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Gravitational decoherence, alternative quantum theories and semiclassical gravity

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Abstract. In this report we discuss three aspects: 1) Semiclassical gravity theory (SCG): 4 levels of theories describing the interaction of quantum matter with classical gravity. 2) Alternative Quantum Theories: Discerning those which are derivable from general relativity (GR) plus quantum field theory (QFT) from those which are not 3) Gravitational Decoherence: derivation of a master equation and examination of the assumptions which led to the claims of observational possibilities. We list three sets of corresponding problems worthy of pursuit: a) Newton-Schrödinger Equations in relation to SCG; b) Master equation of gravity-induced effects serving as discriminator of 2); and c) Role of gravity in macroscopic quantum phenomena.

1. Introduction

There is general agreement that general relativity (GR) is an excellent theory describing the large scale structures of spacetime and quantum field theory (QFT) a highly successful theory for matter down to the verifiable subnuclear levels. Yet, it is equally well-accepted that intrinsic contradictions between general relativity and quantum theories exist. There are many serious efforts to reconcile or unify them in the search of *a theory for the microscopic structures of spacetime*, which is what **quantum gravity** (QG) entails – and carries no other meaning, specifically not quantizing general relativity (see, e.g., [1])– but it is fair to say to date no one school can show definitive and complete success in this goal.

1.1. Semiclassical gravity

A modest yet no less productive attempt is to place these two theories together: Q $\oplus$ G, not Q $\otimes$ G, which we don't yet quite understand how to do – GR being a classical theory for the macroscopic realm while QFT a quantum theory for the microscopic world, and see what discrepancies this union may reveal, such as in their mathematical structures, and what new physical insights we may gain. This was the goal set in **quantum field theory in curved spacetime** (QFTCST) [2, 3, 4] which began in the late 60's with cosmological particle creation studies [5] and epitomized in Hawking's 1974 [6] discovery of black hole radiance. Focused efforts in seeking ways to regularize or renormalize the stress energy tensor of quantum fields made it possible to tackle the so-called 'backreaction problem' [7, 8] in finding how quantum matter fields affect the dynamics of spacetime.

Solving the backreaction problem is at the core of **semiclassical gravity** theory (SCG) [9] developed in the 80's based on the *semiclassical Einstein equation* (SCE). Discovery in the 90s of a lawful place for the fluctuations of quantum fields promoted this to the *Einstein-Langevin equation* [10] which enables one to solve for the induced metric fluctuations (Wheeler's poetic 'spacetime foam'). This ushered in a new theory known as **stochastic gravity** [11, 12]. Both theories have since been developed extensively and applied to strong field situations such as structure formation in the early universe and black hole fluctuation and backreaction issues.

As a summary remark, the validity of semiclassical gravity in the form first proposed by Moller and Rosenfeld [13] in the 60's is often raised by authors of Newton-Schrödinger equation, citing the arguments by Page and Geilker [14], Eppley and Hannah [15][1]. Leaving aside the question of whether gravity should be quantized, which had seen much broader and deeper discussions since then, the internal consistency of relativistic semiclassical gravity by itself had since been investigated further and there are better responses to the challenges posed in the early 80s (read e.g. papers by Kibble & Randjbar-Daemi and Duff in [16]). We refer to two substantive papers, one by Flanagan and Wald [17] on semiclassical gravity, the other by Hu, Roura and Verdaguer [18], which also considered the role of the induced metric fluctuations in the criteria.

1.2. Alternative quantum theories (AQT)
General relativists following this vein have probed the interplay between gravity and quantum largely from the angle of how quantum matter affects spacetime (Q $\rightarrow$ G). Asking the question in the other direction (G$\rightarrow$ Q), namely, how gravity could have an effect on quantum phenomena, has been going on for just as long (e.g., [19]) mainly by quantum foundation theorists. The foremost issue is why macroscopic objects are found sharply localized in space (their wave functions "collapsed" on definite locales) while those of microscopic objects extend over space. This contradiction is captured in the celebrated Cat of Schrödinger [2]. One can very coarsely place these theories in three groups: The Girahdi-Remini-Weber (GRW)- Pearle models [20] of continuous spontaneous localization (CSL), the Diósi-Penrose theories [21, 22] invoking gravitational decoherence, and the recent trace dynamics theory of Adler [23] which attempts to provide a sub-stratum theory from which quantum mechanics emerges. A nice description of these theories can be found in a recent review by Bassi et al [24].

1.3. Gravitational decoherence
One important process where the interplay of gravity with quantum manifest is gravitational decoherence – the mechanism where the quantum coherence of a particle is diminished due to its interaction with an environment, in this case provided by the gravitational field. (The differences between quantum, intrinsic and gravitational decoherence are explained in the Introduction of [28]. See also [29, 30]). Gravitational decoherence is invoked in an important class of AQTs, that by the name of Diósi-Penrose theories. We want to find out the special features of gravitational decoherence, such as the decoherence rate and the associated basis and how gravitational decoherence differs from decoherence by a non-gravitational environment. To perform quantitative analysis of this effect one needs a master equation which has not been

[1] Since many of these theories which combine classical gravity with quantum mechanics are now also referred to as semiclassical gravity by many practitioners, to distinguish from them (which we see below are important) we will call theories based on quantum field theory (second quantized, permitting particle creation) in curved spacetime, which is a well-established form of GR+QFT, Relativistic Semi-Classical Gravity (RSCG).

[2] Note that 'cat-states' have been found for atoms whereas entangled 'dead' and 'alive' state for real cats, which are a little bigger than atoms, have not. The difference between micro and macro objects is crucial insofar as their quantum behaviors are concerned. A missing basic ingredient is nonequilibrium statistical mechanics which helps us interpolate between micro/few body effects and macro/many-body phenomena. Here lies the importance of macroscopic quantum phenomena (MQP), a subject hitherto overlooked by theorists but I feel is essential in understanding why cat-states can never be found for real cats.

derived from first principles until only recently (see papers by Blencowe and by Ananstopoulos & Hu below). Earlier equations have been reasoned out rather than derived from known microscopic physics. The 'reasoning out' process admits inputs based on phenomenological arguments according to the proponents' particular wishes. Whether gravity can be an effective source of decoherence is a reasonable motivation to work out a master equation for such analysis. More of this in Sec. 4.

1.4. Experimental possibilities

While the early universe and black holes are the natural arena where strong field quantum processes play out, which necessitate QFTCST and RSCG, the weak field and nonrelativistic limits are certainly more within the reach of what laboratory experiments can measure. Rapidly improving precision levels of observational possibilities in molecule interferometry, optomechanics and mirco-trap experiments [31] are pushing this closer to reality. While tests of AQTs are understandably high on the agenda of the proponents of such theories, the more modest yet equally if not more important task is to "put SCG to test" (note SCG here is used in the weak field nonrelativistic context) as exemplified by Chen's group's recent derivation of a NS equation for many particles and estimating the predictions of their NS equation as different from that of the standard Schrödinger equation [32] (see below).

