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# Investigations of fundamental phenomena in quantum mechanics with neutrons

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Abstract. Neutron interferometer and polarimeter are used for the experimental investigations of quantum mechanical phenomena. Interferometry exhibits clear evidence of quantumcontextuality and polarimetry demonstrates conflicts of a contextual model of quantum mechanics á la Leggett. In these experiments, entanglements are achieved between degrees of freedom in a single-particle: spin, path and energy degrees of freedom are manipulated coherently and entangled. Both experiments manifest the fact that quantum contextuality is valid for phenomena with matter waves with high precision. In addition, another experiment is described which deals with error-disturbance uncertainty relation: we have experimentally tested error-disturbance uncertainty relations, one is derived by Heisenberg and the other by Ozawa. Experimental results confirm the fact that the Heisenberg's uncertainty relation is often violated and that the new relation by Ozawa is always larger than the limit. At last, as an example of a counterfactual phenomenon of quantum mechanics, observation of so-called quantum Cheshire Cat is carried out by using neutron interferometer. Experimental results suggest that pre- and post-selected neutrons travel through one of the arms of the interferometer while their magnetic moment is located in the other arm.

## 1. Introduction

Quantum mechanics (QM) is one of the most successful theories in physics and its predictions are verified with high precision in a wide range of the field by experiments. From the beginning, however, QM is providing an extraordinary and strange view of nature, which is different from that in classical physics. For instance, a particle such as an electron, a neutron and a positron can behave as non-locally located wave: wave-particle duality postulates that all particles exhibit both wave and particle properties. The deBroglie relation,  $\lambda = \hbar/mv$  connects the wavelength  $\lambda$ with the mass m and the velocity v of the particle. Furthermore, the uncertainty principle forms the basis of indeterminacy in QM. It prohibits and describes the limitations of simultaneous measurements of certain pairs of observables [1]: there are physical properties, which can be accessed only with limitation. Moreover, superposition of macroscopic systems like Schrödinger's cat [2] as well as the non-local correlation suggested by Einstein, Podolsky, and Rosen (EPR) [3] are not intuitively understandable and seem to be paradoxical at first sight. Both are proposed as thought experiments to derive (seemingly) contradictory conclusions. Fundamental investigations of QM have been revealing its peculiarities, some are regarded as a resource of new technology in future.

From the very beginning, neutron interferometer experiments are established as one of the most powerful tools for investigations of quantum mechanical phenomena on a very fundamental

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basis [4, 5]. Over the last decades the neutron interferometry has provided excellent opportunities for many different types of interferometer experiments with neutrons, ranging from fundamental quantum investigations to application measurements, such as precise measurements of coherent The former exploits the neutron interferometry as a matterneutron scattering lengths. wave interference experiment and the latter is frequently required for other neutron scattering spectroscopy. Consequences of the nonrelativistic Schrödinger equation for matter-waves can be studied, for instance, with electrons, atoms, ions, and molecules. Features of neutron interferometry, such as macroscopic-scale experiments, high detector-efficiency, low decoherencerate, and high-efficiency manipulation rate, make it a unique strategy for quantum mechanical investigations. Recently neutron polarimeter experiments have turned out to serve as another tool to verify the basic concepts of quantum mechanics. There, basis used in the experiment are spanned not by two paths  $|I\rangle$  and  $|II\rangle$ , but by spin eigenstate  $|\uparrow\rangle$  and  $|\downarrow\rangle$ . With this device, for instance, the noncommutation of the Pauli spin operator [6] and a number of geometric phase measurements [7] are carried out. The implicit polarization interference scheme allows us to perform textbook like demonstrations of QM with high efficiency and stability.

In this paper, recent experiments with the neutron interferometer and the polarimeter are presented. The interferometry studies properties of quantum system triply-entangled between degrees of freedom [8] and the polarimetry investigates a contextual model á la Leggett [9]. In addition, successive measurements of neutron's spin [10, 11] exhibits the violation of the naive error-disturbance relation by Heisenberg and confirms the validity of a new universally valid error-disturbance relation by Ozawa [12, 13]. Furthermore, a recent experiment to observe a quantum Cheshire Cat [14, 15] in neutron interferometer setup is described: neutron and its magnetic moment are observed to be disembodied and spatially separated [63].

