu_0}{2\hbar} B L_z, \quad \mathcal{H}^+_S = -\mu_{Sz} B = \frac{\mu_0}{\hbar} B S_z .$

$$\boxed{\mathcal{H}^+ = \mathcal{H}^+_L + \mathcal{H}^+_S = \frac{\mu_0}{2\hbar} B \left(L_z + 2 S_z \right) .}$$

g=2 !

3. Further consequences of the zpf

3c. Quantum nonlocality

The local momentum fluctuations $\sigma_p^2(x) = -\left(\hbar^2/4\right)\left(\partial^2 \ln \rho/\partial x^2\right)$

give rise to an extra kinetic energy term (‘quantum potential’)

$$V_Q = \frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\frac{\partial^2\sqrt{\rho}}{\partial x^2},$$

responsible for the quantum nonlocality (even for one particle).

Two-particle system: $\rho(x_1, x_2) = \rho_1(x_1)\rho_2(x_2)\rho_{12}(x_1, x_2)$,

$$V_{Q1}(1,2) = -\frac{\hbar^2}{2m}\left[\frac{\partial_1^2\sqrt{\rho_1}}{\sqrt{\rho_1}} + \underline{\underline{\frac{\partial_1^2\sqrt{\rho_{12}}}{\sqrt{\rho_{12}}}}} + \underline{\underline{\frac{1}{2}\frac{\partial_1\rho_1}{\rho_1}\frac{\partial_1\rho_{12}}{\rho_{12}}}}\right].$$

3. Further consequences of the zpf
3c. Entanglement

For two noninteracting particles $i, j,$

$$m_i \ddot{x}_i = f_i(x_i) + m_i \tau_i \dddot{x}_i + e_i \left(E_i(t) + \frac{1}{e_j} m_j \tau_j \dddot{x}_j \right)$$

each particle is affected by the presence of the other, and the a_i affect both x_i and x_j. Hence (for particle i/j in state α/α')

$$(FG)^{(i)}_{A=(\alpha,\alpha')} = \sum_{\beta,\beta'} \hat{F}_{\alpha\beta} \hat{G}_{\alpha'\beta'} a^{(i)}_{i\alpha\beta} a^{(i)}_{j\alpha'\beta'} e^{i(\omega_{\alpha\beta}+\omega_{\alpha'\beta'})t}$$

$a_{\alpha\beta} a_{\alpha'\beta'}$ characterize the common background field;

$\omega_{AB} = \omega_{\alpha\beta} + \omega_{\alpha'\beta'} = \mathcal{E}_A - \mathcal{E}_B$ are transition frequencies of the bipartite system, with $\mathcal{E}_A = \mathcal{E}_\alpha + \mathcal{E}_{\alpha'}$.

4. Further consequences of the zpf
4c. Entanglement

When the particles have a common res. frequency, $\omega_{\delta\kappa} = \omega_{\kappa'\delta'}$,
$(FG)_A$ contains nonfactorizable Fourier coefficients,

$$\hat{F}_{\alpha\delta}\hat{G}_{\alpha'\delta'} + \lambda_{DK}\hat{F}_{\alpha\kappa}\hat{G}_{\alpha'\kappa'},$$

with $\lambda_{DK} = \langle a_i(\omega_{\delta\kappa}) a_j(\omega_{\delta'\kappa'})\rangle$.

This implies correlations between the particles. In the Hilbert-space description,

$\rightarrow \quad |\delta\rangle_i |\delta'\rangle_i + \lambda_{DK} |\kappa\rangle_i |\kappa'\rangle_j$, with $\omega_{\delta\kappa} = \omega_{\kappa'\delta'}$.

For identical particles all res. frequencies are common. Invariance of Γ_{FG} under interchange of $a's$ and of particle states leads to totally (anti)symmetric state vectors, $\lambda = \pm 1$.

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Some conclusions

- **The quantum phenomenon is not intrinsic to the particle or the field**
  ...but emerges from the matter-field interaction
  ...as the system evolves towards the nondissipative (quantum) regime
- In this regime the electrons move in stationary orbits, determined by the energy balance condition
- Absolute stationarity is reached only in the ground state, *with* $\rho_{\mathrm{ZPF}} \sim \omega^3$
- The stationary solutions have an ergodic behavior
- In the quantum regime the material system responds linearly to the background field
- The theory leads to (nonrelativistic) qed
- The spin of the electron emerges from its interaction with the (circularly polarized) zpf
- The correct spin gyromagnetic ratio *g*=2 is predicted

Some more conclusions

- Interactions between `independent' particles are mediated by the zpf
- For bipartite systems with degeneracy (common resonance frequencies), entangled states are produced
- For identical particles, stationary state vectors are (anti)symmetric
- The (stochastic) particle trajectories do not disappear
- Quantum nonlocality is not ontological (no superluminal transmission,
  no action at a distance)
- By eliminating the zpf, the quantum description appears as acausal
- The complete theory is causal, local, realist and objective
- **The dynamics of the transition from 'classical' to 'quantum' needs further investigation**

  ...as well as the extension to the entire field-particle system.
- **There is ample opportunity for new physics!**

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