Designs of experimental setups and tests are already underway which involve theorists working in QFTCST. Witness the increased activities in precision measurement of Casimir effect, dynamical Casimir effect, for understanding the vacuum energy modified by boundaries, and the parametric amplification thereof, which is the mechanism underlying cosmological particle creation. Jets and bursts of atoms from the controlled collapse of a BEC ("Bosenova") can be used to understand particle creation in inflationary universe [33], an example of laboratory cosmology. Accelerating atoms for testing the Unruh effect and finding analogs of Hawking effect in BECs and moving mirrors are being pursued. These activities belong to the realm of a new field called analog gravity [34]. Theoretical results obtained in the 70s are now being improved on and applied to designs for possible experimental verifications. In our opinion, tests related to RSCG will come next. Now is a good time for researchers in the 80s (on RSCG) and 90s (on semiclassical stochastic gravity) to join force with experimental activities so their expertise can enrich our understanding of the interplay between quantum matter and classical gravity.

1.5. Theoretical preparatory work for observing gravitational decoherence and testing AQTs

AQTs have been in existence for 3 decades, but the tests of such theories are put in practice only recently thanks to the increasing precision required of the measurements of these gravity-induced effects (see, e.g., [31]). Indications of the timeliness for these investigations are seen also in concentrated recent activities, e.g., 6 papers of substance have appeared in the last 6 months. We list five of them here with a short description because we will refer to all of them in this report. Noteworthy also is Adler's recent work incorporating gravity into his trace dynamics theory [35]. Increased effort in theoretical investigations of NS equations and gravitational decoherence are needed to better prepare the ground for measurement possibilities.

1) Dyson's 2012 essay, [36] "Is a graviton detectable?" will be useful for our discussions of gravitational decoherence, e.g., whether a thermal bath of gravitons is for real.

2) Review by Bassi et al [24] has a lucid summary of alternative quantum theories and a clear description of the experimental results in progress (See Table 1, also [37].)

3) Chen's group [32], while assessing experimental observability, attempts to mark out the differences between NS and ordinary QM predictions and give a theoretical derivation of a NS equation for N particles. (**Version C**.) We will show in Sec. 3 the differences between their

equation and that obtained from taking the nonrelativistic limit of SCE equation. We find that the single or many particle NS equations are not from GR + QFT.

4) Anastopoulos and Hu [28] derived a master equation for gravitational decoherence based solely on GR+QFT. We call this theory **Version A**. The procedure and results of this paper are summarized in Section 4.

5) Blencowe [38] derived a master equation using the influence functional method and made claims to the effect that gravitational decoherence is strong enough to soon be within observational range. (We call his master equation **Version B**.) We have reservations in his claim which invokes two assumptions: a thermal bath of gravitons and the increase of decoherence strength with mass. We will comment on these two assumptions in Section 7.

In all, our present work attempts to address the core issues in the interplay between gravity and quantum, places the Newton-Schrödinger equation in the context of relativistic semiclassical gravity and explores the theoretical base for observational possibilities of gravitational decoherence. Our hope is that the theoretical structure developed based on known physics from general relativity and quantum field theory and the results obtained here can provide a definitive standard for all alternative quantum theories to compare in their reasonings and predictions. The new results reported here on the master equation for gravitational decoherence and the NS equation in relation to SCG are based on two recent papers [28, 39]

2. Quantum matter interacting with classical gravity: 4 levels of inquiry

Newton-Schrödinger (NS) equations describe the motion of nonrelativistic quantum particle(s) in a weak gravitational field potential. There are many forms, justified with different rationales, originated from different motivations. Some are in conflict with general relativity as they are not the nonrelativistic limit of relativistic semiclassical gravity (RSCG). From their formal appearance one may view the NS equation as the nonrelativistic limit of the Einstein-Klein-Gordon equation, but NS in structure is closer to the Hartree-Fock equations. Yet, despite the similarity in structure between the NS eqn and Hartree-Fock equations, there are differences between gravito- and electro-statics) in the self energy for a many body system. These issues need be clarified so that one knows what his/her results of calculation pertain to in relation to other theories and in comparison with experiments.

2.1. Level 0: Newton-Schrödinger / Schrödinger-Poisson equations: non-relativistic quantum particle in a weak gravitational field

This is the arena where most of the activities in finding / showing the overt / hidden effects of gravity on quantum mechanics takes place and proposals of alternative quantum theories reside. Because it is for non-relativistic particles in a weak gravitational field, this is also the domain where laboratory experiments are carried out. Most researchers working in the 70s on quantum field theory in curved spacetime (QFTCST) and in the 80s working on (relativistic) semiclassical gravity (RSCG) (see below) have largely ignored this level of activities going on at the same time, albeit sparingly, as much as researchers working on quantum foundational issues largely were unaware of developments in RSCG in the last four decades. The former group's attention was focused on quantum effects in strong gravitational fields as in the early universe and in black holes while the latter group was focusing on proposing alternative quantum theories (AQTs) and their experimental verification possibilities. The latter group's definition of SCG is largely within the realm of Level 0.

So what are the problems of interest at this Ground Level? One can start with an almost textbook-like example of calculating the dispersion of a Gaussian wave packet with initial spread a_0 of a massive (m) particle according to the Schrödinger equation $i\hbar\partial\psi/\partial t = -\frac{\hbar^2}{2m}\nabla^2\psi - mV\psi$ when it moves in a gravitational potential V sourced by its own wavefunction $\nabla^2 V = 4\pi Gm|\psi|^2$.

This set of Newton-Schrödinger equation is imbued with a tension illustrated in this simple example between the natural quantum dispersion of a wavepacket against the gravitational collapse due to its own mass. The rather low critical mass obtained by Salzman and Carlip [40, 41] in 2006 was contested by Giulini and Grossardt in [42] in 2010. By introducing a length scale ℓ the SN equation can be written in terms of a dimensionless coupling constant, $K = 2(\ell/\ell_P)(m/m_P)^3$ where ℓ_P, m_P are the Planck length and mass respectively. Giulini and Grossardt in [42] showed that inhibition of the dispersion becomes significant when the dimensionless coupling constant K becomes of order unity. Their conclusion (quoting from [24])".. leads to an important inference: The models of Karolyhazy, Diósi, and Penrose all agree that if the width of the quantum state associated with an object of mass m becomes greater than of the order $\hbar^2/(Gm^3)$, the quantum-to-classical transition sets in. For the experimentally interesting $a = 0.5\mu m$ this gives m of about 10^9 amu." This estimate puts the actual measurement of this effect beyond today's experimental capability but the intellectual challenge and excitement towards realization of this goal are certainly growing. More examples to expound the interplay between effects of quantum dispersion and gravitational pull in wave packets and more complex systems can be explored at the next level (Level 1) with the help of quantum field theory in curved spacetime techniques.