# 2. Entanglement between degrees of freedom of single neutrons

## 2.1. Entanglement studied with neutron interferometer

It was Einstein, Podolsky, and Rosen (EPR) [3] and afterwards Bell [17] who shed light on the non-local properties between subsystems in quantum mechanics. Bell inequalities are constraints imposed by local hidden-variable theories (LHVTs) on the results of spacelike separated experiments on distant systems. The conflict between LHVTs and QM is even more apparent in tri- or multi-partite quantum systems which was analysed by Greenberger, Horne and Zeilinger (GHZ) [18, 19]. There, the contradiction leads to nonstatistical predictions in contrast to common Bell-inequalities. Mermin [20] showed that this conflict can be converted into a larger violation of a Bell-like inequality between three or more separated systems. Experimental tests of these inequalities were reported with photons [21, 22] as well as with ions [23]. Kochen and Specker [24] were the first to analyse the concept of contextuality in QM and find striking phenomena predicted by quantum theory. Note that LHVTs form a subset of a larger class of hidden-variable theories known as noncontextual hidden-variable theories (NCHVTs): in NCHVTs the result of a measurement of an observable is assumed to be predetermined and not affected by a (previous or simultaneous) suitable measurement of any other compatible or co-measurable observable.

In single particle systems, different degrees of freedom (DOFs) can be entangled. In this scenario, the conflict arises not between QM and LHVTs but a violation confirms the impossibility of NCHVTs [25]. The violation of Bell-like inequality is reported by using entanglement between spin and path DOF of single neutrons [26, 5], as well as single photons [27], both confirming quantum contextuality. In addition, tests of Kochen-Specker theorem are carried out [28, 29], which again favor quantum contextuality. Later, quantum tomographic analysis of the Bell-like state is accomplished [30], approving the quality of the generated Bell-like state of DOFs. For further investigations of entanglement of DOFs, it is essential to development methods to manipulate other DOFs: the use of interaction between neutron's



Figure 1. Setup for the study of tripartite entangled GHZ-like state in neutron interferometry.

spin and a time-dependent magnetic field enables a coherent energy-manipulation scheme. The experiment confirms high quality of the method [31], which is expected to allow advanced studies of multipartite entanglements in single particles.

# 2.1.1. Tripartite entangled Greenebrger-Horne-Zeilinger (GHZ) state

Here, we describe the experimental realization of tripartite entanglement for single neutrons [8] where one external degree of freedom (path states in the interferometer) is entangled with two internal degrees of freedom (spin and energy) leading to a violation of a Mermin-like inequality [20]. It was Greenberger, Horne and Zeilinger who first proposed the Greenebrger-Horne-Zeilinger (GHZ) state for four spin-1/2 particles [18, 19]. Later Mermin [32] presented a version with three spin-1/2 particles; in Ref. [33] he pointed out the use of the GHZ-states to reveal the relation between Kochen-Specker theorem [24] and Bell theorem [17].

Imagine a perfect crystal neutron interferometer experiment [4, 5], where the up-polarized incident neutron beam passes through the beam-splitter. Here, the state of neutron's path is transformed into a superposition of two path-states,  $\frac{1}{\sqrt{2}}(|I\rangle + |II\rangle)$ . In the interferometer, a RF spin-flipper is inserted in the path II, which flips neutron's spin by a time-dependent interaction: this induces energy transitions from the initial energy state  $|E_0\rangle$  to states  $|E_0 - \hbar\omega\rangle$  by photon exchange [34]. Consequently, one can generate neutrons in a triply entangled GHZ-like state, given by

$$|\Psi_N^{GHZ}\rangle = \frac{1}{\sqrt{2}} \Big(|\uparrow\rangle \otimes |\mathbf{I}\rangle \otimes |E_0\rangle + |\downarrow\rangle \otimes |\mathbf{II}\rangle \otimes |E_0 - \hbar\omega\rangle \Big). \tag{1}$$

The state of neutrons is characterized by three (the spin, path and energy) DOFs: all of them are described simply by two-level quantum systems. Measurement represented by operators  $\sigma_x$  and  $\sigma_y$  in each DOF are accomplished simply by phase manipulations between the basis states in each DOF. In particular,

(i) The spin-phase  $\alpha$  is adjusted by a magnetic field oriented along +z direction ('accelerator' coil) together with a DC-flipper in  $\pi/2$ -flipping mode.

