

# Probing the Sub-Quantum with Weak Measurements

Boris Braverman

# Outline

- Some history
- Bohmian Mechanics
- Weak measurement as a probe
  - Bohmian trajectories: double-slit interference
  - Entangled particles: double double-slit & nonlocality
  - Other quantum features
- Conclusion

# Is Quantum Mechanics “The Final Theory”?

1. Is it correct?
2. Is it complete?
3. Are we thinking about it in the best way?

# Search for a Deterministic Theory

- EPR (1935): Quantum mechanics is incomplete. We should look for HV's (implicitly: local ones)
- Von Neumann (1932): No deterministic HV theories!
- Bohm (1952): Explicit hidden variable model inspired by De Broglie's pilot wave. Highly non-local.
- Bell (1964): Does every hidden-variable model have to be non-local like Bohm's? (Yes!)

ON THE IMPOSSIBLE PILOT WAVE

A B S T R A C T

J.S. Bell  
CERN - Geneva

The strange story of the von Neumann impossibility proof is recalled, and the even stranger story of later impossibility proofs, and how the impossible was done by de Broglie and Bohm. Morals are drawn.

Dedicated to Louis de Broglie  
on the occasion of his ninetieth birthday

But in 1952 I saw the impossible done. It was in papers by David Bohm<sup>(5)</sup>. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the "observer", could be eliminated.

Moreover the essential idea was one that had been advanced already by de Broglie<sup>(6)</sup> in 1927, in his "pilot wave" picture.

# Deterministic Extensions to QM

- *A deterministic theory must*
  - Satisfy Bell's theorem (Bell, 1964)
  - Exhibit contextuality (Kochen and Specker, 1967)
  - Have microscopic superluminal signaling (Colbeck and Renner, 2011)
  - Include the wavefunction as a variable (Pusey, Barret, Rudolph, 2012)

ARTICLE

Received 17 Jan 2011 | Accepted 30 Jun 2011 | Published 2 Aug 2011

DOI: 10.1038/ncomms1416

No extension of quantum theory can have improved predictive power

Roger Colbeck<sup>1</sup> & Renato Renner<sup>2</sup>

nature  
physics

ARTICLES

PUBLISHED ONLINE: 6 MAY 2012 | DOI: 10.1038/NPHYS2309

**On the reality of the quantum state**

Matthew F. Pusey<sup>1\*</sup>, Jonathan Barrett<sup>2</sup> and Terry Rudolph<sup>1</sup>

# Bohmian Mechanics

- Define  $R, S$  real such that  $\psi = R e^{iS/\hbar}; P = R^2$
- Schrodinger's equation implies:
  - Hamilton-Jacobi equation with a quantum potential
  - Continuity equation:

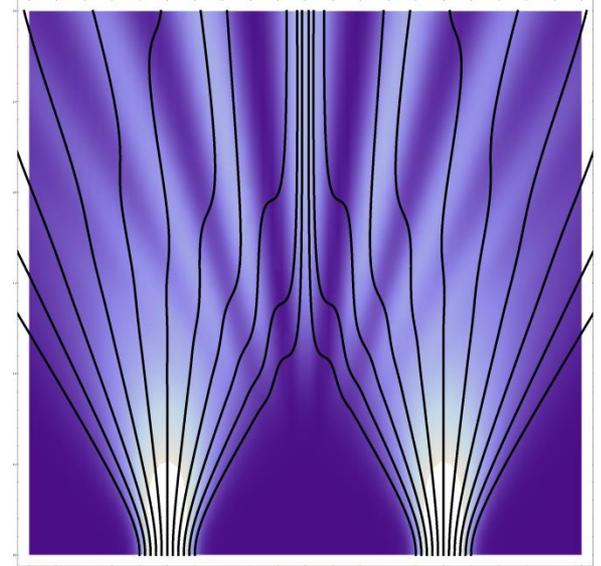
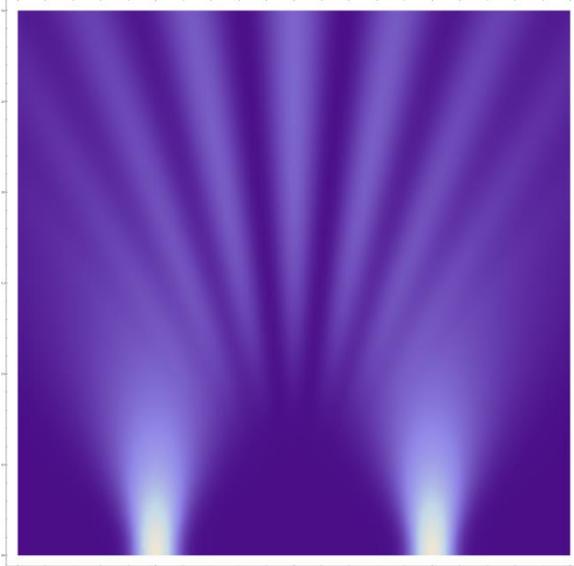
$$\frac{\partial P}{\partial t} + \nabla \cdot (P \vec{v}) = 0$$

- The Bohmian velocity is given by

$$\vec{v} = \frac{\vec{j}}{P} = \frac{\vec{\nabla} S}{m}$$

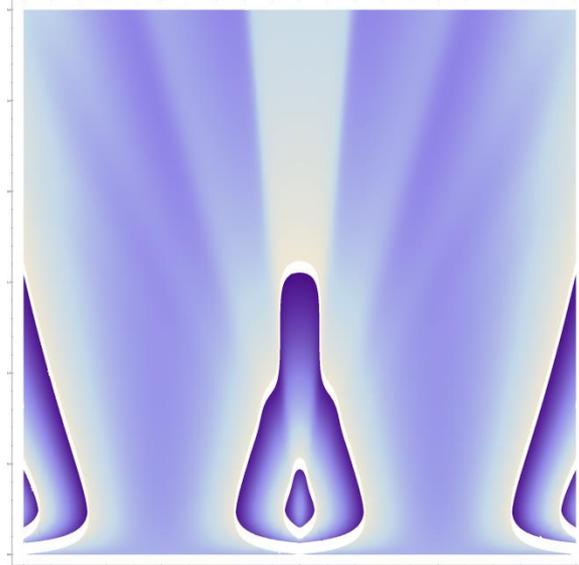
D. Bohm, Phys. Rev. **85**, 166 and 180 (1952)

# Bohmian Trajectories

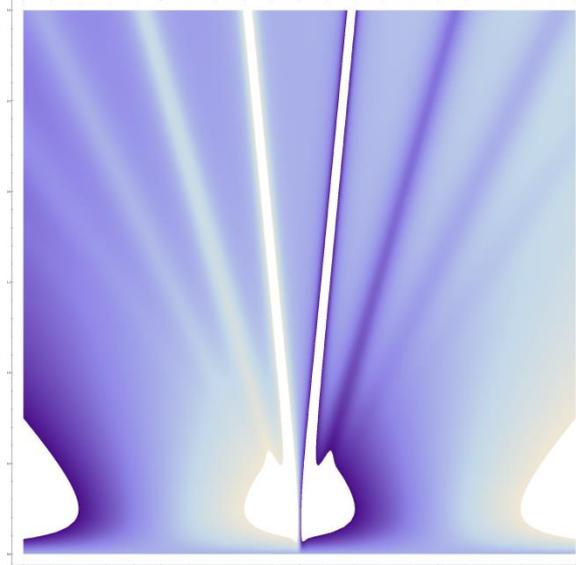


$|\psi|^2$

$S$



$V$



# Weak Measurement

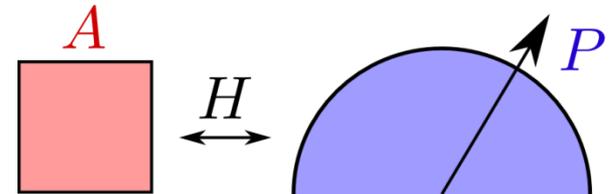
- Use a pointer with a big uncertainty (relative to the strength of the measurement interaction)
- Allows one to obtain information, without creating entanglement between system and pointer (effective "collapse")
- Can be used as a tool for probing and understanding subtle quantum features