2.2. Level 1: Relativistic matter fields in strong gravitational field. Einstein-Klein-Gordon equation

We now enter the relativistic realm, both for the quantum field and for gravity: Schrödinger equation is upgraded to Klein-Gordon equation for scalar particles or Dirac equation for spinors. The effect of curvature enters in the wave equation for a scalar field through the Laplace-Beltrami operator $\nabla^2\Phi - m^2\Phi = 0$. This theory has been applied to treat self-gravitating particles [43] or boson stars [44]. Looking for a solution of the metric tensor sourced by the relativistic field requires that the Einstein equation and the KG equation be solved simultaneously in a self-consistent manner, namely, $G_{\mu\nu} = 8\pi G T_{\mu\nu}(\Phi)$ where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}(\Phi)$ is the stress energy tensor of the scalar field. Note at this level the field can be viewed either as first quantized or second quantized. As a first quantized field Guzman et al have shown that the E-KG equation reduces in the non-relativistic, weak field limit to the NS equation. Spherically symmetric solutions of the NS equation have been found by e.g., Moroz et al [45].

At the second quantized level truly quantum field theoretical effects like particle creation from the vacuum begin to show up. At the 'test field' level in which a quantum field propagates in a fixed given curved spacetime, this is the realm of quantum field theory in curved spacetime (QFTCST) [2, 4]. How this field affects the background spacetime is described by the semiclassical Einstein equation. One now enters the realm of relativistic semiclassical gravity.

2.3. Level 2: Relativistic semiclassical gravity (RSCG): semiclassical Einstein equation (SCE), including backreaction of quantum matter field

Semiclassical gravity (SCG) has been used for a wide range of theories where gravity is treated classically and the matter field quantum mechanically, including the nonrelativistic NS equation. But the treatment of matter varies from one particle to many-particle systems to quantum fields, and even for quantum, there is a difference between first quantized and second quantized. To avoid confusion we add the word relativistic to SCG to refer to fully relativistic Einstein's theory for gravity valid under strong field conditions, and fully relativistic quantum fields at the second quantized level for matter. One known example of such a theory is that based on the semiclassical Einstein equation: $G_{\mu\nu} = 8\pi G < T_{\mu\nu}(\Phi) >$ where $T_{\mu\nu}(\Phi)$ is the stress energy tensor of the matter field, here represented by a scalar field Φ, and $<>$ denotes taking the expectation value with respect to a certain quantum state.

Note in general because of the sum over all modes there is ultraviolet divergence in this expression. Much of the effort in the field of QFTCST in the mid-70s focused on finding ways to regularize or renormalize these divergences. By 1978 when the results obtained by different regularization approaches more or less converged serious studies of RSCG began, under the theme of "backreaction problems" which went on to the 80's – e.g., the backreaction of vacuum energy of quantum fields (such as the Casimir effect) and particle creation (from the vacuum) on the dynamics of the spacetime. This requires a self consistent solution of both the semiclassical Einstein equation governing the spacetime dynamics and the quantum matter field equation.

From physical considerations, the backreaction of quantum fields brings forth dissipation in the dynamics of spacetime through the SCE Eq. How to reckon the appearance of non-unitary terms in an otherwise unitary evolution dictated by Einstein's equation was the first conceptual challenge. Understanding this issue from the open quantum system viewpoint was helpful in discovering the next level of structure, in the Einstein-Langevin equation.

2.4. Level 3: Stochastic SC gravity (SSCG): Einstein-Langevin equation, including fluctuations in quantum field and metric

Including fluctuations of quantum field as a source driving the semiclassical Einstein equation faces another challenge. Why and how should a noise term appear in the SCE equation? These two issues were resolved by borrowing concepts in nonequilibrium statistical mechanics, namely, the existence of fluctuation-dissipation relations and the use of the (Feynman-Vernon) influence functional formalism to provide an analytic basis for the description of quantum noise. This was how semiclassical stochastic gravity theory came into being [12]. Further proof by Verdaguer et al [46] that the noise can be written in a covariant form and satisfies the divergence-free condition ensures its rightful place in the Einstein-Langevin equation [10].

3. Newton-Schrödinger equation and semiclassical gravity

The Newton-Schrödinger (NS) equations play a prominent role in alternative quantum theories (AQT)[24], emergent quantum mechanics [23], macroscopic quantum mechanics [27], gravitational decoherence [28, 38](as in the Diósi-Penrose models [21, 22]) and semiclassical gravity [12]. The class of theories built upon these equations have drawn increasing attention because experimentalists often use it as the conceptual framework and technical platform for understanding the interaction of quantum matter with classical gravity and to compare their prospective laboratory results. It is thus timely and necessary to explore the assumptions entering into the construction of these equations and the soundness of the theories built upon them, especially in their relations to general relativity (GR) and quantum field theory (QFT), the two well-tested theories governing the dynamics of classical spacetimes and quantum matter.

Since NS are often simplistically conjured as the weak field (WF) limit of GR and the nonrelativistic (NR) limit of QFT, their viability is usually conveniently assumed by proxy, courtesy their well-accepted progenitor theories. We are not convinced of this. In a recent paper [39] Anastopoulos and I show that NSEs do not follow from general relativity (GR) and quantum field theory (QFT), and there are no 'many-particle' NSEs, such as derived recently in [32]. We come to this conclusion from two considerations: 1) Working out a model (see [28]) with matter described by a scalar field interacting with weak gravity, with a procedure the same as in deriving the NR limit of quantum electrodynamics (QED). 2) Taking the NR limit of the semiclassical Einstein equation (SCE), the central equation of relativistic semiclassical gravity (RSCG) (see last section for the four levels of SCG), a fully covariant theory based on GR+QFT with self-consistent backreaction of quantum matter on the spacetime dynamics [12]. The key points are summarized in [47].

Before we explain the differences between theories based on NSE and those obtained from GR+QFT it may be useful to first highlight their differences in physical predictions:

3.1. Problems with the Newton-Schrödinger equations (NSE)

We mention three aspects here.

A. In NSE the *gravitational self-energy* defines non-linear terms in Schrödinger's equation. In Diósi's theory [21], the gravitational self-energy defines a stochastic term in the master equation. With GR+QFT gravitational self-energy only contributes to mass renormalization, at least in the weak field (WF) limit.The Newtonian interaction term at the field level induces a divergent self-energy contribution to the single-particle Hamiltonian. It does not induce nonlinear terms to the Schrödinger equation for any number of particles.