(ii) The phase manipulation of the path is accomplished with an auxiliary phase shifter  $\chi$  made of a parallel-sided Si plate and the last plate of the interferometer.

(iii) The so-called zero-field precession phase  $\gamma$  [35] is employed for the phase manipulation of the energy. The second RF-flipper together with a DC-flipper are used. (An experimentally convenient method to manipulate individually the Larmor phase  $\alpha$  and the zero-field phase  $\gamma$ was found and reported in [36].)

Mermin derived an inequality suitable for experimental tests to distinguish between predictions of QM and LHVTs [20]. In a similar way, assuming a tripartite system and taking



Figure 2. Typical intensity modulations by scanning the phase shifter at various angles for spin ( $\alpha$ ) and energy ( $\gamma$ ) degrees of freedom.

$\sigma^s_x\sigma^p_x\sigma^e_x)$	$\sigma_x^s\sigma_y^p\sigma_y^e)$	$\sigma_y^s\sigma_x^p\sigma_y^e)$	$\sigma_y^s\sigma_y^p\sigma_x^e)$
+0.659(2)	-0.632(2)	-0.603(2)	-0.664(2)

Table 1. Finally obtained expectation values

the assumption in the conditionally independent form due to NCHVTs instead of LHVTs, the border for a sum of expectation values of certain product observables is obtained. The sum of expectation values of product observables, called M, is defined as

$$M \equiv E[\sigma_x^s \sigma_x^p \sigma_x^e] - E[\sigma_x^s \sigma_y^p \sigma_y^e] - E[\sigma_y^s \sigma_x^p \sigma_y^e] - E[\sigma_y^s \sigma_y^p \sigma_x^e]$$
(2)

where  $E[\ldots]$  represents expectation values, and  $\sigma_j^s$ ,  $\sigma_j^p$ , and  $\sigma_j^e$  represent Pauli operators for the two-level systems in the spin, path, and energy DOF, respectively. NCHVTs demands  $|M| \leq 2$ , while quantum theory predicts an upper bound of 4: any measured value of M that is larger than 2 decides in favor of quantum contextuality.

The experiment was carried out at the neutron-interferometer beam line S18 at the high flux reactor at the Institute Laue Langevin (ILL) [8]. A schematic view of the experimental setup is shown in Fig.1. To determine the expectation values in M in Eq.(2), we performed 16 independent path phase  $\chi$  scans by tuning the spin phase  $\alpha$ , and the energy phase  $\gamma$  each at 0,  $\pi/2$ ,  $\pi$  and  $3\pi/2$ . Typical sinusoidal intensity modulations are depicted in Fig.2. The finally obtained expectation values are shown in Tab.1. From these values, the final M-value was calculated as  $M = 2.558 \pm 0.004$ , which clearly exhibits a violation of the Mermin-like inequality,  $|M| \leq 2$ , and confirms the invalidity of the assumption of non-contextuality. The deviation from the ideal value of 4 is solely due to the reduced contrast of the interferograms. Recently a further study of tripartite entanglement of neutron's DOF appeared [37]: experimental evidence of the generation of distinct types of genuine multipartite entanglement is confirmed. By using appropriate nonlinear witness, even finer properties of the type of the genuine multipartite entanglement are revealed. In the analysis, the extraordinarily high fidelity of the generated entangled states is verified.



Figure 3. Setup of the neutron polarimetric test of contextual realistic model á la Leggett.

# 2.2. Entanglement studied with neutron polarimeter

For investigations on the foundations of quantum mechanics, the neutron interferometer is established as an ideal tool of studies with matter waves. Although such an interferometer is suitable for highly intuitive proof-of-principle demonstrations of quantum phenomena [4, 5], the device is extremely delicate: interferogram can easily disappear due to vibrations and thermal disturbance. An alternative approach to perform quantum optical experiments with neutrons is the use of neutron polarimeter, where the interference between spin eigenstates is observed. The advantange of this approach is (i) instrument is very robust: pretty insensitive to the environmental disturbances, (ii) highly efficient (> 99%) manipulations are possible: final contrast reaching at about 99%. Neutron polarimetry has been used to demonstrate fundamental quantum-mechanical properties. For studies of entanglement, A Bell-test with bipartite entanglement [38] as well as GHZ-state with tripartite enantanglement [39] are reported.