Aharonov, Albert, Vaidman, PRL **60**, 1351 (1988)

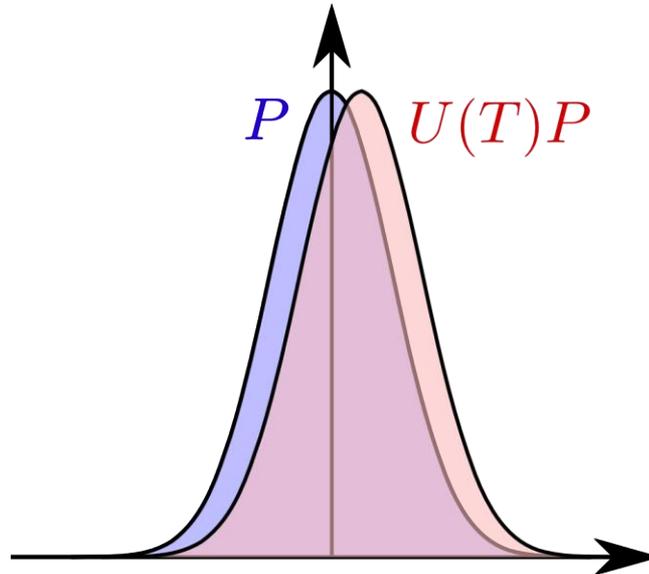
# Weak Measurement

- Pointer-system interaction:

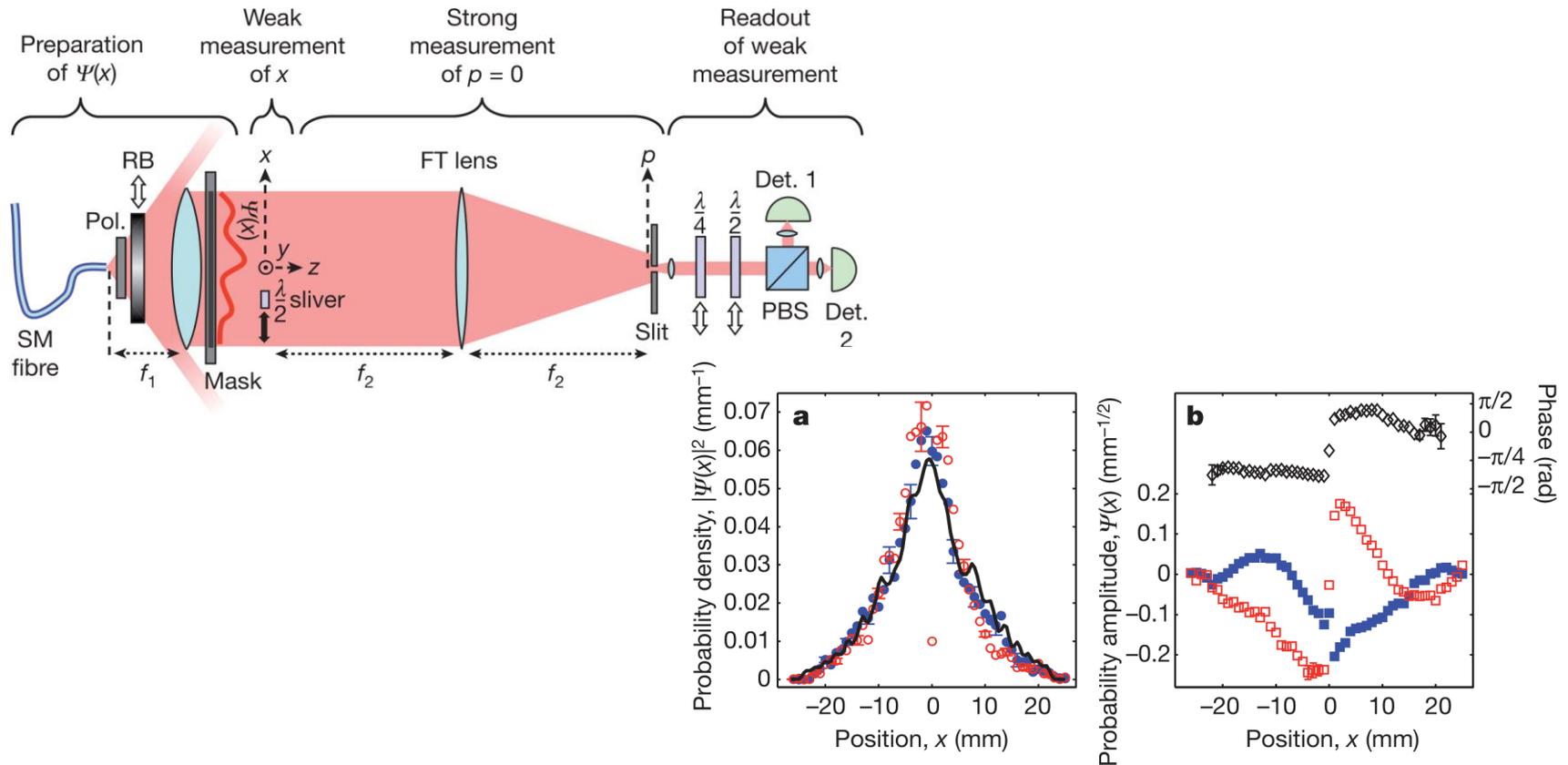
$$H = \chi P A$$



- $\chi T$  needs to be small



# Direct Measurement of the Wavefunction



J. S. Lundeen et al., Nature **474**, 188 (2011)

# Measuring $X$ and $P$ Simultaneously

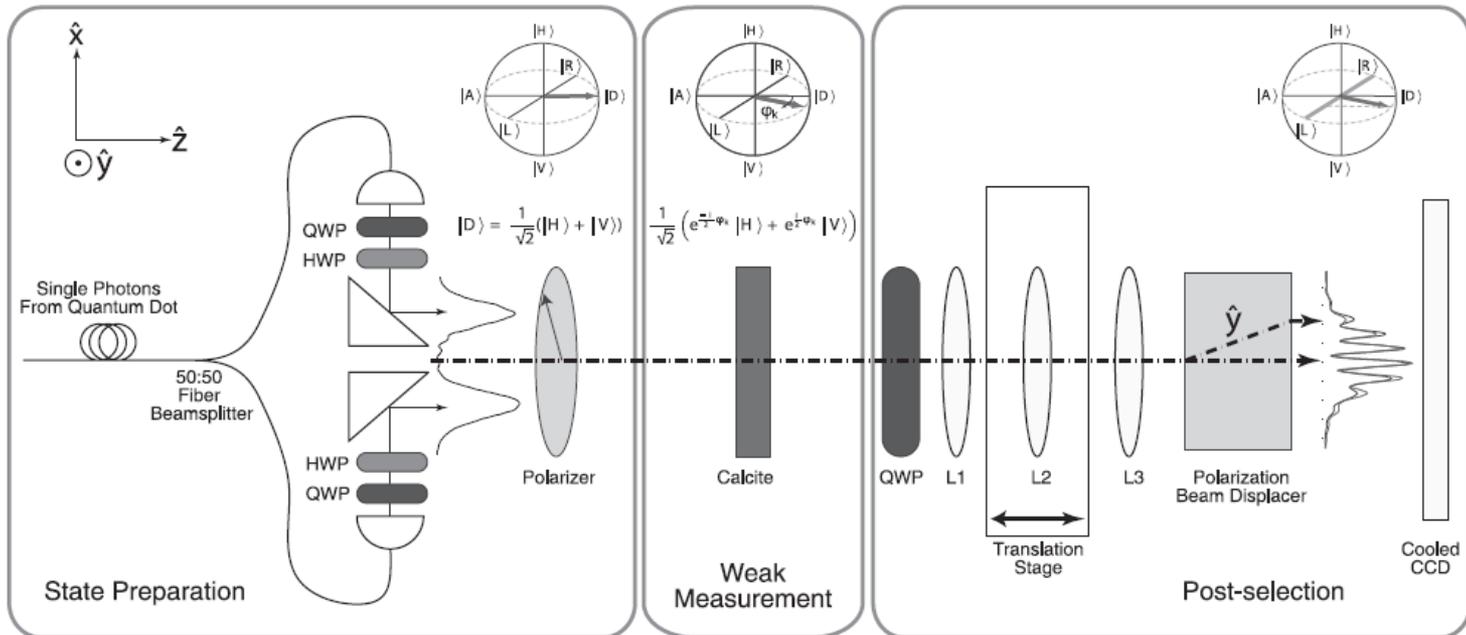
- Weak measurements can be used to define a “*naively observable velocity*” (Wiseman, 2007)
- Perform a weak measurement of  $P$ , followed by a strong measurement of  $X$ :

$$v(x) = \frac{\langle x | \hat{P} | \psi \rangle}{\langle x | \psi \rangle}$$

- This precisely equals the Bohmian velocity!

$$v = \frac{j}{P}$$

# Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer



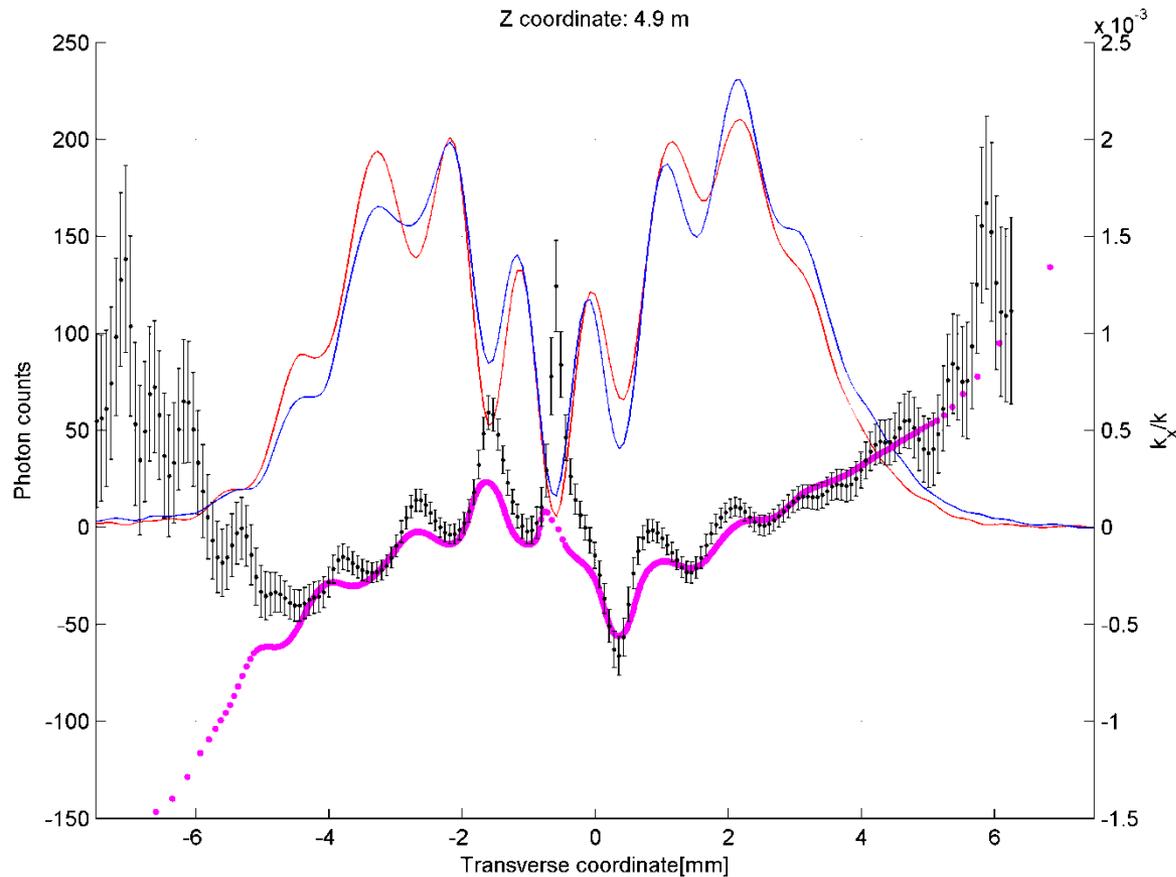
$$\varphi(k_x) = \zeta \frac{k_x}{|\mathbf{k}|} + \varphi_0$$

$$I_R \propto [1 + \sin\varphi(k_x)]$$

$$I_L \propto [1 - \sin\varphi(k_x)]$$

Kocsis et al., Science **332**, 1170 (2011)