B. The *one-particle NS equation* appears as the Hartree approximation for N particle states as $N \to \infty$. Consider the ansatz $|\Psi\rangle = |\chi\rangle \otimes |\chi\rangle \ldots \otimes |\chi\rangle$ for a N-particle system. At the limit $N \to \infty$ the generation of particle correlations in time is suppressed and one gets an equation which reduces to the NS equation for χ [51, 35][3]. . However, in the Hartree approximation, $\chi(\mathbf{r})$ is *not* the wave-function $\psi(\mathbf{r})$ of a single particle, but a *collective variable* that describes a system of N particles under a mean field approximation.

C. A severe problem of the NSE when applied to a single-particle wave function is its *probabilistic interpretation.* Consider two statistical ensembles of particles one of which is described by the wave-function $\psi_1(\mathbf{r})$ and the other by the wave function $\psi_2(\mathbf{r})$. The ensemble obtained from mixing these ensembles with equal weight is described in standard quantum theory by the density matrix $\rho(\mathbf{r}, \mathbf{r}') = \frac{1}{2}[\psi_1(\mathbf{r})\psi_1^*(\mathbf{r}') + \psi_2(\mathbf{r})\psi_2^*(\mathbf{r}')]$. The usual Schrödinger evolution guarantees that the probabilistic interpretation of the density matrix remains consistent under time evolution $\rho_t(\mathbf{r}, \mathbf{r}') = \frac{1}{2}[\psi_1(\mathbf{r}, t)\psi_1^*(\mathbf{r}', t) + \frac{1}{2}[\psi_2(\mathbf{r}, t)\psi_2^*(\mathbf{r}', t)]$. This property does not apply for non-linear evolutions of the wave-functions. The problem of nonlinearity in quantum mechanics is an old issue which many AQTs are aware of, so we will just mention it here without further pursuit.

In what follows we will show that the only meaningful description of quantum matter interacting with classical gravity is if the matter degrees of freedom are described in terms of quantum fields, *not* in terms of single-particle wave functions in quantum mechanics.

3.2. NS equation not from GR + QFT

The NS equation governing the wave function of a single particle $\psi(\mathbf{r}, t)$ is of the form

$$i\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + m^2 V_N[\psi]\psi, \tag{1}$$

where $V_N(\mathbf{r})$ is the (normalized) gravitational (Newtonian) potential given by

$$V_N(\mathbf{r}, t) = -\int d\mathbf{r}' \frac{|\psi(\mathbf{r}', t)|^2}{|\mathbf{r} - \mathbf{r}'|}. \tag{2}$$

It satisfies the Poisson equation

$$\nabla^2 V_N = 4\pi G\mu, \tag{3}$$

where $\mu = m|\psi(\mathbf{r}, t)|^2$ is the mass density, the nonrelativistic (slow motion) limit of energy density $\varepsilon = T_{00}$ (see below).

The Newton-Schrödinger equation predicts spatial localization of the wave-function, and decoherence only as a consequence of spatial localization. This "collapse of the wave function"

[3] Note it is long known [49] that RSCG is the theory obtained as the large N limit of N component quantum fields living in a curved spacetime. Roura and Verdaguer [50] further showed that the next to leading order large N expansion produces stochastic semiclassical gravity.

in space for macroscopic objects is a big 'selling-point' of NS equations in many AQTs. Its desirable attributes aside, the logical foundation of the Newton-Schrödinger equation seems shaky to us. The naive identification of Newton as weak field limit of GR and Schrödinger equation as the nonrelativistic limit of QFT is likely behind the justification of NS equations. Here, one should exercise caution, as illustrated below: E.g., on the GR side, not to identify gravitational potential as dynamical variables, and on the QFT side, not to mistake a field as a collection of particles described by single particle wave functions.

3.3. Non-relativistic weak field limit of SCE equation

The central equation of relativistic semiclassical gravity (RSCG) is the semiclassical Einstein equation (SCE), and when quantum field fluctuations are included, the Einstein-Langevin equation, the centerpiece of stochastic semiclassical gravity [12]. We examine the nonrelativistic limit of SCE and show that it is qualitatively different from the 'many-particle' NS equation derived in [32].

The SCE Equation is in the form [4] $G_{\mu\nu} = 8\pi G\langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle$, where $\langle\hat{T}_{\mu\nu}\rangle$ is the expectation value of the stress energy density operator $\hat{T}_{\mu\nu}$ with respect to a given (Heisenberg-picture) quantum state $|\Psi\rangle$ of the field.

In the weak field limit the spacetime metric has the form $ds^2 = (1-2V)dt^2 - d\mathbf{r}^2$, and the non-relativistic limit of the semi-classical Einstein equation takes the form

$$\nabla^2 V = 4\pi G\langle\hat{\varepsilon}\rangle, \tag{4}$$

where $\hat{\varepsilon} = \hat{T}_{00}$ is the energy density operator. This can be solved to yield

$$V(\mathbf{r}) = -G\int d\mathbf{r}'\frac{\langle\hat{\varepsilon}(\mathbf{r}')\rangle}{|\mathbf{r}-\mathbf{r}'|}. \tag{5}$$

The expectation value of the stress energy tensor in general has ultraviolet divergences and need be regularized. The procedures have been established since the mid-70's (see, e.g., [2]).

Two key differences between the NR limit of SCE and NSE are: i) the energy density $\hat{\varepsilon}(\mathbf{r})$ is an operator, not a c-number. The Newtonian potential is not a dynamical object in GR, but subject to constraint conditions. ii) the state $|\Psi\rangle$ of a field is a N-particle wave function. Quantum matter is coupled to classical gravity as a mean-field theory, well defined only when N is sufficiently large.

The (misplaced) procedure leading one from SCE to a NS equation is the treatment of $m|\psi(\mathbf{r},t)|^2$ as a mass density for a single particle, while in fact it is a quantum observable that corresponds to an operator $\hat{\epsilon}(\mathbf{r}) = \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})$ in the QFT Hilbert space when the matter degrees of freedom are treated as quantum fields $\hat{\psi}(\mathbf{r})$ and $\hat{\psi}^\dagger(\mathbf{r})$, as they need be. Not treating these quantities as operators bears the consequences A and B.