## 2.2.1. Leggett's model tested with matter waves

Here, we describe a falsification of a contextual realistic model á la Leggett using neutrons [9]. In 2003, Leggett proposed a non-local realistic model [40], and an extension of Leggett's model appeared later [41]. The experiments with photons show conflicts with Leggett's model [42, 43, 44, 45], but all experimental tests were performed with photons. Thus, it is important to test a model á la Leggett with neutrons, in particular with matter waves. It is worth noting here that this experiment demands extremely high contrast of interferograms, i.e., more than 97.4%; such a high contrast is only accessible with polarimeter in neutron optics.

In our polarimetric test, we followed the criteria used in the first experimental study by Gröblacher et al. [42]. In particular, the contextual theory to be tested here is based on the following assumptions. (i) All the values of measurements are predetermined. (ii) States are a statistical mixture of subensembles. (iii) The expectation values taken for each subensemble obey cosine dependence. Assumptions (i) and (ii) are common to experimental tests of ordinary non-contextual theories and assumption (iii) is a peculiarity of this model. Here, the result of the final measurement of B (A) depends on the setting of the previous measurement of A (B): a realistic contextual model is tested in our experiment. Denoting the measurement settings for observables A and B by  $\vec{a}_1$ ,  $\vec{a}_2$  and  $\vec{b}_1$ ,  $\vec{b}_2$ , respectively, the Leggett-like inequality is given by

$$S_{\text{Legg}} \equiv \left| E_1(\vec{a}_1; \phi) + E_1(\vec{a}_1; 0) \right| + \left| E_2(\vec{a}_2; \phi) + E_2(\vec{a}_2; 0) \right| \le 4 - \frac{4}{\pi} \left| \sin \frac{\phi}{2} \right|, \tag{3}$$

where  $E_i(\vec{a}_i; \phi)$  represent expectation values of joint (correlation) measurements at settings  $\vec{a}_i$ 



Figure 4. S-values as a function of the deviation angle  $\phi$  to test Leggett's alternative model.

and  $\vec{b}_j$  with relative angle  $\phi$ . We assume settings  $\vec{a}_1$ ,  $\vec{a}_2$ , and  $\vec{b}_1$  to lie in a single (equatorial) plane and only  $\vec{b}_2$  lies in a plane perpendicular to it: expectation values  $E_1$  and  $E_2$  are given by correlations in planes perpendicular to each other. Maximum violation is expected at  $\phi_{\text{max}} \sim 0.1\pi$ , resulting in a bound of the Leggett-like inequality  $S_{\text{Legg}} = 3.797$  and a quantum value of  $S_{\text{QM}} = 3.899$ .

The experiment has been carried out at the research reactor facility TRIGA Mark II of the Vienna University of Technology. The experimental setup is depicted in Fig.3. Passing through a bent Co-Ti super mirror array, the beam is highly polarized. The same technique is employed to analyze the polarization. Two identical radio-frequency (RF) spin rotators are employed. Both are put in a homogeneous and static magnetic guide field. In the present experiment, a maximally entangled Bell-like state

$$|\Psi_{\rm N}^{\rm Bell}\rangle \propto \frac{1}{\sqrt{2}} \Big(|\uparrow\rangle|E_0\rangle - |\downarrow\rangle|E_0 - \hbar\omega\rangle\Big) ,$$
(4)

with spin basis states,  $|\uparrow\rangle$  and  $|\downarrow\rangle$ , as well as energy basis states,  $|E_0\rangle$  and  $|E_0 - \hbar\omega\rangle$ , is generated and affected by successive energy and spin measurements. Initial Larmor-precession scan exhibit sinusoidal intensity modulations with more than 99% contrast. By tuning the rotation angle of RF1 to  $\pi/2$ , a maximally entangled Bell-like state  $|\Psi_N^{\text{Bell}}\rangle$  is generated. Denoting unit vectors representing measurement directions as  $\vec{x}[\theta, \varphi]$  with polar angle  $\theta$  and azimuthal angle  $\varphi$ , the analyzer and RF2 enable to set  $\vec{a}_i[\pi/2, \gamma_i]$  for energy and  $\vec{b}_k[\alpha_k, \beta_k]$  for spin.