# Momentum Measurement



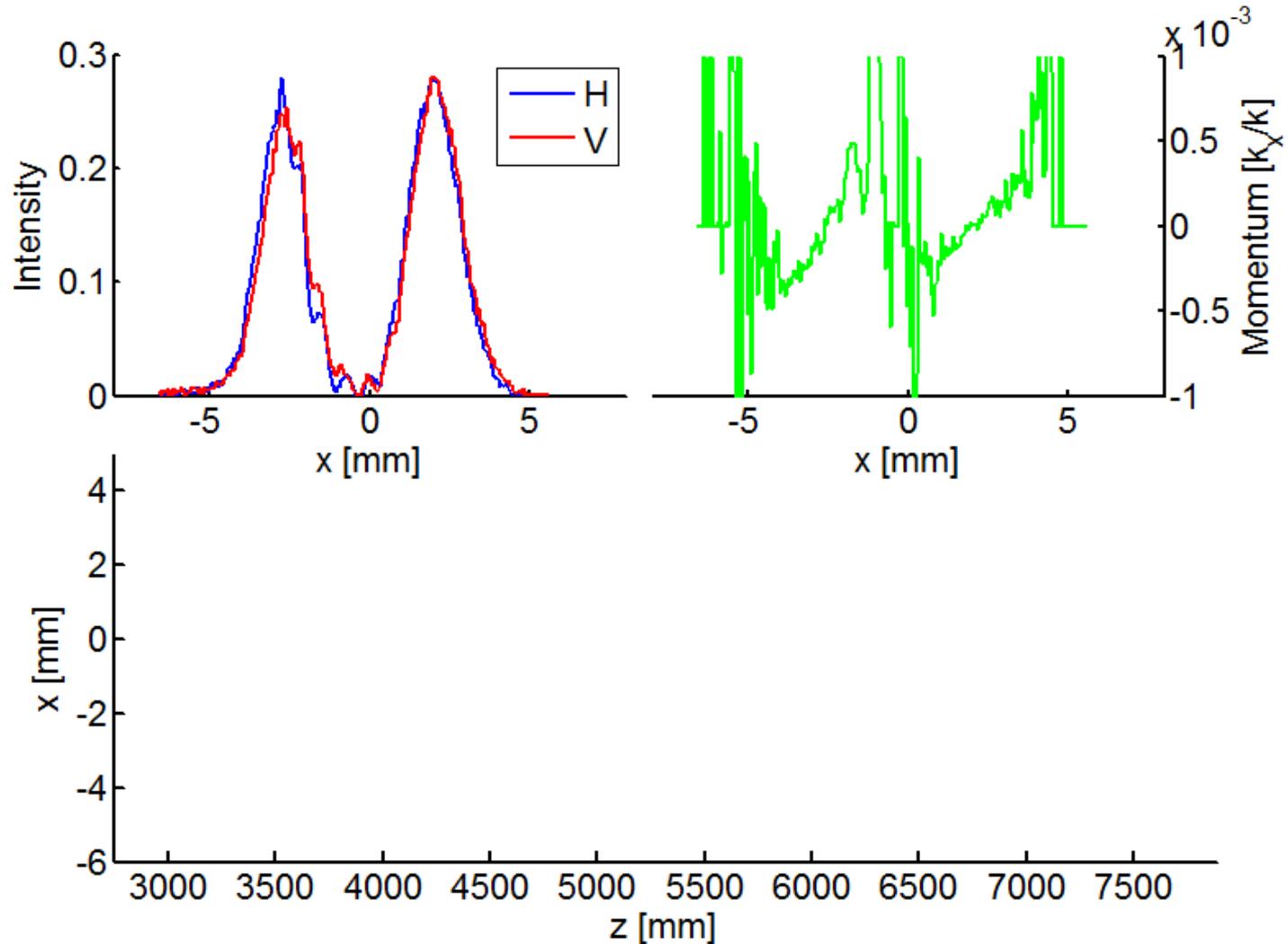
$P_R(x)$

$P_L(x)$

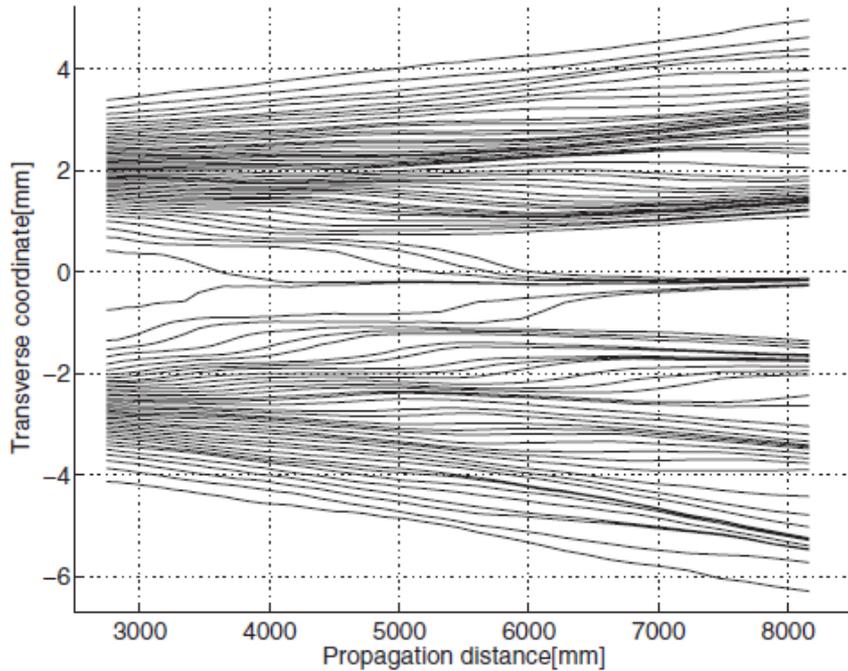
$k_x(x)$  –  
Weak val.

$k_x(x)$  –  
Prob. Cons.