3.4. Analog to the nonrelativistic limit of QED

To cross check these observations we have carried out an independent calculation for matter described by a scalar field interacting with weak gravity, following the same procedures laid out in [28], namely, solve the constraint, canonically quantize the system, then take the nonrelativistic limit. This procedure is same as in obtaining the non-relativistic limit of QED. We obtain the Schrödinger equation

$$i\partial|\psi\rangle/\partial t = \hat{H}|\psi\rangle, \tag{6}$$

[4] We prefer calling this the semiclassical Einstein equation over the Moller-Rosenfeld equation because, after all, it is Einstein's equation with a quantum matter source.

with

$$\hat{H} = -\frac{\hbar^2}{2m}\int d\mathbf{r}\hat{\psi}^\dagger(\mathbf{r})\nabla^2\hat{\psi}(\mathbf{r}) - G\int\int d\mathbf{r}d\mathbf{r}'\frac{(\hat{\psi}^\dagger\hat{\psi})(\mathbf{r})(\hat{\psi}^\dagger\hat{\psi})(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \tag{7}$$

The electromagnetic analog of this equation with the Coulomb potential replacing the gravitational potential here is widely used in condensed matter physics (see [39] for details).

The matrix elements of the operator (7) on the single-particle states $|\chi\rangle$ define the single-particle Hamiltonian:

$$\langle\chi_2|\hat{H}|\chi_1\rangle = -\frac{\hbar^2}{2m}\int d\mathbf{r}\chi_2^*(\mathbf{r})\nabla^2\chi_1(\mathbf{r}) - G\int d\mathbf{r}d\mathbf{r}'\frac{\chi_2^*(\mathbf{r}')\chi_1(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \tag{8}$$

It is clear that Eq. (7) is very different from the NS equation (1) when considering a single particle state. For single-particle states the gravitational interaction leads only to a mass-renormalization term (similar to mass renormalization in QED). This is point A we made above. Using the Hartree approximation to Eq. (4) leads to the same result as the NR WF limit of SCE, not NSE. This is Point B we made earlier. Details of this calculation are in [39].

Our analysis via two routes based on GR+QFT shows that NSEs are not derivable from them. Coupling of classical gravity with quantum matter can only be via mean fields. There are no N-particle NSEs. Theories based on Newton-Schrödinger equations assume unknown physics.

4. Gravitational decoherence

4.1. Master equations from GR + QFT: Our analysis and main results

The procedures we took in [28] are as follows:

First step: Start with the classical action of a massive scalar field interacting with gravity described by the Einstein-Hilbert action. Linearize the Einstein-Hilbert action around the Minkowski spacetime. Look at the weak-field regime. We do this for two reasons: a) we want results which can be tested in laboratory experiments at today's low energy (in contrast to strong field conditions, as found in the early universe or late time black holes). b) In the derivation of the master equation for consideration of gravitational decoherence the tracing-out of the gravitational field is not technically feasible, except for linearized gravitational perturbations.

The second step is to perform a 3+1 decomposition of the action and construct the associated Hamiltonian. Identify the constraints of the system and solve them at the classical level, expressing the Hamiltonian in term of the true physical degrees of freedom of the theory, namely, the transverse-traceless perturbations for gravity and the scalar field. The third step is to canonically quantize the scalar field and the gravitational perturbations together, to ensure the consistency between these two sectors from the beginning.

The fourth step (after Eq. (21) of [28]) is to trace over the gravitational field acting as its environment to obtain a master equation for the reduced density matrix of the quantum matter field, including the backreaction of the gravitational degrees of freedom. The system under consideration is formally similar to a quantum Brownian motion (QBM) model [54, 55].

The master equation for the reduced density matrix $\hat{\rho}_1$ of one non-relativistic quantum particle in 3D interacting with weak perturbative gravity, valid to first order in $\kappa = 8\pi G$, is given by

$$\frac{\partial\hat{\rho}_1}{\partial t} = -\frac{i}{2m_R}[\hat{\mathbf{p}}^2, \hat{\rho}_1] - \frac{\kappa\Theta}{18m_R^2}(\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl})[\hat{p}_i\hat{p}_j, [\hat{p}_k\hat{p}_l, \hat{\rho}_1]] \tag{9}$$

where p_i are the momentum components of the particle, m_R is renormalized mass and Θ has meaning explained below. This master equation enables gravitational decoherence studies and other related tasks. The fifth and final step is to project to a one particle state, then take the non-relativistic limit. We then use this nonrelativistic master equation for the analysis of gravitational decoherence in a single quantum particle.

Main Results

(i) A special feature of decoherence by the gravitational field (in the non-relativistic limit) is the decoherence in the energy (momentum squared) basis, but not (directly) to decoherence in the position basis. This is a direct challenge to theories (such as that proposed by Diósi and concurred by Penrose), which assume a potential energy term so that decoherence occurs in the position basis. Our analysis shows that this class of theories violates the principles of general relativity.

(ii) Many approaches to gravitational or fundamental decoherence proceed by modeling temporal or spatial fluctuations in terms of stochastic processes. However, such fluctuations correspond to time or space reparameterizations, which are pure gauge variables, with no dynamical content, according to classical GR. The assignment of dynamical content to such reparameterizations implicitly presupposes an underlying theory that violates the fundamental symmetry of classical GR.

(iii) The decoherence rate depends not only on the matter-gravity coupling, but also on the intrinsic properties of the environment, such as its spectral density which reflects to some extent the characteristics of its sub-constituents composition. Measurement of the gravitational decoherence rate, if this effect due to gravity can be cleanly separated from other sources, may provide valuable information about the statistical properties of the sub-constituents, or what we called the "textures", of spacetime.

5. Constraining alternative quantum theories (AQT)

Diósi's theory – Version D [21]

Diósi proposed a master equation of the form

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}, \hat{\rho}] - \frac{1}{4}\kappa G \int d\mathbf{r}_1 d\mathbf{r}_2 [\hat{\mu}(\mathbf{r}_1), [\hat{\mu}(\mathbf{r}_2), \hat{\rho}]] \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|}, \tag{10}$$

where $\hat{\mu}(\mathbf{r})$ is the mass density operator for the system and κ a constant of order unity. Diósi's master equation predicts decoherence of superpositions of macroscopically distinct states X and Y with a decoherence time $\tau_{dec} = 2\hbar/[2U_D(X,Y) - U_D(X,X) - U_D(Y,Y)]$, where

$$U_D(X,Y) = -G \int d\mathbf{r}_1 d\mathbf{r}_2 \frac{f(\mathbf{r}_1; X) f(\mathbf{r}_2; Y)}{|\mathbf{r}_1 - \mathbf{r}_2|}, \tag{11}$$

with X and Y parameterizing the distributions μ. Typically one thinks of X and Y as centers of mass, whence the theory predicts decoherence in position.