For a measurement of the Leggett-like inequality, Eq. (3), we require correlation measurements between settings outside the single plane. The maximum discrepancy between Leggett's model and quantum mechanics is expected at the angle  $\phi_{\text{max}} \sim 0.1\pi$  with directions  $\vec{a}_1[\pi/2,0]$ ,  $\vec{a}_2[\pi/2,\pi/2]$ ,  $\vec{b}_1[\pi/2,-\phi_{\text{max}}]$  and  $\vec{b}_2[\pi/2-\phi_{\text{max}},\pi/2]$ . The final  $S_{\text{Legg}}$ -value of the Leggett-like inequality is determined as  $S_{\text{Legg}} = 3.8387(61)$  at  $\phi = 0.104\pi$ , which is clearly larger than the boundary 3.7921: the violation is more than 7.6 standard deviations. In order to see the tendency of the violations, the parameter  $\phi$  is tuned to 8 different values between 0 and  $0.226\pi$ . Figure 4 shows a plot of the experimentally determined  $S_{\text{Legg}}$  together with the limit of the contextual model as well as the quantum mechanical prediction, calculated for a contrast of 99%. The experimental values follow the quantum mechanical prediction, and this clearly confirms the violation of Leggett's model.

## 3. Error-disturbance uncertainty relation

The uncertainty principle refers to intrinsic indeterminacy of quantum mechanics and ranks among the most famous statements of modern physics [46]. It was Heisenberg who first formulated the uncertainty relation as a limitation of accuracies of position and momentum measurements [1]. Later on, the uncertainty relation was reformulated in terms of standard deviations[47, 48], which denotes only the statistical quantity and neglects neither the disturbance due to interactions in a quantum measurement nor measurement error. It was known that the validity of Heisenberg's original relation is justified only under limited circumstances and Ozawa proposed a new universally valid error-disturbance uncertainty relation [12, 13]. Here, we describe a successive spin measurement of neutrons that allows determining the error of a spincomponent measurement and the disturbance caused on another spin-component measurement [10]. The results confirm that both error and disturbance completely obey the Ozawa's relation but often violate the Heisenberg's relation.

# 3.1. From Heisenberg to universally valid uncertainty relation

In 1927, Heisenberg proposed the uncertainty relation for the error  $\epsilon(Q)$  of an electron's position measurement and the disturbance  $\eta(P)$  of the momentum measurement in a form  $\epsilon(Q)\eta(P) \sim \frac{\hbar}{2}$ where  $\hbar$  is Planck's constant divided by  $2\pi$  [1] (here, we use  $\frac{\hbar}{2}$  for consistency with modern treatments). The reciprocal relation  $\sigma(Q)\sigma(P) \geq \frac{\hbar}{2}$  for standard deviations  $\sigma(Q)$ ,  $\sigma(P)$  of position and momentum was proved by Kennnard [47], which was generalized to arbitrary pairs of observables A, B by Robertson[48] as  $\sigma(A)\sigma(B) \geq \frac{1}{2} |\langle \psi|[A, B]|\psi\rangle|$ , in any states. Here, [A, B] represents the commutator [A, B] = AB - BA and the standard deviation is defined as  $\sigma(A)^2 = \langle \psi | A^2 | \psi \rangle - \langle \psi | A | \psi \rangle^2$ . Robertson's relation with standard deviations has a mathematical basis. Nevertheless, the proof of the reciprocal relation for the error, a generalized form of Heisenberg's error-disturbance uncertainty relation

$$\epsilon(A)\eta(B) \ge \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|, \tag{5}$$

is not straightforward. Recently, rigorous and general theoretical treatments of quantum measurements have revealed the failure of Heisenberg's relation Eq.(5), and derived a new universally valid uncertainty relation [12, 13] given by

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \ge \frac{1}{2}|\langle \psi|[A,B]|\psi\rangle|.$$
(6)

Here, the error  $\epsilon(A)$  is defined as the root-mean-square (rms) of the difference between the output operator  $O_A$  actually measured and the observable A to be measured, whereas the disturbance  $\eta(B)$  is defined as the rms of the change in the observable B during the measurement. Note that the additional second and third terms imply a new accuracy limitation, which does not necessarily follow the trade off relation of error and disturbance.