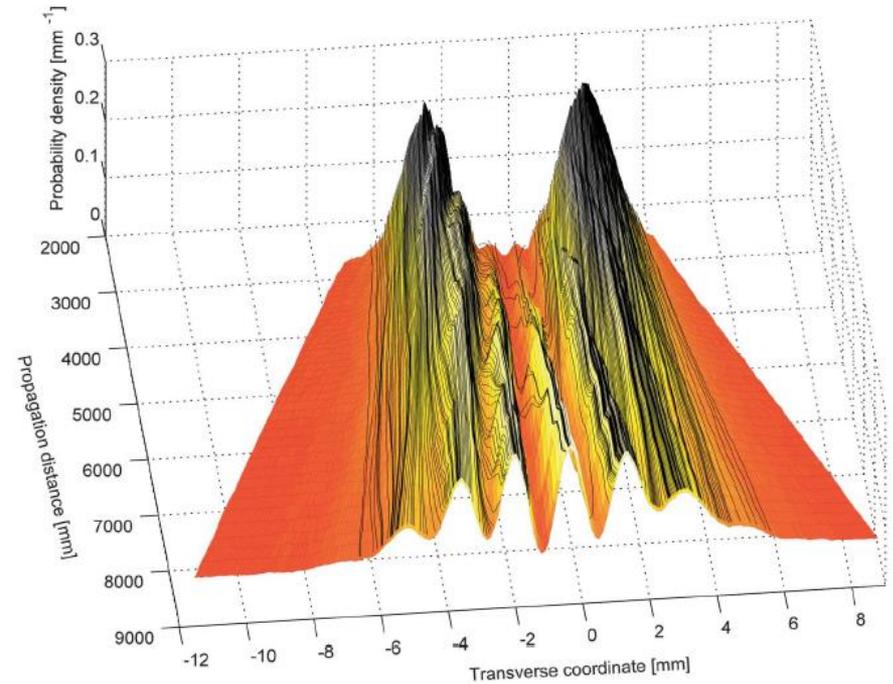
# Trajectory Reconstruction



# Trajectory Reconstruction

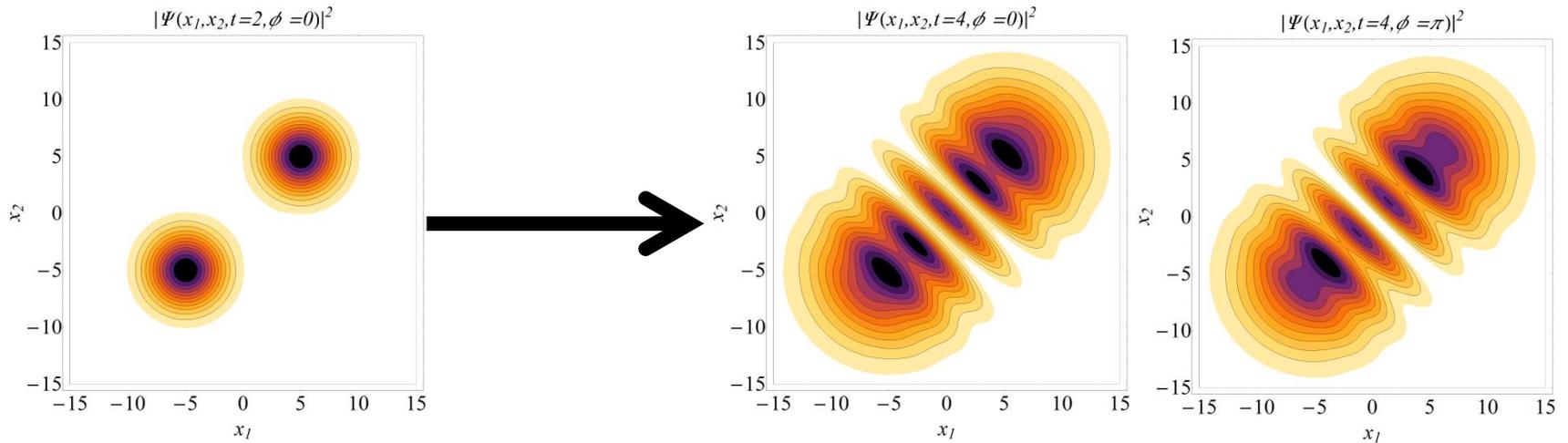
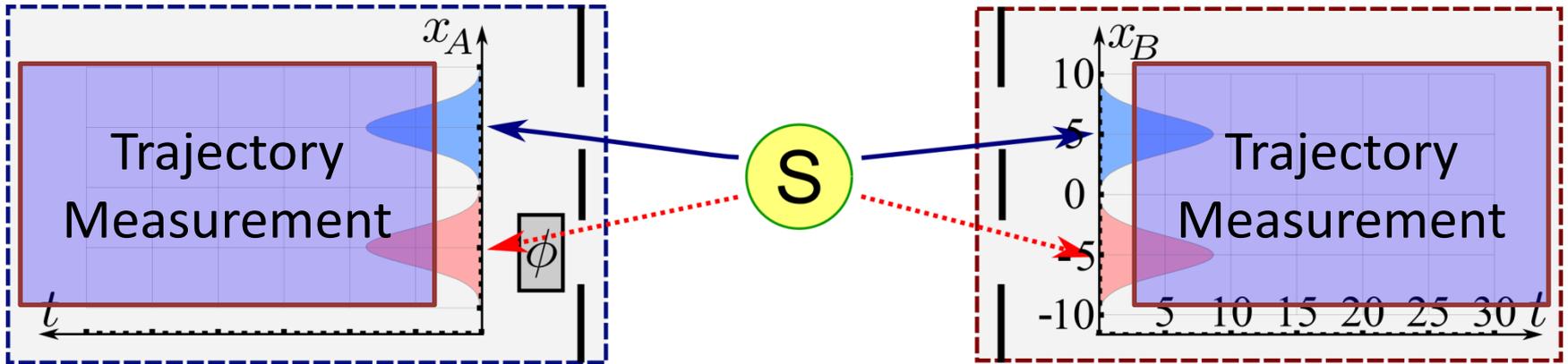


Reconstructed trajectories



Trajectories on top of probability distribution

# Double Double-Slit



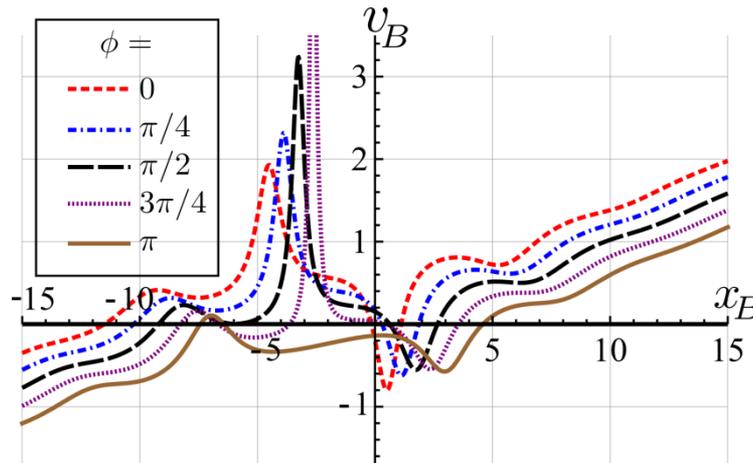
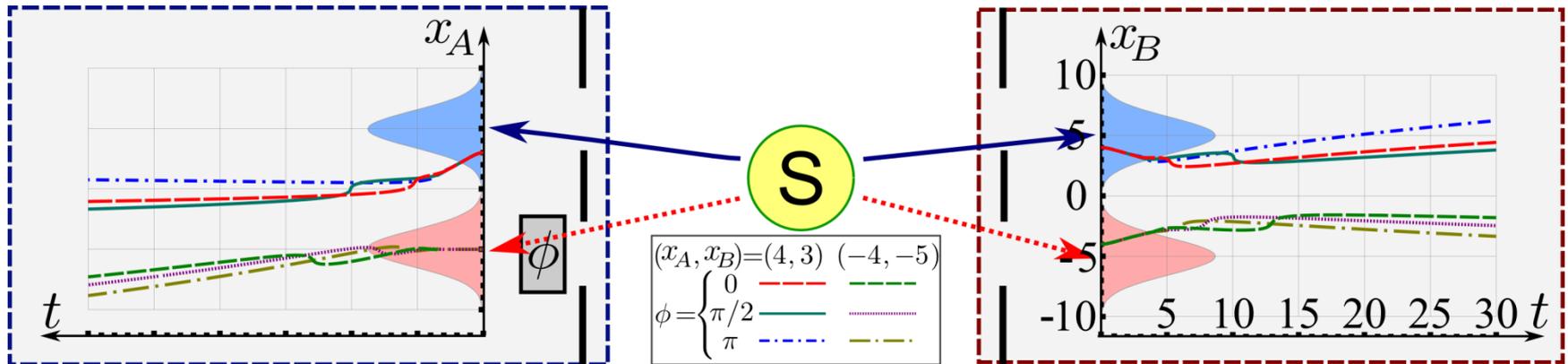
Braverman and Simon, PRL **110**, 060406 (2013)