One consequence of our investigation is the observation that Diósi's master equation cannot be derived from the framework of GR+QFT. It comes from the following considerations: General Relativity implies that the Newtonian interaction follows from the theory's Hamiltonian constraint. The solution of the constraint leads to a modification of the Hamiltonian through the addition of a Newtonian interaction term (in the non-relativistic limit): $H = H_0 - G \int d\mathbf{r}_1 d\mathbf{r}_2 \frac{f(\mathbf{r}_1) f(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|}$. Hence, the consistent quantization of the theory should place the Newtonian interaction term as a part of the quantum *Hamiltonian*, not as part of the non-unitary dynamics. There is no reason to structure the postulated non-unitary terms as a Newtonian interaction term, as is the case in Diósi's master equation. This is forced upon as an assumption which contradicts general relativity.

More generally, we feel that general relativity suggests a very different class of fundamental decoherence models with different reduction basis and type of noise. Working this out explicitly can make the comparison with the alternative models more quantifiable.

Other alternative theories of quantum mechanics 'aided' (or 'interceded') by gravitational effects – at low energy (in contrast to the Planck scale) include the so called 'continuous spontaneous localization' (CSL) models of Girahdi-Remini-Weber (GRW)- Pearle [20] (see also work of Bassi et al [24]). The state reduction in these schemes is often facilitated by considering stochastic processes on the quantum system's Hilbert space and stochastic Schrödinger equations are often suggested as an alternative to quantum mechanics. We will not address them here because the source of noise is phenomenologically motivated. We focus in the above on the Diósi-Penrose theories because it highlights the conflicts between gravity with quantum in a more transparent way - even if finally proven wrong, either way.

6. Role of gravity in macroscopic quantum phenomena

Historically a primary motivation for introducing the continuous collapse models (CLS) is trying to make sense of the readily collapsed wave function of macroscopic objects while preserving the wave function in the microscopic realm. There are two main features of this class of models. They are (from [24] p. 482): nonlinearity (which we also see in the SCE eqn), 2) stochasticity (which we see in stochastic gravity where the noise originates from quantum matter fields). There are also two requirements: 3) no superluminal signaling – this is forbidden from the start in RSCG since the basic principles of quantum field theory and relativity are observed. 4) an amplification mechanism – an important issue which we feel has not been explored enough. This is one aim of Chen's program on macroscopic quantum mechanics (MQM) [27] and our work on macroscopic quantum phenomena [56, 57, 58, 59]. We now turn to this issue.

The main motivation on the theoretical side of the recent work by Chen's group is to derive a NS equation for many particles. They looked into the interaction between particles, the separation of scales in the dynamics of the center of mass variable from other variables. A similar concern was raised in the paper by Chou Hu and Yu [60] where they set out to find the conditions where the "Center of Mass Axiom" is observed and a master equation for the N particles can be derived. Their key finding is, for interaction potentials dependent only on the separation between any two oscillators, the master equation for N oscillators has the same form as the HPZ master equation [55] for a single oscillator. Studying N oscillator systems will enable one to see how their interaction affects the outcome.

Technically the master equation for gravitational decoherence we derived recently [28] applies to configurations with any number of particles. This is because we first derived a master equation for quantum matter fields before projecting it to the single-particle subspace. One can easily project it to any particle number state to obtain a master equation for N particles.

7. Observing gravitational decoherence – contributing factors

A major factor in the surge of attention paid to gravitational decoherence is because several well respected experimental groups showed interest in the measurement of such effects [31]. This is corroborated by some theorists' claims that their predicted values are close to current observable experimental precision levels. It is thus important to examine carefully the assumptions made in theories which assert a significant effect in gravity's power to decohere a quantum particle. For example in the recent paper of Blencowe [38] (which we referred to as Version B) two assumptions were made, as follows:

7.1. Is thermal graviton bath a tenable assumption?

Gravitational decoherence depends strongly on assumptions about the nature of gravitational perturbations. The usual assumption that Minkowski spacetime is the ground state of quantum gravity would imply that gravitational perturbations are very weak and cannot lead to decoherence. This is the result we obtained in [28] where the source of gravitational decoherence is due to weak perturbations off the Minkowski spacetime. However, if general relativity is a hydrodynamic theory and gravity is in the nature of thermodynamics, Minkowski spacetime could presumably be identified with a macrostate (i.e., a coarse-grained state of the micro-structures). In this case, the perturbations are expected to be much stronger and they may act efficiently as agents of decoherence. (This information is contained in the Θ parameter in [28], the former case has $\Theta = 0$, the latter case some large value.)

The underlying issue is whether gravitons are thermalized, and if so what is the graviton bath temperature? The source of gravitons can either be from weak gravitational perturbations in the experiment's environment or as remnants from the early universe. This is not a new issue. For gravitons as quantized perturbations off Minkowski space, which provide the lowest common denominator for a gravitational source in the consideration of decoherence, one can take the graviton scattering processes (see e.g., Papini's review [61]) and calculate their cross sections. Because of the extremely weak nature of their interactions, it will be very small. For gravitons of cosmological origin, Blencowe took the value of 1 degree K citing Kolb and Turner's book [5]. Dyson's lecture [36] also addresses these points and is a good source of reference and comparison.

A somewhat equivalent way to look at this issue is the difference between (in the classical view) a superposition of gravitational waves and (in the quantum view) a mixed state of such superpositions. This essential point need be explicated mathematically.

In a more probing and elaborate investigation of this issue some earlier calculations may be helpful, e.g., by Calzetta and Hu [62] and others on the conditions of thermalization in a $\lambda\phi^4$ theory. (This had been cross-examined by particle physicists before they did the same calculation for non-Abelian theories in heavy ion collision and quark gluon plasma processes.) We can replace the λ by the graviton interaction constant obtained above to get an estimate of whether it makes sense to assign a temperature to gravitons. It may turn out that a 'graviton bath' is quite remote from reality in today's environment.

7.2. Does simple scaling up of quantum attributes apply to macroscopic objects?

In addition to the assumption of a thermal bath for gravitons the other main reason why Blencowe obtained a large number (compared to ours) for the gravitationally induced decoherence rate is because he uses a simple scaling from a quantum particle to a massive object. For an initial superposition of ground and excited states of a single atom the decoherence rate from his formula is $\sim 10^{-45}/sec$. This small rate (meaning it takes a very long time) is commensurate with our claim that for a weak gravitational perturbations background (at zero temperature) there is essentially no gravitational decoherence effect. However, Blencowe continues, "For a matter system comprising an Avogadro's number of atoms $\sim$ 1 gram in a quantum superposition where all of the atoms are either in their ground state or all in their excited state," he got a decoherence rate of $\sim 10^2$/sec. "For a system with mass $\sim$ 1 kg in such a superposition state, the gravitationally induced decoherence rate projects is $\sim 10^8$/sec. We believe Blencowe made an implicit assumption in how a macroscopic system's quantum behavior is directly related to the quantum features of its microscopic constituents. This is a largely unexplored topic under the general subject of macroscopic quantum phenomena (MQP).