#### 3.2. Uncertainty relation in successive spin measurements

Here, the validities of two forms of error-disturbance relations, Eqs.(5) and (6) are experimentally tested with neutron's successive spin-measurements. The experimental setup is depicted in Fig.5. Observables A and B are set as  $\sigma_x$  and  $\sigma_{\phi_B}$  (an observable lying on the equator with the azimuthal angle  $\phi_B$  of the Bloch sphere). The initial state  $|\Psi\rangle$  is set to be +z spin state,  $|+z\rangle$ . In order to observe dependence of the error  $\epsilon(A)$  and the disturbance  $\eta(B)$  on the output observable,  $O_A = \sigma_x \cos \phi_{OA} + \sigma_y \sin \phi_{OA}$  (instead of exactly measuring  $A = \sigma_x$ ), the apparatus M1 is designed to actually carry out measurements of adjustable observables. For determination of the error  $\epsilon(A)$  and the disturbance  $\eta(B)$ , the method proposed in Ref. [13] is used.

The experiment was carried out at the research reactor facility TRIGA Mark II of the TU-Vienna. The monochromatic neutron beam is polarized crossing a super-mirror polarizer and two other super-mirrors are used as analyzers. The guide field together with four DC spin



Figure 5. Setup of the neutron optical test of error-disturbance uncertainty relation

rotator coils, induces Larmor precession to allow state preparation and projective measurements of  $O_A$  in M1 and B in M2. To test the error-disturbance uncertainty relation in Eqs.(5) and (6), the standard deviations  $\sigma(A)$ ,  $\sigma(B)$ , the error  $\epsilon(A)$  and the disturbance  $\eta(B)$  are determined from the experimentally obtained data:the measurements of the standard deviations  $\sigma(A)$  and  $\sigma(B)$  are carried out by M1 and M2 separately, whereas error  $\epsilon(A)$  and disturbance  $\eta(B)$  are determined by successive projective measurements utilizing M1 and M2.

Results of the measurements are shown in two cases: (i)  $B = \sigma_y$  and (ii)  $B = \sigma_x \cos(5\pi/6) + \sigma_y \sin(5\pi/6)$ . The azimuthal angle of  $\phi_{OA}$  of the output observable  $O_A$  is varied from 0 to  $2\pi$ . From the obtained values of error  $\epsilon(A)$ , disturbance  $\eta(B)$ , standard deviations  $\sigma(A)$  and  $\sigma(B)$ , the Heisenberg error-disturbance product  $\epsilon(A)\eta(B)$  and the universally valid expression  $\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B)$  are plotted as a function of the detuned azimuthal angle  $\phi_{OA}$  in Fig.6. The figures illustrate the fact that the universally valid expression is always larger than the limit whereas the Heisenberg product is often below the limit. In particular in the range  $\phi_{OA} = [0, \pi/2]$  of Fig.6(a), a trade-off relation between the error and the disturbance is observed and the Heisenberg product is observed where the universal expression actually touches the limit, which corresponds to the case where the equal sign of the inequality Eq. (6) really occurs.