October 6, 2013

Boris Braverman, EmQM2013

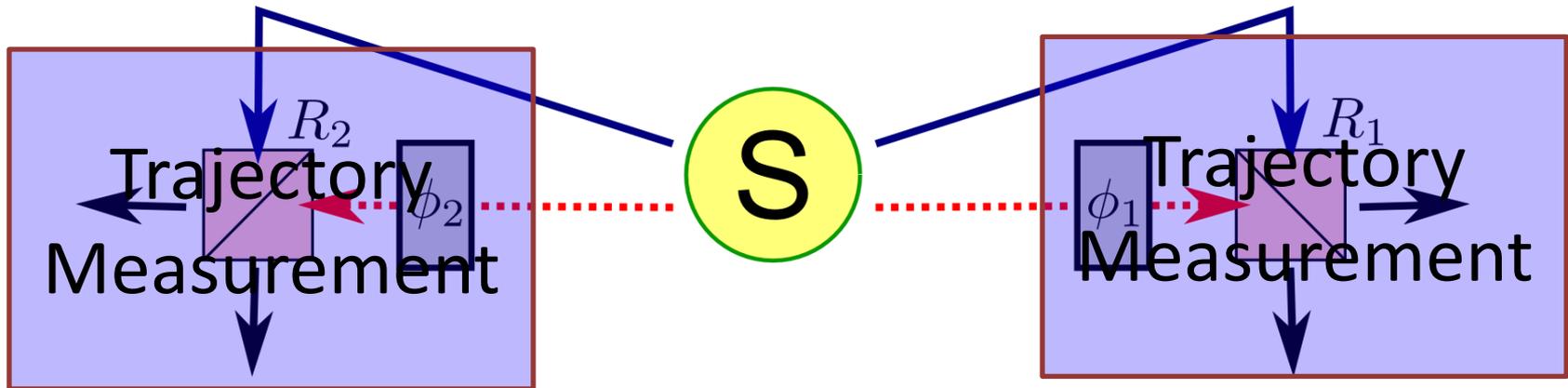


# Nonlocal Trajectories



# Bell Tests

- Direct probing of Bohmian trajectories in a Bell test experiment



# Contextuality

- Kochen-Specker theorem (1967): Hidden-variable theories must be *contextual*.
- Let  $\lambda$  be the set of HV's.
- Then,  $\hat{A}(\lambda)$  depends on what other commuting observables are measured together with it.

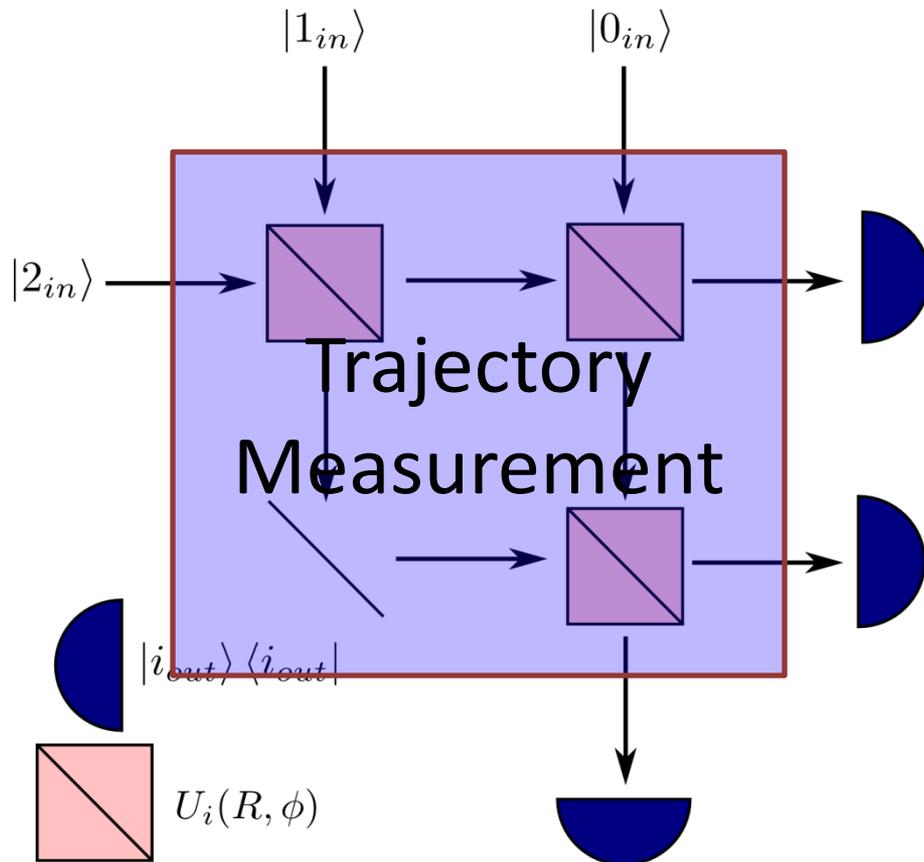
$$\hat{A}(\lambda, \{\hat{B}, \hat{C}\}) \neq \hat{A}(\lambda, \{\hat{D}, \hat{E}\})$$

- Related to the von Neumann proof:

$$\widehat{A + B}(\lambda) \neq \hat{A}(\lambda) + \hat{B}(\lambda)$$

# Contextuality

- Construct a qutrit analyzer



- Measurements of  $|0\rangle$ ,  $|2\rangle$  are non-contextual
- $|1\rangle$  is contextual!
- Can directly observe trajectories change

# Surrealistic Bohm Trajectories

Berthold-Georg Englert<sup>1,2</sup>, Marlan O. Scully<sup>3</sup>, Georg Süssmann, and Herbert Walther<sup>1,2</sup>

<sup>1</sup> Sektion Physik, Universität München, Am Coulombwall 1, D-8046 Garching, Germany

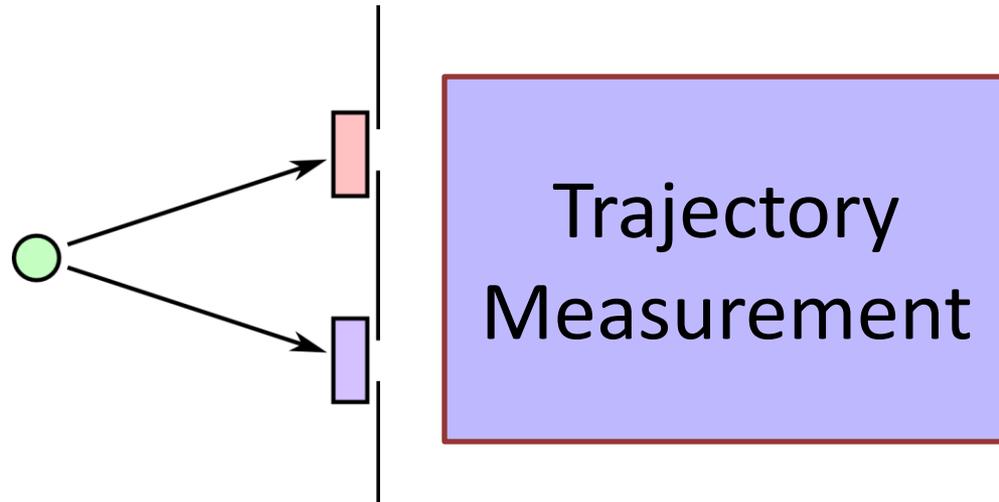
<sup>2</sup> Max-Planck-Institut für Quantenoptik, Ludwig-Prandtl-Straße 10, W-8046 Garching.

<sup>3</sup> Department of Physics, Texas A & M University, College Station, TX 77843-4242.