[5] We are not sure how that value came about. The likely path of argument is to draw an analogy with neutrinos (e.g., from Weinberg's 1971 book). But neutrinos also interact very weakly. Thus this 1K value needs closer scrutiny.

This issue needs to be addressed before one can assuringly take the results for micro quantum objects and scale it up to macro domains. Interaction strength and quantum coherence amongst the sub-constituents are expected to play a role.

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References

[1] B. L. Hu, Emergent /Quantum Gravity: Macro/Micro Structures of Spacetime J. Phys. Conf. Ser. 174 (2009) 012015 [arXiv:0903.0878]
[2] N. D. Birrell and P. C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, U.K., 1982).
[3] R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics* (University of Chicago Press, 1994)
[4] L. Parker and D. J. Toms, *Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity* (Cambridge University Press, 2009)
[5] L. Parker, Phys. Rev. 183, 1057 (1969).
[6] S. W. Hawking, Nature **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).
[7] Ya. Zel'dovich and A. Starobinsky, Zh. Eksp. Teor. Fiz **61**, 2161 (1971) [Sov. Phys.- JETP **34**, 1159 (1971)] L. Grishchuk, Ann. N. Y. Acad. Sci. 302, 439 (1976). B. L. Hu and L. Parker, Phys. Lett. 63A, 217 (1977). B. L. Hu and L. Parker, Phys. Rev. **D17**, 933 (1978). F. V. Fischetti, J. B. Hartle and B. L. Hu, Phys. Rev. **D20**, 1757 (1979). J. B. Hartle and B. L. Hu, Phys. Rev. **D20**, 1772 (1979). **21**, 2756 (1980). J. B. Hartle, Phys. Rev. D23, 2121 (1981). P. A. Anderson, Phys. Rev. D28, 271 (1983); D29, 615 (1984).
[8] J. W. York, Jr. "Black hole in thermal equilibrium with a scalar field: The back-reaction" *Phys. Rev.* D **31**, 775 (1985). P. R. Anderson, W. A. Hiscock and D. A. Samuel, Phys. Rev. D **51**, 4337 (1995). W. A. Hiscock, S. L. Larson and P. A. Anderson, "Semiclassical effects in black hole interiors" gr-qc/9701004 Phys. Rev. D **56**, 3571 (1997). S. Bose, L. Parker and Y. Peleg, Phys. Rev. Lett. 76, 861 (1996) Phys. Rev. D53, 7089 (1996) and erarlier references therein. F. C. Lombardo, F. D. Mazzitelli, Phys. Rev. D58 (1998) 024009; F. C. Lombardo, F. D. Mazzitelli and J. G. Russo, Phys. Rev. D59 (1999) 064007.
[9] Semiclassical gravity entails the backreaction problem studied in [7] and [8].
[10] E. Calzetta and B.-L. Hu, *Phys. Rev.* D **49**, 6636 (1994). B. L. Hu and A. Matacz, Phys. Rev. D **51**, 1577 (1995); B. L. Hu and S. Sinha, Phys. Rev. D **51**, 1587 (1995); A. Campos and E. Verdaguer, Phys. Rev. D **53**, 1927 (1996); F. C. Lombardo and F. D. Mazzitelli, Phys. Rev. D **55**, 3889 (1997). R. Martin and E. Verdaguer, Phys. Rev. D **60**, 084008 (1999); D **61**, 084008 (1999)
[11] B. L. Hu and E. Verdaguer,Class. Quant. Grav. 20 (2003) R1-R42 [gr-qc/0211090]
[12] B. L. Hu and E. Verdaguer, Living Review Relativity, 11 (2008) 3 [arXiv:0802.0658]
[13] C. Moller, Les Theories Relativistes de la Gravitation Colloques Internationaux CNRX 91 edited by A Lichnerowicz and M.-A. Tonnelat (Paris: CNRS) (1962). L. Rosenfeld, Nucl. Phys. 40, 353 (1963).
[14] D. N. Page and C. D. Geilker, "Indirect Evidence for Quantum Gravity", Phys. Rev. Lett., 47, 979-982 (1981).
[15] K. Eppley and E. Hannah, "On the Logical Necessity of Quantizing the Gravitational Field", Found. Phys. 7, 51-68 (1977).
[16] C. J. Isham, Roger Penrose, Dennis William Sciama, eds Quantum Gravity 2 (Clarendon Press, 1981)
[17] E. E. Flanagan and R. M. Wald, Phys. Rev. D 54, 6233 (1996)
[18] B. L. Hu, Albert Roura and E. Verdaguer, "Induced Metric Fluctuations and Validity of Semiclassical Gravity", Physical Review D 70 (2004) 044002
[19] F. Karolyhazy, Nuovo Cimento A 42 (1966) 390.
[20] G. C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D34, 470 (1986). P. Pearle, Phys. Rev. D13, 857 (1976); 29, 235 (1984); 33, 2240 (1986). G. C. Ghirardi, P. Pearle and A. Rimini, Phys. Rev. A42, 78 (1990); G. C. Ghirardi and A. BassiRMP, Phys. Rep. 379, 257 (2003); R. Tumulka, Rev. Math. Phys. 21, 155 (2009).
[21] L. Diósi, Phys. Lett. A105, 199 (1984); Phys. Lett. 120, 377 (1987); Phys. Rev. A40, 1165 (1989).
[22] R. Penrose, in *Quantum Concepts in Space and Time*, R. Penrose and C. J. Isham editors, (Oxford, 1986, Clarendon Press); Gen. Rel. Grav. 28, 581 (1996); Phil. Trans. Roy. Soc. (London) A, 356, 1927 (1998).
[23] S. L. Adler, *Quantum Theory as an Emergent Phenomenon* (Cambridge University Press, 2004).