After the publication of the first experimental confirmation of the new error-disturbance uncertainty relation by us [10], several papers have followed: for instance, experimental demonstration with photonic system [49, 50] as well as theoretical improvement of the inequality [51] appeared. Note that, among them, a theoretical letter was published, which claims that, by defining error and disturbance in different manners, one can consider that the Heisenberg'e relation is still valid [52]. New definition used in the letter is "state independent, each giving the worst-case estimates across all states": this allows overestimate of the measurement error and disturbance. This claim is immediately criticized by two papers of some authors of the experimental papers [53, 54]: physical analysis in the former figures out the new definition as "disturbance power" and mathematical consideration in the latter reveals breakdowns of the



**Figure 6.** Experimentally determined values of the universally valid uncertainty relation: (i)  $\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B)$  (orange) and (ii)  $\epsilon(A)\eta(B)$  (red) are plotted as a function of the detuning angle  $\phi$  for the cases (a) $B = \sigma_y$  (left) and (b)  $B = \sigma_x \cos(5\pi/6) + \sigma_y \sin(5\pi/6)$  (right).

new definition. It should be emphasized here that, in practical circumstances, one can easily assume (and actually one has pretty often) a measurement apparatus, which is *optimized* for measurements in certain region of the result and/or of certain state: in particular, the device actually measures *only* certain regions (or states) of the sample. In such cases, it is rather natural to consider that the error and disturbance of this kind of device has no influence of measurements not optimized for the device or what actually will not be done, i.e. they can be *state-dependent*. From these considerations, we believe that the worst-case estimates is unsuitable measure of error and disturbance in practice: Ozawa's treatment and definition of error and disturbance reflect the sprit of the original argument of uncertainty relation by Heisenberg.

#### 4. Quantum Cheshire cat: paradoxical phenomenon in quantum mechanics

Quantum mechanics is still capable to exhibit counterfactual phenomena. For instance, Hardy paradox describes a contradiction between classical picture and the outcome of quantum mechanics [55]: joint weak measurements of trajectories of a photon pair on a post-selected state in a pair of Mach-Zehnder interferometers reveal a negative value for a joint probability of locations [56, 57]. Here, a weak measurement is a technique proposed by Aharonov, Albert and Vaidman (AAV) [58], where a weak value defined as  $\hat{A}_w = \langle \Psi_{fin} | \hat{A} | \Psi_{ini} \rangle / \langle \Psi_{fin} | \Psi_{ini} \rangle$ can be obtained with certain pre- and post-selected systems, represented by  $|\Psi_{ini}\rangle$  and  $|\Psi_{fin}\rangle$ with minimal disturbance on the measured system. Note that weak values lie over the range of eigenvalues [59, 60] and may even be complex [61]. The weak value was also used as an amplifier to discover new physical effects that could not be otherwise detected[62]. Recently, another counterfactual paradox, called quantum Cheshire cat, attracted attention: in pre- and post-selected circumstances, a cat, i.e. a particle, is in one place and its grin, e.g. a spin is in another [14, 15].

#### 4.1. Quantum Cheshire cat in a neutron interferometer experiment

In our neutron interferometer experiment, neutron plays a role of the cat and its spin does the grin [63]. The concept of quantum Cheshire cat as well as the experimental setup is depicted in Fig.7. The pre- and post-selected states are set as

$$\begin{cases} |\Psi_i\rangle = \frac{1}{\sqrt{2}} \{ |+x\rangle |I\rangle + |-x\rangle |II\rangle \} \\ |\Psi_f\rangle = \frac{1}{\sqrt{2}} |-x\rangle \{ |I\rangle + |II\rangle \}. \end{cases}$$
(7)



Figure 7. Quantum Cheshire cat in concept (on the left) and the experimental setup with neutron interferometer (on the right). In the Mach-Zehnder interferometer, a cat is in the upper beam path while the grin is in the lower beam path. In the neutron interferometric version, an incident beam is polarized and falls on the interferometer. The pre-selected state  $|\Psi_i\rangle$  is generated in the interferometer and the post selection on the state  $|\Psi_f\rangle$  is carried out on the beam leaving the interferometer.

To characterize the neutron's population in the interferometer and the location of its spin, weak values of the observables,  $\hat{\Pi}_j \equiv |j\rangle\langle j|$  and  $\langle \hat{\sigma}_z^s \hat{\Pi}_j \rangle$  with j = I and II are determined. Theory predicts the values,  $\langle \hat{\Pi}_I \rangle_w = 0$ ,  $\langle \hat{\Pi}_{II} \rangle_w = 1$ ,  $\langle \hat{\sigma}_z^s \hat{\Pi}_I \rangle_w = 1$ , and  $\langle \hat{\sigma}_z^s \hat{\Pi}_{II} \rangle_w = 0$ .