Z. Naturforsch. 47a, 1175–1186 (1992); received September 22, 1992

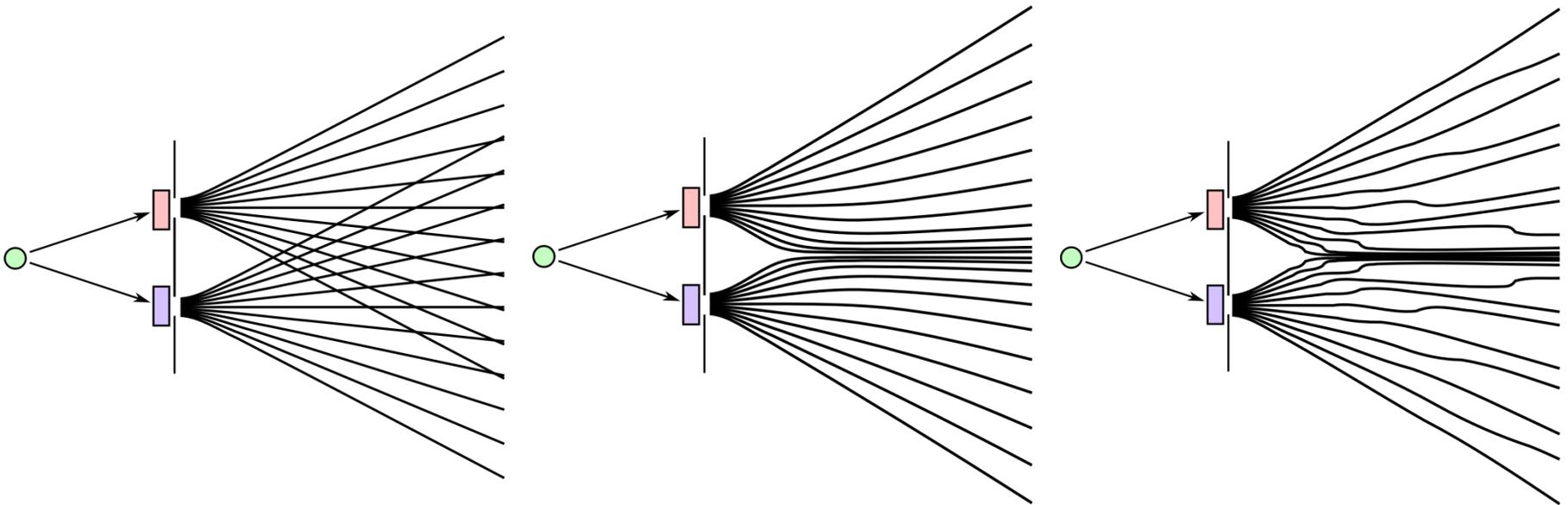
A study of interferometers with one-bit which-way detectors demonstrates that the trajectories, which David Bohm invented in his attempt at a *realistic* interpretation of quantum mechanics, are in fact *surrealistic*, because they may be macroscopically at variance with the observed track of the particle. We consider a two-slit interferometer and an incomplete Stern-Gerlach interferometer, and propose an experimentum crucis based on the latter.

- Perform a which-way experiment with a double slit:



# Surrealist Trajectories

- Which outcome will be observed?



# Summary

- Deterministic extensions to QM are strongly constrained, but are nonetheless appealing
- Weak measurement can be used as a probe of quantum and, in the context of deterministic theories, sub-quantum features such as trajectories
- By combining these two paradigms, we can piece together a different perspective on many quantum behaviours

# Extras

# On the reality of the quantum state

Matthew F. Pusey<sup>1\*</sup>, Jonathan Barrett<sup>2</sup> and Terry Rudolph<sup>1</sup>

Quantum states are the key mathematical objects in quantum theory. It is therefore surprising that physicists have been unable to agree on what a quantum state truly represents. One possibility is that a pure quantum state corresponds directly to reality. However, there is a long history of suggestions that a quantum state (even a pure state) represents only knowledge or information about some aspect of reality. Here we show that any model in which a quantum state represents mere information about an underlying physical state of the system, and in which systems that are prepared independently have independent physical states, must make predictions that contradict those of quantum theory.

- No theory can have multiple underlying states of reality  $\lambda$  correspond to the same QM wavefunction  $\psi$
- Therefore,  $\lambda$  uniquely determines  $\psi$
- $\psi$  is one of the “variables” of the model

# No extension of quantum theory can have improved predictive power

Roger Colbeck<sup>1</sup> & Renato Renner<sup>2</sup>

We remark that several other attempts to extend quantum theory have been presented in the literature, the de Broglie–Bohm theory<sup>16,17</sup> being a prominent example (this model recreates the quantum correlations in a deterministic way but uses non-local hidden variables, see ref. 18 for a summary). Our result implies that such theories necessarily come at the expense of violating assumption *FR*.

the negative: no extension of quantum theory can give more information about the outcomes of future measurements than quantum theory itself. Our result has significance for the foundations of quantum mechanics, as well as applications to tasks that exploit the inherent randomness in quantum theory, such as quantum cryptography.

# Bohmian Mechanics

$$i\hbar\partial\psi/\partial t = -(\hbar^2/2m)\nabla^2\psi + V(\mathbf{x})\psi.$$

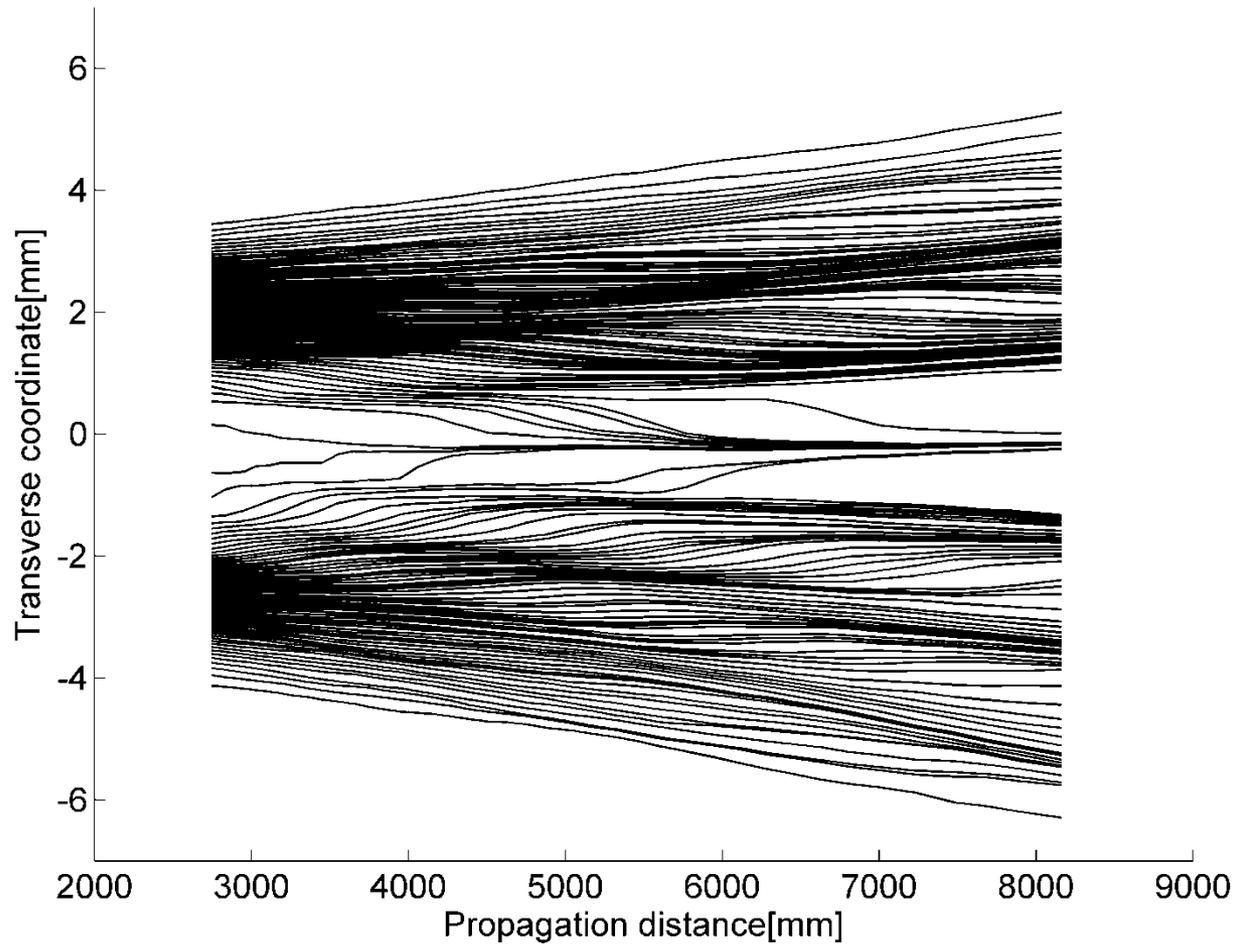
$$\psi = R \exp(iS/\hbar) \quad P(\mathbf{x}) = R^2(\mathbf{x})$$