[24] Angelo Bassi, K. Lochan, S. Satin, T. P. Singh and H. Ulbricht, "Models of wave-function collapse, underlying theories, and experimental tests", Rev. Mod. Phys. 85, 471- 527 (2013)
[25] W. H. Zurek, Phys. Rev. D24, 1516 (1981); D26, 1862 (1982); in *Frontiers of Nonequilibrium Statistical Physics*, ed. G. T. Moore and M. O. Scully (Plenum, N. Y., 1986); Physics Today 44, 36 (1991); W. H. Zurek, Prog. Theor. Phys. 89, 281 (1993); E. Joos and H. D. Zeh, Z. Phys. B59, 223 (1985); A. O. Caldeira and A. J. Leggett, Phys. Rev. A31, 1059 (1985); W. G. Unruh and W. H. Zurek, Phys. Rev. D40, 1071 (1989); B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. D 45, 2843 (1992); J. P. Paz, S. Habib and W. H. Zurek, Phys. Rev. D 47, 488 (1993).
[26] A. J. Leggett, J. Phys. C: Solid State Phys. 14 R415 (2002); Phys. Scr. 102, 69 (2002).
[27] Yanbei Chen, "Macroscopic quantum mechanics: theory and experimental concepts of optomechanics" J. Phys. B: At. Mol. Opt. Phys. 46 (2013) 104001
[28] C. Anastopoulos and B. L. Hu, "A Master Equation for Gravitational Decoherence: Probing the Textures of Spacetime" Class. Quant. Grav. 30, 165007 (2013)
[29] C. Anastopoulos and B. L. Hu, "Decoherence in Quantum Gravity: Issues and Critiques" J. Phys. Conf. Ser. 67, 012012 (2007)[gr-qc/0703137]
[30] C. Anastopoulos and B. L. Hu, "Intrinsic and Fundamental Decoherence: Issues and Problems" Class. Quant. Grav. (special issue) 25, 154003 (2008) [arXiv:0803.3447
[31] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw and A. Zeilinger,' *Nature* **401**, 680 (1999). A. D. Armour, M. P. Blencowe and K. C. Schwab, Phys. Rev. lett. **88**, 148301 (2002). W. Marshall, C. Simon, R. Penrose and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003). M. P. Blencowe, "Nanomechanical Resonators" Phys. Rep. 395, 159-222 (2004). O. Romero-Isart, A. C. Pflanzer, F. Blaser, R. Kaltenbaek, N. Kiesel, M. Aspelmeyer and J. I. Cirac, Phys. Rev. Lett. 107, 020405 (2011). J. D. Teufel, T. Donner, D. Li, J.W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K.W. Lehnert and R.W. Simmonds, Nature (London) 475, 359 (2011). J. Chan, T. P. Mayer Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Groblacher, M. Aspelmeyer and O. Painter, Nature (London) 478, 89 (2011). M. Aspelmeyer, P. Meystre and K. Schwab, Phys. Today 65, 29 (2012).
[32] Huan Yang, Haixing Miao, Da-Shin Lee, Bassam Helou and Yanbei Chen, "Macroscopic Quantum Mechanics in a Classical Spacetime" Phys. Rev. Lett. 110, 170401 (2013)
[33] E. Calzetta and B. L. Hu, Phys. Rev. A68, 043625 (2003)
[34] C. Barcelo, S. Liberati and M. Visser, Analogue gravity, Living Rev. Relativity 14, 3 (2011). G. E. Volovik, The Universe in a Helium Droplet, (Oxford University Press, Oxford, U.K., 2003).
[35] S. L. Adler, Class. Quan. Grav. 30, 195015 (2013)
[36] Freeman Dyson, "Is a Graviton Detectable?" Poincare Prize Lecture presented at the International Congress of Mathematical Physics, Aalborg, Denmark, August 6, 2012.
[37] O. Romero-Isart, Phys. Rev. A 84, 052121 (2011).
[38] M. P. Blencowe, Phys. Rev. Lett. 111, 021302 (2013)
[39] C. Anastopoulos and B. L. Hu, "Newton-Schrödinger Equation and Semiclassical Gravity"
[40] Carlip, S., 2008, Classical Quantum Gravity 25, 154010.
[41] Salzman, P. and S. Carlip, 2006, arXiv:gr-qc/0606120.
[42] Giulini, D. and A. Grossardt, 2011, Classical Quantum Gravity 28, 195026.
[43] Ruffini, R. and S. Bonazzola, 1969, Phys. Rev. 187, 1767.
[44] F. S. Guzman and L. A. Urẽna-Lopez, Phys. Rev. D 68, 024023 (2003); 69, 124033 (2004).
[45] I. M. Moroz, R. Penrose and P. Tod, Classical Quantum Gravity 15, 2733 (1999). R. Harrison, I. Moroz and K. P. Tod, Nonlinearity 16, 101 (2002).
[46] R. Martin and E. Verdaguer, Phys. Rev. D **60**, 084008 (1999); Phys. Rev. D **61**, 124024 (2000).
[47] C. Anastopoulos and B. L. Hu, Comment on [32]
[48] S. L. Adler, J. Phys. A 40, 2935 (2007)
[49] J. B. Hartle and G. T. Horowitz, Phys. Rev. D **24**, 257 (1981).
[50] A. Roura and E. Verdaguer, unpublished.
[51] R. Alicki and J. Messer, J. Stat. Phys. 32, 299 (1983).
[52] H. Spohn, Rev.Mod.Phys. 53, 569 (1980);
C. Bardos, F. Golse, N. Mauser, Methods Appl.Anal. 2, 275 (2000)
[53] C. Bardos, L. Erds, F. Golse, N. Mauser and H-T Yau, C.R. Acad. Sci. Paris 334, 515 (2002).
[54] See, e.g., A. O. Caldeira and A. J. Leggett, Physica A 121, 587 (1983)
[55] B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. D **45**, 2843 (1992); **D 47**, 1576 (1993).
[56] C. H. Chou, B. L. Hu and Y. Subasi, "Macroscopic Quantum Phenomena from the Large N Perspective" in the 5th DICE meeting September 2010, Castello Pasquini/ Castiglioncello (Tuscany), Italy, J. Phys. Conf. Ser. 306 012002 (2011) [arXiv:1106.0556]
[57] C. H. Chou, B. L. Hu and Y. Subasi, " Macroscopic Quantum Phenomena from the Correlation, Coupling

and Criticality Perspectives" in the 2010 IARD Conference, May 2010 Hualien, Taiwan, J. Phys. Conf. Ser. 330, 012003 (2011) [arXiv:1107.3008]
[58] B. L. Hu and Y. Subasi, "Pathways toward understanding Macroscopic Quantum Phenomena" DICE 2012, J. Phys. Conf. Series 442, 012010 (2013) [arXiv:1304.7839]
[59] C. H. Chou, Y. Subasi and B. L. Hu, "Macroscopic Quantum Phenomena from the Coupling Pattern and Entanglement Structure Perspective" submitted to Physica A [arXiv:1308.4225]
[60] C. H. Chou, B. L. Hu and Ting Yu,"Quantum Brownian Motion of a Macroscopic Object in a General Environment", Physica A387 (2008) 432- 444 [arXiv:0708.0882]
[61] G. Papini and S.R. Valluri, Physics Reports 33C, 51 (1977)
[62] E. Calzetta, B. L. Hu, "Thermalization of an Interacting Quantum Field in the CTP-2PI Next-to-leading-order Large N Scheme" (unpublished) [hep-ph/0205271]