In the experiment, the incident neutron beam is polarized by using magnetic prisms and the initial state  $|\Psi_i\rangle$  is generated: a pair of water-cooled spin-rotators are employed in the interferometer [64]. After the relative phase  $\chi$  between the two beams is adjusted by the phase shifter and the beams are recombined at the last plate of the interferometer, the O-beam (interfering beam leaving the interferometer in forward direction) is affected by spin analysis: the post-selection is carried out in combination of the phase shifter and the spin analysis system. Weak measurements of the neutron's population and the spin's location are performed by the use of absorbers and additional spin rotation in one of the beams in the interferometer.

Here, we explain the experimental results qualitatively. First, in the measurements of neutron's population, an absorber is inserted in one of the beam paths in the interferometer. Typical results are shown in Fig. 8: the absorbers in the beam path I (lower path) does not affect the final intensity of the O-beam with a spin-analysis, while intensity of the path II (upper path) decreases according to the strength of the absorber. This suggests that neutrons are traveling through the interferometer, following the beam path I. Next, in the measurement of the location of the neutron's spin, a fairly weak magnetic field is applied in one of the beam path II (upper path) does not affect the final intensity of the O-beam with a spin-analysis, while sinusoidal intensity modulation appears by applying the magnetic field in the path I (lower path). This suggests that neutron's spin, in turn, is traveling through the interferometer, following the beam path II. These results are consistent with the theoretical prediction [15]: neutrons, which are affected by appropriate pre- and post-selection, seem to travel in one of the paths, while their spin is disembodied and located in the other path.

#### 5. Concluding remarks

Experimental tests of quantum mechanics and demonstrations of counterfactual phenomena in quantum mechanics are performed with neutron interferometer and polarimeter: efficient manipulations of the neutron's quantum system enable measurements with high precision. Quantum contextuality is manifested in the studies of bi-partite and tri-partite entanglements between degrees of freedom. In particular, entanglements between different degrees of freedom



Figure 8. Weak measurements of the neutron's population of the path in the interferometer. Absorbers with transmissivity of 1, 0.8 and 0.6 are inserted in one of the beam path in the interferometer. In the upper panel: absorbers in the path I (lower path) affect the intensity. In the lower panel, the intensity of the path II (upper path) decreases by inserting stronger absorbers. Now, one finds neutrons in the path I.



Figure 9. Weak measurements of the neutron's population of the path in the interferometer. Absorbers with transmissivity of 1, 0.8 and 0.6 are inserted in one of the beam path in the interferometer. In the upper panel: absorbers in the path I (lower path) affect the intensity. In the lower panel, the intensity of the path II (upper path) decreases by inserting stronger absorbers. Now, one finds neutrons in the path I.

in single particles are realized and various forms of quantum contextuality have been tested: our investigations with matter waves falsified various hidden-variable theories. Studies of quantum entanglement with matter waves can be extended to generate and utilize a W-like state, a general mixture and even more. The results agreed well with predictions of standard quantum mechanics, although some interpretations are still under discussion. The test of the error-disturbance uncertainty relation, presented here, is actually the first experimental test of this kind: the demonstration is the first experimental evidence for the invalidity of the old (by Heisenberg) and validity of the new (by Ozawa) uncertainty relation. It should be emphasized here that this experiment opens up a new era of uncertainty relation where, after more than 80 years of the Heisenberg's original publication, the topic has revived and come again under hot discussions both from the theoretical and experimental point of view. Our result clarifies a long-standing problem of describing the relation between measurement accuracy and disturbance, and sheds light on fundamental limitations of quantum measurements. One cannot too much emphasize the importance of amending such a fundamental concept not only from a purely academic but also a practical point of view. The studies of counterfactual phenomena of quantum mechanics are presented, where quantum Cheshire cat is generated and observed by using a neutron interferometer setup: neutron and its spin are disembodied. Note that weak values, which are defined and obtained by pre- and post-selection together with special estimation strategy of the intermediate state, allow to create the quantum Cheshire cat. Further investigations of fundamental questions, both conceptual and applied ones, concerning the quantum Cheshire cat, will be possible in the near future.

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