$$\frac{\partial P}{\partial t} + \nabla \cdot \left( P \frac{\nabla S}{m} \right) = 0,$$

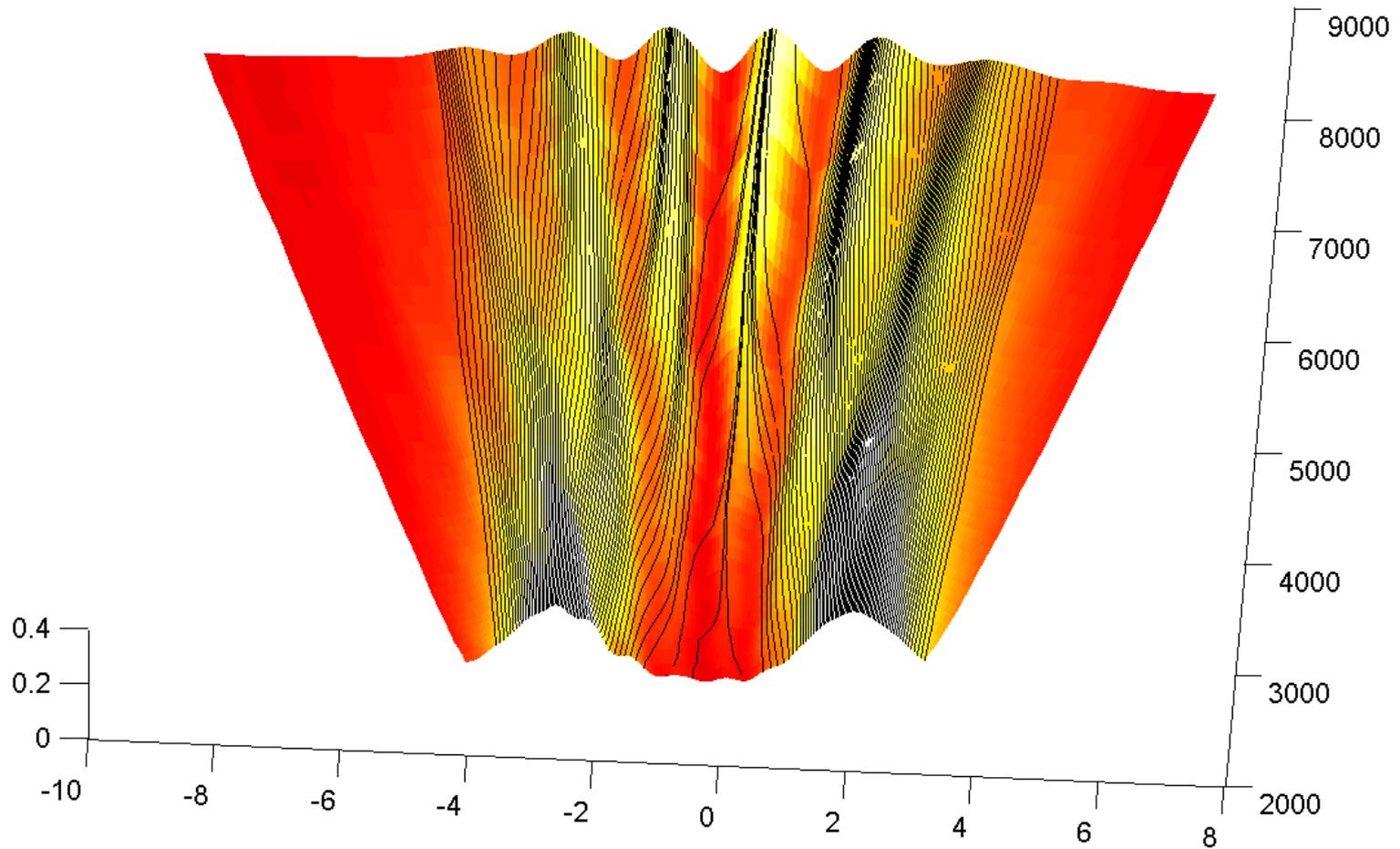
$$\partial P/\partial t + \nabla \cdot (P\mathbf{v}) = 0.$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\mathbf{x}) - \frac{\hbar^2}{4m} \left[ \frac{\nabla^2 P}{P} - \frac{1}{2} \frac{(\nabla P)^2}{P^2} \right] = 0.$$

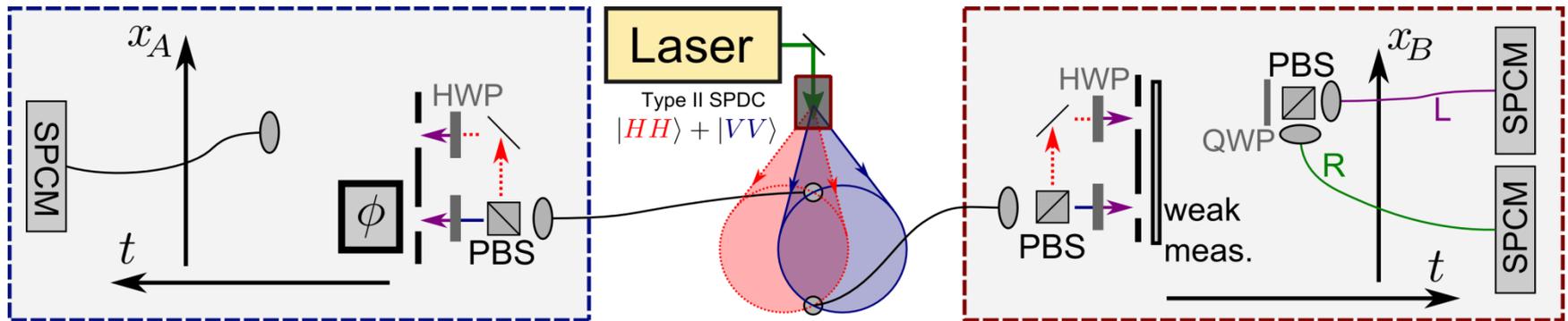
# Trajectory Plots



# Trajectory Plots



# Double-Double Slit Implementation



- To get complete trajectories, need about 18h at realistic experimental parameters